Spectral Characteristics and Oscillation Dynamics in Neodymium Aluminum-Yttrium Garnet Quasicontinuous Lasers

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The frequency and angular spectra and the oscillation dynamics are investigated in neodymium yttrium-aluminum garnet (YAG) quasicontinuous lasers operating at frequencies up to 50 Hz. Quasicontinuous operations conditions (pulse duration $\geq 10^{-4}$ sec, noise amplitude modulation coefficient less than 10%) have been attained in two types of lasers—in a multimode laser with a quasiconcentric spherical cavity (up to 10^4 transverse modes could simultaneously be excited) and a single-mode laser with a plane-spherical cavity with selectors. A pulse power output up to 3 kW has been attained in the first type; the spectral line width is less than 0.02 cm^{-1} and frequency instability is of the same order of magnitude. In the single-mode laser the output power of a beam with diffraction divergence reached 500 W; the line width was less than 0.01 cm^{-1}. Mechanisms of spike excitation in YAG lasers are investigated. The possibility of strongly exceeding the threshold (by as much as 100 times in our generator) in YAG lasers permits one to obtain new information regarding the dynamics of quasicontinuous lasers. In particular it was possible to obtain spikeless generation even in a multimode (with respect to longitudinal modes) laser providing the pumping level was high. The possibilities of observing nonlinear optical phenomena quasicontinuous lasers are discussed. Thus by using a laser with a quasiconcentric cavity one may investigate the effect of spatial coherence of radiation on nonlinear optical processes. The power developed in a single mode generator is sufficient for observing quasicontinuous, stimulated scattering and for pumping quasicontinuous, single-resonator parametric light generators which may be of great interest in ultra-high resolution laser spectroscopy

1. INTRODUCTION

A new trend in nonlinear optics is observation of threshold effects in the field of continuous and quasi-continuous lasers. The first success in this field was the attainment of parametric light generation with an argon-laser pump[1], observation of quasicontinuous stimulated Raman scattering (SRS) in carbon disulfide[2], thermal self-focusing of radiation of an argon laser in glass[3], and observation of SRS in semiconductors pumped with a CO laser[4].

The foregoing experiments were performed with pump sources of 1–10 W power; if the threshold of the effect is determined by the power density, then the use of focusing makes it possible to obtain, with the indicated sources, power densities $\sim 5 \times 10^5 - 10^6$ W/cm^2. However, more powerful pump sources (1–100 kW) are needed to observe a number of nonlinear effects. These effects include the following:

a) quasi-continuous parametric generation of light in the single-resonator scheme, which is most promising for applications;

b) investigation of stimulated Mandel'shtam-Brillouin scattering (SMBS) at low temperatures in the regime of generation of intense hypersonic waves[1)];

c) investigation of the process of establishment of inverse SMBS in the presence of the sequence of giant pulses of scattered light[2)].

[1)]Estimates made in[5] for quartz have shown that it is possible to observe hypersound generation at pump power densities ~1–10 MW/cm^2 and light-pulse durations $\tau_p > \tau_{ac}$ ($\tau_{ac} \sim 10^{-5}$ sec is the time required for the sound to travel through a sample 1 cm long); we recall that in experiments on SMBS from giant pulses ($\tau_p \sim 10^{-8}$ sec) the hypersound intensity is quite low.

[2)]According to [6], at $\tau_p > (10^{-4} - 10^{-5})$ sec, repeated generation of inverse spikes of SMBS goes over in practice into the regime of stationary generation; this effect pertains to a considerable degree also to other types of scattering.

In addition to the foregoing examples, it is possible to indicate a larger number of applications of quasi-continuous lasers having a narrow spectral line, for high and ultra-high resolution spectroscopy (particularly for spectroscopy of direct and inverse stimulated scattering). The use of such lasers, while not leading to new physical results, greatly facilitates the performance of the experiment. Many problems of nonlinear optics could be solved by developing a quasi-continuous ($\tau_p \sim 10^{-3} - 10^{-5}$ sec) repeating (10–50 Hz) laser with a pulse power ~1–100 kW[3)]. To investigate stimulated scattering it is necessary to have in this case a generation line width on the order of 0.01 cm^{-1}. An important factor here is uniformity of the distribution of the intensity in the beam cross section in the near and far fields.

The indicated energy and space-time characteristics can be obtained with the aid of solid-state lasers operating in the free-running mode. The results of experiments performed to date (see, e.g.,[7,11]) show that a spikeless free-running regime is easiest to realize in a single-mode laser and in a laser with a large number of frequency-degenerate transverse modes. In the single-mode (with respect to all three indices) regime, the lasing process has the character of damped pulsations; the duration of the transient process can be reduced in this case by increasing the pump power[8]. Usually the single-mode generation regime can be easily obtained at near-threshold pumping; the requirements with respect to the quality of the selection become more stringent if the threshold is appreciably exceeded.

Another interesting variant of a quasi-continuous laser is a laser with a spherical resonator, in which a large number of frequency-degenerate transverse mode is excited[10,11]. As noted in[12], the properties of a laser with a quasi-concentric resonator approach the

[3)]An interesting application of such generators would be observation of Kerr self-focusing of long pulses.

properties of a laser with non-resonant feedback; as a result one should expect here excitation of a rather narrow-band generation of stable frequency. We report below the results of an experimental investigation of lasers of both types; the use of garnet with neodymium in these lasers makes it possible to study their characteristics in a wide range of excesses of pump power over threshold (we recall in the previously published investigations ruby and neodymium glass were used in such lasers). In both cases, the purpose of the study was to develop a quasi-continuous laser suitable for the observation of nonlinear optical effects; a unique requirement imposed on the choice of the resonator systems and operating regimes was that it be possible to obtain a pulse energy ~1 J[4)]. It is shown below that these requirements can be satisfied at perfectly acceptable spectrum widths and amplitude-fluctuation levels; thus, repeating quasi-continuous lasers using garnets are promising sources for nonlinear optics.

2. LASER WITH QUASICONCENTRIC SPHERICAL RESONATOR

1. Conditions for Obtaining Quasicontinuous Generation

Let us consider a homogeneous dielectric active element of diameter 2a and length l with bleached ends, placed in a spherical resonator with mirrors of equal curvature radius R. The total number $N_\perp$ of the transverse modes in the interval between neighboring longitudinal modes $\Delta\nu_q$ obviously coincides with the total number of transverse modes spanned by the aperture of the resonator. Using the results of[13,14], we have

$$N_\perp = {}^1/_{12}(t_0^2+3t_0-2\sqrt{2t_0})+{}^1/_2, \qquad (1)$$

where

$$t_0=\frac{2a^2k\sqrt{L^*(2R-L^*)}}{L^*(2R-L^*)+4(y^*)^2}\,,\quad L^*=\mathscr{L}-l(1-1/n),$$

$\mathscr{L}$ is the resonator length; n is the refractive index of the active medium; $y^* = yn^{-1}$ if the center of the resonator is located inside the active element and $y^* = y - l(1 - 1/n)$ if it is located outside the resonator; y is the distance from the diaphragming end of the element to the center of the resonator, and $k = 2\pi n\lambda^{-1}$.

At $t_0 \gg 1$ (excitation of high-order transverse modes) we obtain $N_\perp \approx t_0^2/12$ (see[15]). The maximum value of $N_\perp$ is reached at $L^*_{1,2} = R \pm [R^2 - 4(y^*)^2]^{1/2}$, and the minimum one at $L^* = R$, i.e., in a confocal resonator. The value $L^* \approx 2R$ ($n \approx 1$, $L^* \approx \mathscr{L}$) corresponds to an almost-concentric resonator. With decreasing parameter y^*, the maximum value of the transverse-mode density increases (the minimum value $y^*_{\min} = l/2n \ll R$ corresponds to coincidence of the centers of the active element and the resonator). The foregoing is illustrated by the diagrams of Fig. 1.

In accord with the foregoing, one should expect in an almost-concentric resonator at $y^* \ll R$ a strong condensation of the transverse modes and a smoothing out of the temporal and spatial distributions of the generation, and consequently a sharp decrease in the width of the radiation line[5)].

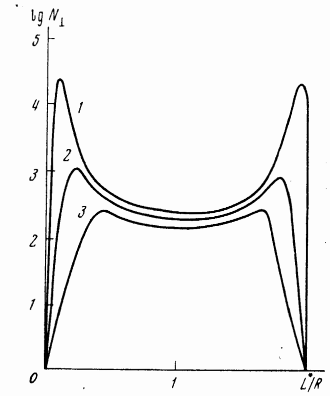

FIG. 1. Dependence of log $N_\perp$ on L^*/R at y^*/R equal to 0.04 (curve 1), 0.1 (2), and 0.3 (3).

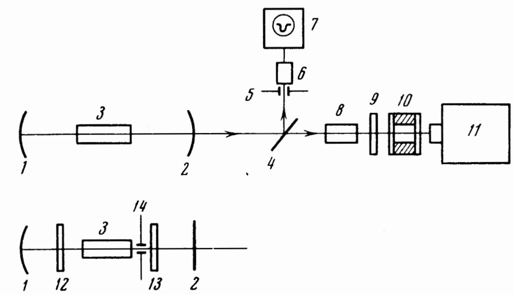

FIG. 2. Diagram of experimental setup (the generator with mode selection is shown separately). The active element 3 is pumped by a lamp IFP-800; 1, 2—total-reflection and output mirrors; 4—glass plate; 5—diaphragm; 6—FEU-28 photomultiplier; 7—S1-8A oscilloscope; 8—$LiNbO_3$ frequency doubler; 9—light filter S3S-21; 10—Fabry-Perot etalon; 11—high-speed camera SFR-2M; 12, 13—anisotropic felspar plates; 14—diaphragm.

2. Experimental Setup and Measurement Procedure

The experimental setup is shown in Fig. 2. The laser contains an illuminating quartz monoblock with active element of YAG with neodymium and a pump lamp placed in the center of the resonator; the resonator mirrors have reflection coefficients 99.6% ("total-reflection" mirror) and 30% (output mirror). The reflection coefficient of the output mirror is close to optimal; the end faces of the active element are bleached for the generation wavelength (λ = 1064.2 nm). The supply source ensures operation of the pump lamp with a pulse energy 13—800 J and a pulse repetition frequency 0—50 Hz. The upper limit of the pump energy is determined by the allowable average power of the pump lamp, and the lower limit is determined by the pump-lamp ignition voltage, which greatly exceeds the generation threshold voltage.

The temporal characteristics of both the radiation as a whole and of the beam at each point of its cross section were investigated with the aid of a photomultiplier and an oscilloscope. In the latter case, a diaphragm measuring 11 × 40 μ was placed ahead of the photo-

[4)]Such an energy corresponds to a power ~ 1-5 kW; amplification of such a pulse to an energy of 20 J and attainment of a pulse power reaching 100 kW is a relatively simple matter.

[5)]The foregoing is valid if the Q of the resonator is approximately the same at all three modes or if the pump energy greatly exceeds the threshold. In the case of mode discrimination, the conditions for smoothing the generation and for narrowing the spectrum become much worse.

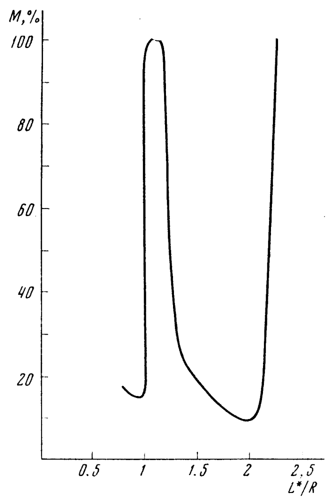

FIG. 3. Dependence of the depth of modulation of the generation pulse on the value of L*/R for a spherical resonator at y*/R = 0.04.

multiplier, and the photomultiplier with the diaphragm scanned the cross section of the beam in steps of $\sim 10\ \mu$.

To investigate the spectral characteristics of the laser radiation, the latter is transformed into the visible region (532 nm) by an $LiNbO_3$ nonlinear crystal and is fed to a Fabry-Perot etalon with base 5 mm. The spectral and temporal characteristics were investigated with the aid of a high speed camera (SFR) operating in the slit scanning mode (slit width 0.4 mm) and with a prime scale on the SFR film of 1.3 μsec/mm.

3. Experimental Results

Plane-parallel resonator. The radiation of an YAG : Nd laser in a plane-parallel resonator ($R_{1,2} = \infty$) at not too large pump values is spiked in character. With increasing pump power, a dc component appears on the oscillograms of the envelope and the depth of its modulation increases with increasing pump, not exceeding 30–40% at a pump energy $W_p = 450$ J (more than 40 times the generation threshold).

Spherical resonator. Figure 3 shows the dependence of the depth of modulation M of the envelope, averaged over the pulse, on the parameter L^*R^{-1} in the case of a symmetrical placement of the active element in the spherical resonator ($y^*R^{-1} \approx 0.04$, $R_{1,2} = 46.3$ cm, $W_p = 200$ J). We see that at $L^*R^{-1} \approx 1.0$–1.2 (almost-confocal resonator) the usual spike regime is observed ($M \approx 100\%$); it was established that this regime corresponds to generation of a small number of transverse modes (Fig. 4a, photograph from the screen of an electron-optical converter). When L^*R^{-1} changes in either direction from this region, the value of M decreases rapidly, the transverse distribution of the intensity becomes more and more uniform (see Figs. 4b and 4c), and the divergence of the radiation increases. The smoothest temporal structure ($M \leq 10\%$) and the most uniform transverse distribution of the intensity (Fig. 4d) occurs in an almost-concentric resonator ($L^*R^{-1} \approx 1.9$–2.0); in this case the divergence reaches $\sim 1°$. The spectrum of the radiation pulsations at all values of L^*R^{-1} corresponds to an average frequency ~ 500 kHz.

The oscillograms of Fig. 5 illustrate the dependence of the duration and of the depth of the pulsations of the transient processes in an almost-concentric resonator on the pump energy. At a pump energy $W_p \approx 100$ J, these quantities reach a minimum, and remain practically unchanged with further increase of W_p.

It should be noted that at all values of L^*R^{-1} the oscillograms of both the radiation as a whole and of each point of the beam, when scanning with a photomultiplier covered by a diaphragm, are qualitatively the same, the exception being the points on the periphery of the beam,

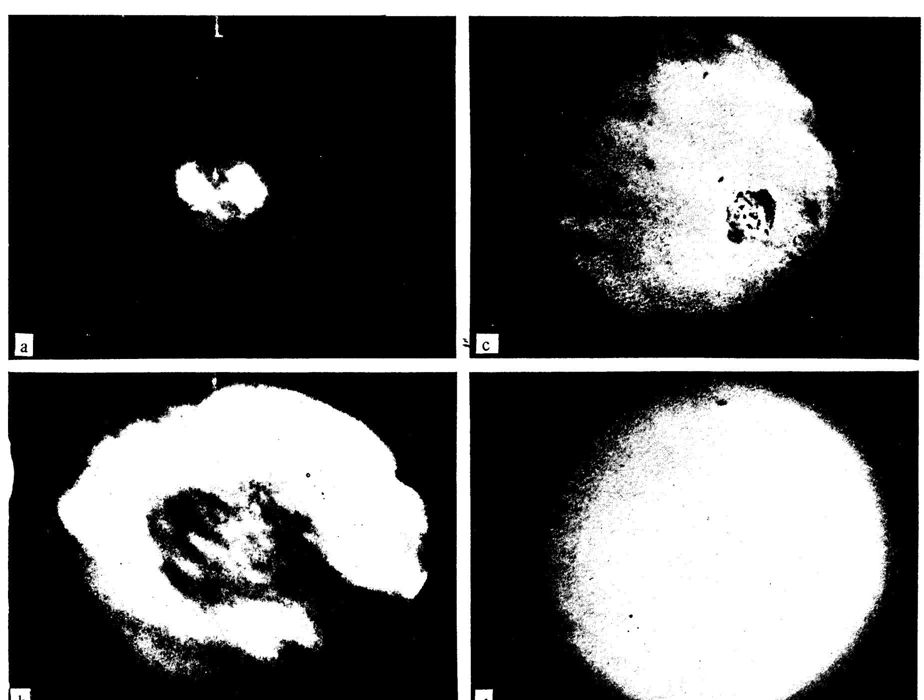

FIG. 4. Transverse distribution of laser radiation intensity for a spherical resonator in the near field at y*/R = 0.04 for values of L*/R of 1.05 (a), 1.20 (b), 0.93 (c), and 1.95 (d).

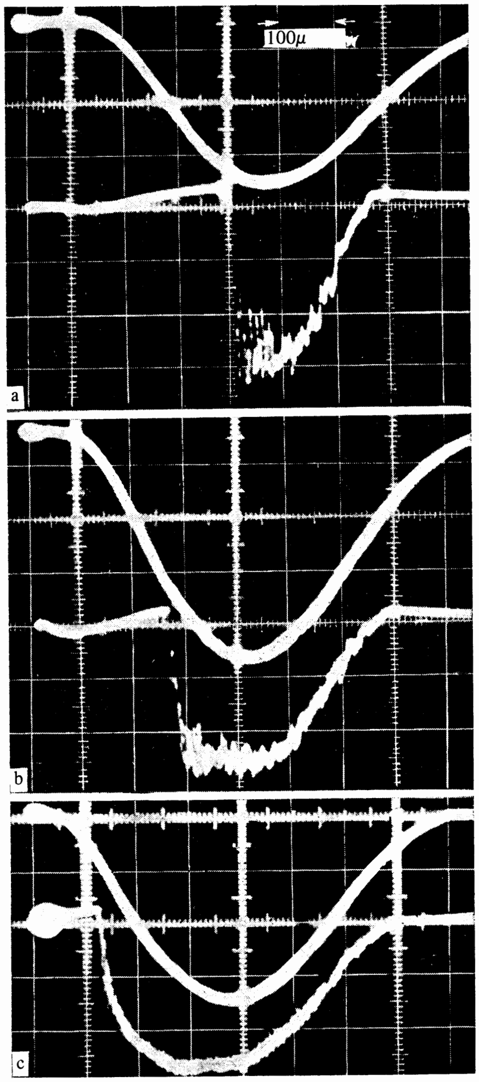

FIG. 5. Shape of generation pulse in a spherical resonator at $y^*/R = 0.04$ and $L^*/R = 1.95$ at different pump energies: a) $W_p = 13$ J, b) $W_p = 50$ J, c) $W_p = 100$ J; in all the figures the upper pulse is the pump pulse.

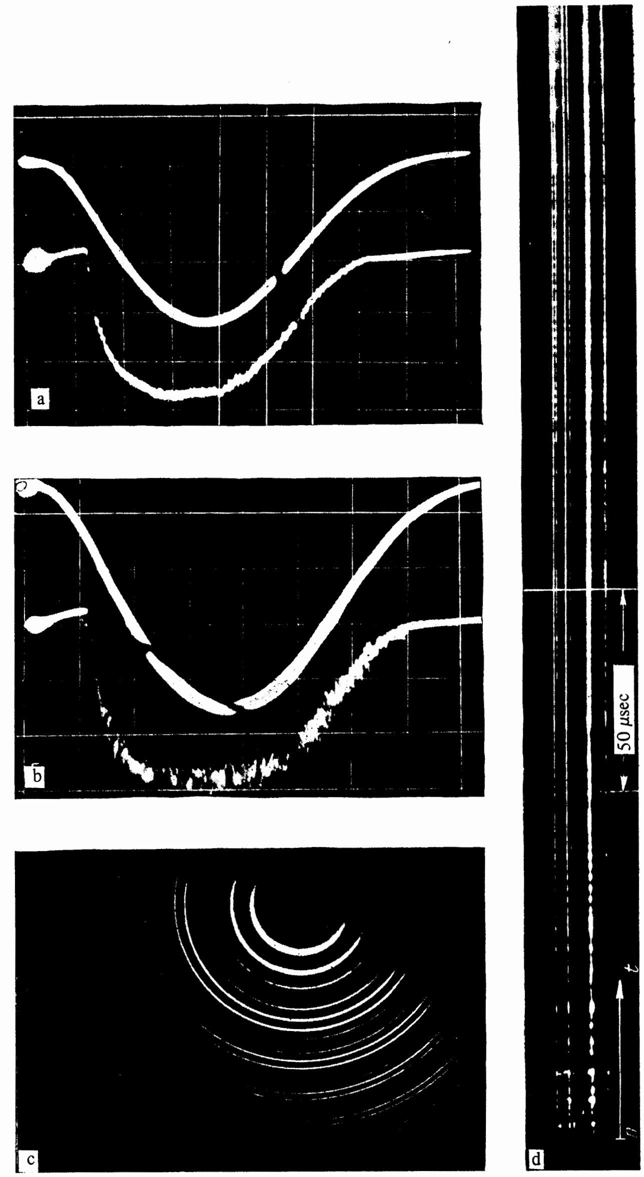

FIG. 6. Characteristics of generation in a hemispherical resonator at a distance 44 cm between mirrors. a—Generation pulse (lower) without a diaphragm ahead of the photomultiplier and pump pulse (upper); b—the same, but with a diaphragm ahead of the photomultiplier; c—generation spectrum integrated over the pulse, Fabry-Perot etalon base 0.5 cm; d—streak photograph of generation spectrum. One division of the grid corresponds to 100 μsec.

where the depth of the pulsations reaches 100% on the oscillograms even at $L^*R^{-1} \approx 1.9$–2.0.

The spectrum of the second harmonic of the fundamental radiation consists at $L^*R^{-1} \approx 1.9$ and $M \approx 10\%$ of one narrow line of width ~0.02 Å, which is quite stable in time (stability not worse than 0.02 cm^{-1}). The time of establishment of such a generation spectrum does not exceed several microseconds in the pump-energy interval $W_p = 13$–450 J. The spectral line has a tendency to become narrower with increasing pump power in the region of low pumping and to remain practically constant at large pump powers.

Semispherical resonator. When the output spherical mirror is replaced by a flat one ($R_2 = \infty$), the dynamics of the generation changes appreciably. The minimum depth of the pulsations (~10%) for the radiation as a whole and 20% in the case of scanning with a diaphragm) is observed at $W_p \geq 100$ J and $L^*R^{-1} \approx 0.9$–1.0 (we recall that the hemispherical resonator is equivalent to a spherical one with a length equal to double the length of the hemispherical resonator). The corresponding oscillograms are shown in Figs. 6a and 6b. The spectrum of the second harmonic consists in this case of several almost equidistant lines with an average distance between them ~0.06 Å and an average half-width ~0.015 Å (Fig. 6c), and does not change in the pump energy interval $W_p = 13$–450 J. The intensity and the spectrum of the radiation (Fig. 6d) at the start of the pulse experience random pulsations that go over into a regime with regular pulsations of the envelope[6)].

Energy characteristics. We had at our disposal active

[6)]We note that a spikeless regime for each mode (see Fig. 6c) was observed in the entire range of pump variation.

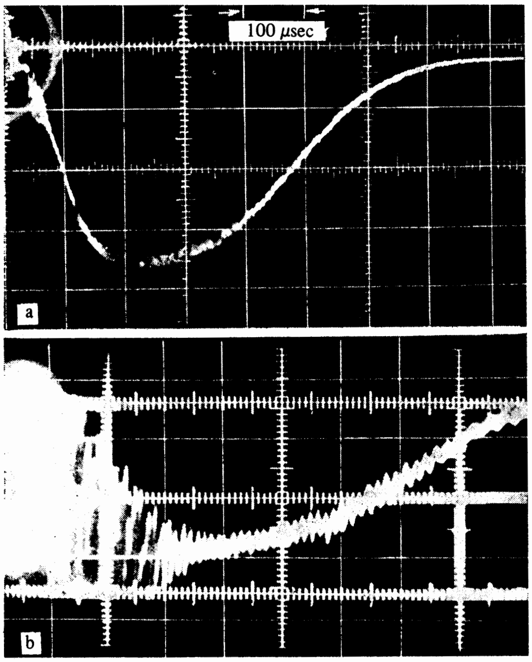

FIG. 7. Oscillograms of envelope of quasicontinuous generator with mode selection at different pump energies: a) W_p = 200 J, b) W_p = 32 J.

elements of YAG crystals with neodymium (concentration ~1 at.%) measuring 4 × 45 mm. The maximum energy in the quasi-continuous regime in the pulse (~1.1 J) was obtained with such elements at a pump energy 450 J (in the entire lamp), corresponding to an average pulse power ~2.5 kW at a duration ~450 μsec. When active elements with dimensions (6–7) × 80 mm were used, the energy in the pulse was 4–5 J, and the power averaged over the pulse was 10 kW.

4. Discussion

The dependence of the depth of the radiation pulsations M on the parameter L^*R^{-1}, measured for a spherical resonator near the value $L^*R \approx 2.0$ (Fig. 3) can be satisfactorily explained by assuming generation of a large number of transverse modes (Fig. 1) and retention of high Q of the resonator. At $L^*R^{-1} = 1.9$, $Y^*R^{-1} = 0.04$, and $R_{1,2} = 46.3$ cm, the interval $\Delta\nu_q = c/2L \approx 1.8 \times 10^8$ Hz between neighboring longitudinal modes subtends over 1.4×10^4 transverse modes. The half-width of an individual mode of the resonator without allowance for the diffraction losses is $\Delta\nu_r \approx 3.2 \times 10^7$ Hz. Thus, the transverse modes in this case overlap significantly, the mode spectrum becomes practically continuous, and the properties of a laser with such a resonator turn out to be similar to the properties of a laser with nonresonant feedback[12] 7).

In an almost-confocal resonator ($L^*R^{-1} \approx 1$) the number of transverse modes, while decreasing significantly in comparison with the concentric resonator (Fig. 1), still remains quite high ($N_\perp \approx 2.5 \times 10^3$); this holds true for an ideal resonator with mirrors of equal curvature. In practical resonator schemes, the mirrors have different curvatures, and the active element introduces considerable distortions into the resonator. This leads to a sharp decrease of the resonator Q, and the generation occurs at several transverse modes of lower order (Fig. 4a); accordingly, the depth of pulsations increases sharply.

A hemispherical resonator, made from a spherical one by replacing the output spherical mirror by a flat one ($R_1 = R$; $R_2 = \infty$), is characterized by parameters $Y^* = 22$ cm, $y^*R^{-1} \approx 0.48$. Accordingly, the aperture of the resonator and the density of the transverse modes turn out in this case to be much smaller than in a spherical resonator. In this regime, longitudinal modes that can be distinctly separated are excited in the laser (see Figs. 6c and 6d). It should be emphasized, however, that the form of the envelope in this multimode regime, with a relatively small number of modes, depends significantly on the pump level. In particular, in the case of large excesses above threshold (Figs. 6a and 6b), the generation regime is practically spikeless.

This circumstance (see also[17]) is an argument favoring the point of view advanced in[18,19] concerning the mechanism of spiked generation ("statistical" model of spiked generation). Indeed, an increase of the pump power increases the "strength" of the limit cycle: a decrease of the relaxation time of the amplitude makes the laser more stable with respect to different perturbations of the resonator and of the pump. We note, incidentally, the interesting fact that the generation spectrum is independent of the pump power when the latter ranges from 13–450 J. We are dealing here apparently with saturation of the spectrum, a phenomenon already discussed in[20].

3. SINGLE-MODE QUASICONTINUOUS GARNET LASER

The diagram of a laser of this type is shown in Fig. 2. Selectors for transverse and longitudinal modes were introduced into the semispherical resonator with the YAG active element already described in Sec. 2. The transverse-mode selector was a diaphragm with variable diameter. In typical regimes, the diaphragm diameter d was 2 mm. The longitudinal-mode selector was either a Fabry-Perot etalon, installed in place of the output mirror, or a selector made up of isotropic plates (see, e.g.,[21]). In the latter case, the role of the polarizer could be played by the active element itself; the anisotropy was produced here as a result of the pump anisotropy. The choice of the selector parameter was based on standard rules. The best characteristics were obtained in a laser in which the frequency selection was effected with the aid of anisotropic plates. Of greatest importance was the circumstance that the narrow-band spikeless generation of such a generator can be obtained at high pump levels. Typical figures characterizing this generator are: pulse duration $\tau_p \approx 4 \times 10^{-4}$ sec, pulse energy W = 0.2 J, width of spectral line ~0.005 cm^{-1}, and divergence $\Delta\theta \approx 4'$ at a beam diameter 2a = 2 mm.

Thus, the described quasicontinuous generator (the operating characteristics remained the same up to a repetition frequency ~50 Hz) delivers in the unfocused beam a power up to 10^4 W/cm^2. The use of focusing

7)We note that actually, owing to mode competition (see[16]), the number of generation modes can decrease, although it remains sufficiently large.

makes it possible to increase this figure without difficulty to 10^7 W/cm^2; when an amplifier is used, a power density $\sim 10^8$ W/cm^2 is reached. Oscillograms of the envelope are shown in Fig. 7. One can see clearly the oscillatory transient regime followed by the quasicontinuous pulse. The time of establishment of the quasicontinuous regime decreased from $\tau_{est} \sim 2 \times 10^{-4}$ sec at a pump energy W_p = 13 J to $\tau_{est} = 10^{-6}$ sec at W_p = 200 J.

It should be noted that the level of the "residual" fluctuations of the generator amplitude, which remained also at $t > \tau_{est}$, depends, as shown by experiments, on the diameter of the diaphragm which separates the transverse modes. A decrease of the diaphragm diameter decreases the level of the amplitude fluctuations. This result also agrees with the ideas developed in[18,19].

4. CONCLUSION

To observe and investigate a number of nonlinear phenomena, it is necessary to have laser radiation sources operating in a quasicontinuous regime ($\tau_p \approx 10^{-5}-10^{-3}$ sec) and with a high average pulse power (1–10 kW). We have demonstrated in this paper the feasibility of producing such a source on the basis of aluminum-yttrium garnets with neodymium. In an almost-continuous resonator, we have obtained quasicontinuous radiation (intensity fluctuations not more than 10%) with pulse duration $(4-5) \times 10^{-4}$ sec, average power in the pulse P_p = 2–3 kW, and repetition frequency up to 50 Hz; the width of the spectral line does not exceed $\delta\nu = 0.02$ cm^{-1}; the instability of the generation wavelength is of the same order. The number of generated transverse modes in the spherical resonator is 10^3-10^4, and if the Q's of all the modes are approximately equal (or if the generation threshold is greatly exceeded) the resonator mode spectrum becomes practically continuous, leading to a smoothing of the temporal and spatial transverse distributions of the intensity. A shortcoming of such a generator is the small radius of spatial coherence r_0 of its radiation (in our generator $r_0 \approx 60\ \mu$; the divergence is $\Delta\theta \sim 1^\circ$), so that it can be effectively utilized for observation of incoherent nonlinear processes, for example stimulated Raman scattering. At the same time, the developed generator makes it possible to carry out a detailed quantitative investigation of the influence of spatial coherence of the interacting beams on nonlinear optical phenomena such as harmonic generation and parametric amplification; this problem has been discussed heretofore only theoretically (see, e.g.,[22]). It should be noted at the same time that, as shown in[23], the width of the angular pump spectrum that becomes involved in the parametric-amplification process is determined not only by the dispersion properties of the nonlinear crystal, but also by the pump power. Estimates show that critical pump powers at which a total pump contained in an angle $\sim 1^\circ$ operates effectively are not very large. Thus, a parametric light generator can be used as a device increasing the brightness of the developed quasicontinuous source. Single-mode YAG lasers described in Sec. 3 of the present paper make it possible to solve the large number of problems listed in the introduction. What is remarkable is the experimentally demonstrated possibility of obtained good energy characteristic with a narrow spectral line and with diffraction divergence of the beam. It is important that in YAG lasers at large values of the pump, as shown by experiment, the generation regime turns out to be spikeless practically independently of the mode structure of the resonator. The latter agrees with the conclusion of[18,19] that one of the main causes of spiked generation is instability of the resonator and pump parameters.

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[1]R. L. Byer, M. K. Oshman, J. F. Young, and S. E. Harris, Appl. Phys. Lett. **13**, 109 (1968).

[2]S. Jppen, Appl. Phys. Lett. **15**, 380 (1969)

[3]S. A. Akhmanov, Yu. G. Gorokhov, D. P. Krindach, A. P. Sukhorukov, and R. V. Khokhlov, Zh. Eksp. Teor. Fiz. **57**, 16 (1969) [Sov. Phys. JETP **30**, 9 (1970)].

[4]A. Mooradian, S. Brucek, and F. Blum, Appl. Phys. Lett. **17**, 481 (1970).

[5]S. A. Akhmanov, Yu. D. Golyaev, V. G. Dmitriev, and Yu. E. D'yakov, V Vsesoyuznaya konferentsiya po nelineĭnoĭ optike (Fifth All-union Conference on Nonlinear Optics), Kishinev, 1970, Abstracts publ. by MGU, 1970.

[6]S. A. Akhmanov, M. A. Bol'shov, Yu. E. D'yakov, and S. A. Zhilkin, ibid.

[7]C. L. Tang, H. Statz, and G. De Mars, J. Appl. Phys. **3**, 2289 (1963).

[8]D. Roess, Proc. IEEE **52**, 206 (1964).

[9]A. Z. Grasyuk, I. G. Zubarev, and V. F. Mulikov, Preprint FIAN, No. 91, 1970.

[10]Yu. V. Naboĭkin, A. M. Ratner, A. I. Rom-Krichevskaya, and Yu. A. Tiunov, in Kvantovaya elektronika (Quantum Electronics) No. 2, Naukova Dumka, 1967, p. 146.

[11]A. M. Ratner, Kvantovaya generatory sveta s bol'shim uglovym raskhozhdeniem (Lasers with Large Angular Divergences), Kiev, 1970.

[12]R. Ambartsumyan, N. G. Basov, S. P. Bazhulin, and V. Letokhov, Zh. Eksp. Teor. Fiz. **58**, 441 (1969) [Sov. Phys. JETP **31**, 234 (1969)].

[13]L. A. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **45**, 684 (1963) [Sov. Phys. JETP **18**, 471 (1964)].

[14]H. Kogelnik and T. Li, Appl. Opt. **5**, 10 (1966).

[15]I. M. Kopzhenevich and A. M. Ratner, Ukr. Fiz. Zh. (Russ. Ed.) **15**, 2 (1970).

[16]A. A. Mak, L. N. Soms, A. I. Stepanov, and V. A. Fromzel', Opt. Spektrosk. **26**, 793 (1969).

[17]M. I. Dzhibladze, E. M. Zolotov, T. M. Murina, and A. M. Prokhorov, Kratkie soobshcheniya po fizike (Brief Reports on Physics), **3**, 41 (1970).

[18]B. L. Lifshitz, Dokl. Akad. Nauk SSSR **194**, 1298 (1970) [Sov. Phys. Dokl. 15, 949 (1971)].

[19]G. N. Vinokurov, I. M. Galaktionova, V. R. Egorova, A. A. Mak, B. M. Sedov, and Ya. I. Khanin, Zh. Eksp. Teor. Fiz. **60**, 489 (1971) [Sov. Phys. JETP 33, 362 (1971)].

[20]B. L. Lifshitz and V. N. Tsikunov, Zh. Eksp. Teor. Fiz. **49**, 1843 (1965) [Sov. Phys. JETP 22, 1260 (1966)].

[21]Yu. I. Belyaev, A. M. Kiselev, and A. M. Novikov, Radiofizika 13, 1405 (1970).

[22]S. A. Akhmanov and A. S. Chirkin, Radiofizika 13, 787 (1970).

[23]S. A. Akhmanov, Yu. E. D'yakov, and A. S. Chirkin, Zh. Eksp. Teor. Fiz., Pis'ma Red. 13, 724 (1971) [JETP Lett. 13, 514 (1971)].

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