Anomalous Hall Effect for Polarized Electrons in Semiconductors

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The anomalous Hall effect for polarized electrons in semiconductors is predicted and calculated in the Kane model. The effect consists in the appearance of an e.m.f. along the y axis on illumination with polarized light and in the z direction of a sample located in an electric field directed along z. The effect is due to asymmetric scattering of polarized electrons by crystal lattice defects resulting from their spin-orbit interaction with a periodic field. The spin-orbit scattering constant is obtained for the case of scattering by charged impurity centers. It is found to be m^2c^2/m^*E_g times greater than the free electron constant (m is the free electron mass, m^* the effective electron mass in the crystal, E_g the forbidden band width, and c the velocity of light). An expression is also found for the spin relaxation time s.

A stationary polarization of the conduction electrons is established in semiconductors under the influence of polarized laser radiation.[1-4] The degree of conduction-electron polarization at high radiation intensity is determined by the ratio of the electron spin-relaxation time τ_s to the non-equilibrium electron lifetime τ_n and can reach 50%.[5] The polarization of the conduction electrons leads to a number of singularities in the kinetic equation.[6] In particular, the asymmetrical polarized-electron scattering due to the presence of strong spin-orbit interaction with the initial field of the lattice should lead to an anomalous Hall effect.[1)] An exact quantitative analysis of this phenomenon was carried out within the framework of the Kane model, and an expression was obtained for the anomalous Hall conductivity σ_s in terms of the known semiconductor parameters, viz., the effective mass m*, the width of the forbidden band E_g, and the spin-orbit splitting constant Δ of the hole band. In the present paper we consider the case when the conduction electrons are scattered by charged impurities. The phonon scattering mechanism will be investigated in a subsequent paper.

1. WAVE FUNCTIONS OF ELECTRONIC STATES OF THE CONDUCTION BAND, PROBABILITY OF ELECTRON SCATTERING BY A CHARGED IMPURITY CENTER

The Hamiltonian for the conduction electron is

$$\hat{H}=\frac{\hat{\mathbf{p}}^2}{2m}+v(\mathbf{r})+\frac{\hbar}{4m^2c^2}\sigma[\nabla v(\mathbf{r}),\hat{\mathbf{p}}]+\sum_j\Phi(\mathbf{r}-\mathbf{r}_j) \qquad (1)$$

$$\equiv\hat{H}_0+\sum_j\Phi(\mathbf{r}-\mathbf{r}_j),\quad \Phi(\mathbf{r}-\mathbf{r}_j)=-\frac{e^2z}{\varkappa|\mathbf{r}-\mathbf{r}_j|},$$

where m is the free-electron mass, $v(\mathbf{r})$ describes the interaction of the electron with the periodic field of the crystal lattice, $\Phi(\mathbf{r}-\mathbf{r}_j)$ describes the interaction of the electron with the impurity center located at the point $\mathbf{r}_j$, ez is the charge of the impurity center, κ is the dielectric constant of the crystal, and σ is the Pauli matrix vector. The interaction with the impurities, which leads to elastic scattering of electrons, will be regarded as a perturbation. The spin-orbit interaction with the periodic lattice field, which in the case of a semiconductor is not small, will be taken into account exactly within the framework of the Kane model.[7]

We seek the eigenfunctions of the Hamiltonian $\hat{H}_0$ in the form $\psi_{nk}(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}u_{nk}(\mathbf{r})$, where $u_{nk}(\mathbf{r})$ is a periodic function. In the case of semiconductors of the III-V type, taking into account only the interaction of the conduction band with the valence band, we obtain for the Bloch amplitudes u_{nk} of the electronic states of the conduction band the following expressions (the direction of the quantization axis is chosen to be the direction of the incident polarized light):

a) for materials where $\Delta\gg E_g$ (such as InSb),

$$u_{k\uparrow}=c_1\left\{|is\uparrow\rangle+\frac{kP}{3E_g}\left|\left[\left(2\frac{k_x}{k}-i\frac{k_y}{k}\right)x+\left(2\frac{k_y}{k}+i\frac{k_x}{k}\right)y+2\frac{k_z}{k}z\right]\uparrow\right\rangle-\frac{kP}{3E_g}\left|\left[\frac{k_z}{k}(x+iy)-\left(\frac{k_x}{k}+i\frac{k_y}{k}\right)z\right]\downarrow\right\rangle\right\},$$

$$u_{k\downarrow}=c_2\left\{|is\downarrow\rangle+\frac{kP}{3E_g}\left|\left[\left(2\frac{k_x}{k}+i\frac{k_y}{k}\right)x+\left(2\frac{k_y}{k}-i\frac{k_x}{k}\right)y+2\frac{k_z}{k}z\right]\downarrow\right\rangle-\frac{kP}{3E_g}\left|\left[-\frac{k_z}{k}(x-iy)+\left(\frac{k_x}{k}-i\frac{k_y}{k}\right)z\right]\uparrow\right\rangle\right\}; \qquad (2)$$

b) for materials where $\Delta\ll E_g$ (such as GaAs),

$$u_{k\uparrow}=c_3\left\{|is\uparrow\rangle+\frac{kP}{E_g}\left|\left[\frac{k_x}{k}x+\frac{k_y}{k}y+\frac{k_z}{k}z\right]\uparrow\right\rangle+\frac{kP}{3E_g}\left(\frac{\Delta}{E_g}+\frac{\hbar^2k^2}{2mE_g}\right)\left|\left[-\frac{k_z}{k}(x+iy)+\left(\frac{k_x}{k}+i\frac{k_y}{k}\right)z\right]\downarrow\right\rangle\right\},$$

$$u_{k\downarrow}=c_4\left\{|is\downarrow\rangle+\frac{kP}{E_g}\left|\left[\frac{k_x}{k}x+\frac{k_y}{k}y+\frac{k_z}{k}z\right]\downarrow\right\rangle+\frac{kP}{3E_g}\left(\frac{\Delta}{E_g}+\frac{\hbar^2k^2}{2mE_g}\right)\left|\left[\left(-\frac{k_x}{k}+i\frac{k_y}{k}\right)z+\frac{k_z}{k}(x-iy)\right]\uparrow\right\rangle\right\}. \qquad (3)$$

The symbols $\uparrow$ and $\downarrow$ at the Bloch amplitudes $u_{k\uparrow}$ and $u_{k\downarrow}$ denote the spin projections corresponding to the given state at the minimum of the conduction band (at $\mathbf{k}=0$), and the symbols $|\ \rangle$ denote the corresponding spinors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. The symmetrical function $|is\rangle$ coincides with the Bloch amplitude (without allowance for spin) of the electronic states at the minimum of the conduction band. The functions $|x\rangle$, $|y\rangle$, and $|z\rangle$ are orthonormalized functions that behave in transformations from the

[1)] The idea of the existence of such an effect was independently advanced by V. G. Fleisher.

crystal symmetry point group like the vector components x, y, z. The parameters Δ and P, in accord with [7], denote matrix elements of the type

$$\Delta = -\frac{3}{4}\frac{i\hbar}{m^2c^2}\langle x|[\nabla v(\mathbf{r})\hat{\mathbf{p}}]_y|z\rangle, \qquad P = \frac{\hbar}{m}\langle is|\hat{p}_x|x\rangle, \tag{4}$$

here Δ determines the spin-orbit splitting in the p-band (i.e., it measures the distance at $\mathbf{k} = 0$ between the bands of the heavy and light holes corresponding to the total angular momentum $^3/_2$, and the split-off band corresponding to a total angular momentum $^1/_2$), while the parameter P is connected with the effective mass m* by the relation

$$\frac{2}{3}\frac{P^2}{E_g} = \frac{\hbar^2}{2m^*} - \frac{\hbar^2}{2m}. \tag{5}$$

The quantity k^2P^2/E_g^2 is a small parameter of the order of T/Eg in nondegenerate semiconductors or ζ/Eg in degenerate ones (T is the temperature and ζ the chemical potential), i.e., on the order of the ratio of the average conduction electron energy to E_g. The coefficients c_1, c_2, c_3, and c_4 are normalization factors equal to unity, accurate to the small quantities K^2P^2/E_g^2 (and Δ/Eg in case (b)). It is seen from (2) and (3) that when $\mathbf{k} \neq 0$ the eigenfunctions of the Hamiltonian H_0 are not states with definite spin projection, as was the case in the absence of the spin-orbit interaction, but remain degenerate in energy and correspond to the eigenvalue

$$\varepsilon_k = \frac{2P^2k^2}{3E_g} + \frac{\hbar^2k^2}{2m} \approx \frac{2}{3}\frac{P^2k^2}{E_g} \tag{6}$$

(inasmuch as $m^* \ll m$).

Let us calculate the matrix element for scattering by a charged impurity center between Bloch functions $u_{ks}e^{i\mathbf{k}\cdot\mathbf{r}}$ (u_{ks} is $u_{k\uparrow}$ or $u_{k\downarrow}$ in formulas (2) in (3)):

$$\langle ks|\Phi(\mathbf{r}-\mathbf{r}_j)|\mathbf{k}'s'\rangle = e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}_j}\varphi(|\mathbf{k}-\mathbf{k}'|) \times\left\{\delta_{ss'}\left[1-\frac{P^2(\mathbf{k}^2-\mathbf{k}\mathbf{k}')}{E_g^2}\right] - i\frac{\gamma P^2}{3E_g^2}\langle s|\sigma[\mathbf{k}\mathbf{k}']|s'\rangle\right\}, \tag{7}*$$

where

$$\gamma = \begin{cases} 1, & \text{if} \quad \Delta \gg E_g \\ 2\Delta/E_g, & \text{if} \quad \Delta \ll E_g \end{cases}, \qquad \varphi(|\mathbf{k}-\mathbf{k}'|) = -\frac{4\pi e^2 z}{V\varkappa(\mathbf{k}-\mathbf{k}')^2}. \tag{8}$$

Here V is the volume of the crystal. In formula (7), the term with the vector product $\mathbf{k}\times\mathbf{k}'$ corresponds to the correction for the spin-orbit interaction in the quasirelativistic electron-scattering theory, with the electron mass replaced by the effective mass m* and mc^2 by Eg/γ, and with the sign of the entire expression reversed. This term can be formally obtained in the effective-mass approximation by adding to the initial Hamiltonian

$$\hat{H}_1 = -\frac{\hbar^2\gamma}{4m^*E_g}\sigma[\nabla\Phi(\mathbf{r}-\mathbf{r}_j), \hat{\mathbf{p}}]. \tag{9}$$

The additional term H_1 has the structure of the Hamiltonian of the spin-orbit interaction of the electron with the scattering center, except that the coupling constant is larger here than the ordinary one by several orders of magnitude.

We shall need in what follows the square of the modulus of the matrix elements of the amplitude for elastic scattering of an electron by a Coulomb center without spin flip $|\langle \mathbf{sk}|t^+(\epsilon_k)|\mathbf{sk}'|\rangle^2$. We recall that in formal scattering theory the amplitude $t^+(E)$ is defined by the relation

$$t^+(E) = \Phi(\mathbf{r}-\mathbf{r}_j)\left(1 - \frac{1}{E-H_0+i\delta}t^+\right). \tag{10}$$

As is well known, the scattering of an electron by a Coulomb center depends on the direction of the electron spin, owing to the spin-orbit interaction. When perturbation theory is used, this dependence appears only in the approximation that follows the Born approximation. We write down $|\langle \mathbf{sk}|t^+(\epsilon_k)|\mathbf{sk}'\rangle|^2$ in this approximation, neglecting the corrections to the spin-independent term:

$$|\langle sk|t^+(\varepsilon_k)|sk'\rangle|^2 = \varphi^2(|\mathbf{k}-\mathbf{k}'|) + \frac{4\pi^2e^6z^3\gamma}{\varkappa^3V^2E_gk^5}\frac{\ln(1/\sin\psi/2)}{\sin^2\psi}\langle s|\sigma[\mathbf{k}\mathbf{k}']|s\rangle, \tag{11}$$

where ψ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$. We note that the term responsible for the asymmetric scattering has the same structure as the analogous term in quasirelativistic theory.[8]

The corrections to the scattering matrix element due to the Bloch amplitudes were taken into account by a number of authors,[9,10] but they, unlike us, were interested in corrections to symmetrical scattering and therefore confined themselves to the Born approximation.

2. KINETIC EQUATION, ANOMALOUS HALL CONDUCTIVITY

We assume that the time τ_ϵ of the photoelectron energy relaxation is much shorter than the photoelectron lifetime τ_n, and the spin relaxation time τ_s is comparable with τ_n. Then stationary illumination of the semiconductor with polarized light establishes in the conduction band a uniform electron-energy distribution with a temperature equal to the lattice temperature T, total concentration n, and degree of polarization α. The degree of polarization is calculated in detail in [4,5].

Assume that a weak electric field E is applied to the semiconductor. Then the density j of the electric current can be represented in the form

$$\mathbf{j} = \mathbf{j}^0 + \mathbf{j}^1 = \sigma\mathbf{E} + \sigma_s[\mathbf{E}s]/s. \tag{12}$$

We shall call the quantity σ_s the anomalous Hall conductivity. The anomalous Hall current j^1 is due to the spin-orbit interaction (9).

The total-current density j is determined by the expression $\mathbf{j} = -eV^{-1}\operatorname{Tr}\rho\hat{\mathbf{v}}$, where ρ is the density matrix and $\hat{\mathbf{v}}$ the electron-velocity operator. In the representation of the Bloch functions $\psi_{ks} = e^{i\mathbf{k}\cdot\mathbf{r}}u_{ks}$

$$\langle sk|\hat{\mathbf{v}}|s'k'\rangle = \frac{\partial\varepsilon_k}{\hbar\partial\mathbf{k}}\delta_{ss'}\delta_{\mathbf{k}\mathbf{k}'} = \frac{\hbar\mathbf{k}}{m^*}\delta_{ss'}\delta_{\mathbf{k}\mathbf{k}'} \tag{13}$$

(ϵ_k is defined by (6)). Consequently, in calculating the current density we need only the diagonal density-matrix elements $\rho_{sk,sk} \equiv f_{sk}$.

We obtain the distribution function f_{sk} by solving the

*$[\mathbf{k}\mathbf{k}'] \equiv \mathbf{k}\times\mathbf{k}'$.

kinetic equation in the approximation linear in the spin-orbit interaction. At low impurity concentration N, the kinetic equation in [11]

$$-\frac{e}{\hbar}\mathbf{E}\frac{\partial f_{s\mathbf{k}}}{\partial \mathbf{k}}=\frac{2\pi}{\hbar}NV\sum_{\mathbf{k}'}|t^{+}_{s\mathbf{k},s\mathbf{k}'}|^2\delta(\varepsilon_k-\varepsilon_{k'})(f_{s\mathbf{k}'}-f_{s\mathbf{k}}). \quad (14)$$

Using (11), we rewrite the kinetic equation as follows:

$$-\frac{e}{\hbar}\mathbf{E}\frac{\partial f_{s\mathbf{k}}}{\partial \mathbf{k}}=\sum_{\mathbf{k}'}(W^{0}_{\mathbf{k}\mathbf{k}'}+W^{1}_{s\mathbf{k},s\mathbf{k}'})f_{s\mathbf{k}'}, \quad (15)$$

where

$$W^{0}_{\mathbf{k}\mathbf{k}'}=\frac{2\pi}{\hbar}NV\varphi^2(|\mathbf{k}-\mathbf{k}'|)\delta(\varepsilon_k-\varepsilon_{k'})(1-\delta_{\mathbf{k}\mathbf{k}'}),$$
$$W^{1}_{s\mathbf{k},s\mathbf{k}'}=\frac{8\pi^3e^6z^3N\gamma}{V\hbar\varkappa^3E_g^2k^5}\sin^{-2}\psi\ln\left[\left(\sin\frac{\psi}{2}\right)^{-1}\right] \quad (16)$$
$$\times\langle s|\sigma[\mathbf{k}\mathbf{k}']|s\rangle\delta(\varepsilon_k-\varepsilon_{k'})(1-\delta_{\mathbf{k}\mathbf{k}'}).$$

In the case of a weak electric field, the distribution function $f_{s\mathbf{k}}$ can be sought in the form

$$f_{s\mathbf{k}}=f^{00}_{s\mathbf{k}}+f^{0}_{s\mathbf{k}}+f^{1}_{s\mathbf{k}},\quad f^{00}_{s\mathbf{k}}=\left\{\exp\left(\frac{\varepsilon_k-\zeta_s}{T}\right)+1\right\}^{-1}, \quad (17)$$

where $f^{00}_{s\mathbf{k}}$ is the equilibrium electron distribution function, $f^{0}_{s\mathbf{k}}$ and $f^{1}_{s\mathbf{k}}$ are corrections proportional to the electric field, and the chemical potentials ζ_s are expressed in terms of the conduction-electron concentration n and the degree of their polarization α from the relations

$$n=\frac{1}{(2\pi)^3}\iiint(f^{00}_{\uparrow k}+f^{00}_{\downarrow k})d^3\mathbf{k},$$
$$\alpha=\frac{1}{(2\pi)^3}\iiint(f^{00}_{\uparrow k}-f^{00}_{\downarrow k})d^3\mathbf{k}. \quad (18)$$

From (15) we obtain the following equations for the determination of $f^{0}_{s\mathbf{k}}$ and $f^{1}_{s\mathbf{k}}$

$$-\frac{e}{\hbar}\mathbf{E}\frac{\partial f^{00}_{s\mathbf{k}}}{\partial \mathbf{k}}=\sum_{\mathbf{k}'}W^{0}_{\mathbf{k}\mathbf{k}'}f^{0}_{s\mathbf{k}'}, \quad (19)$$

$$-\sum_{\mathbf{k}'}W^{1}_{s\mathbf{k},s\mathbf{k}'}f^{0}_{s\mathbf{k}'}=\sum_{\mathbf{k}'}W^{0}_{\mathbf{k}\mathbf{k}'}f^{1}_{s\mathbf{k}'}. \quad (20)$$

Solving (19) in the approximation of the relaxation time $\tau(\mathbf{k})$, we get

$$f^{0}_{s\mathbf{k}}=e(\mathbf{E}\mathbf{k})\frac{\hbar\tau(k)}{m^*}\frac{\partial f^{00}_{s\mathbf{k}}}{\partial\varepsilon_k}, \quad (21)$$

where

$$\tau(k)=\frac{1}{2\pi}\frac{\hbar^3\varkappa^2k^3}{Ne^4z^2m^*\Lambda} \quad (22)$$

is the usual time of momentum relaxation on charged impurities (Λ in (22) is the Coulomb logarithm). A formal solution of (20) is

$$f^{1}_{s\mathbf{k}}=-\sum_{\mathbf{k}'}\{(W^0)^{-1}W^1\}_{s\mathbf{k},s\mathbf{k}'}f^{0}_{s\mathbf{k}'}. \quad (23)$$

Using (13) and (23) we obtain for the anomalous Hall current

$$\mathbf{j}^1=-\frac{e\hbar}{Vm^*}\sum_{\mathbf{k}'',\mathbf{k}',\mathbf{k}}\mathbf{k}''\{(W^0)^{-1}\}_{\mathbf{k}'',\mathbf{k}'}W^{1}_{s\mathbf{k}',s\mathbf{k}}f_{s\mathbf{k}}{}^{0}$$
$$=-\frac{e\hbar}{Vm^*}\sum_{\mathbf{k},\mathbf{k}'}\mathbf{F}_{\mathbf{k}'}W^{1}_{s\mathbf{k}',s\mathbf{k}}f_{s\mathbf{k}}{}^{0}, \quad (24)$$

where the function $\mathbf{F}_{\mathbf{k}'}$ is determined from the equation

$$\mathbf{k}=\sum_{\mathbf{k}'}\mathbf{F}_{\mathbf{k}'}W^{0}_{\mathbf{k}'\mathbf{k}}. \quad (25)$$

Solving this equation, we get

$$\mathbf{F}_{\mathbf{k}}=-\mathbf{k}\tau(k). \quad (26)$$

Substituting (26) and (21) in (24) we obtain for $\mathbf{j}^1$ the expressions

$$\mathbf{j}^1=\frac{e^2\hbar^2}{Vm^{*2}}\sum_{\mathbf{k},\mathbf{k}'}\mathbf{k}'W^{1}_{s\mathbf{k}',s\mathbf{k}}\tau^2(k)(\mathbf{E}\mathbf{k})\frac{\partial f_{s\mathbf{k}}{}^{00}}{\partial\varepsilon_k}. \quad (27)$$

Integrating with respect to $d^3\mathbf{k}'$ and with respect to the angles in the integration with respect to $d^3\mathbf{k}$, we obtain

$$\mathbf{j}^1=\frac{\pi}{3}\frac{e^6z^3Nn\gamma}{\varkappa^3\hbar E_gm^*}\int_0^\infty\sqrt{\varepsilon}\,\tau^2(\varepsilon)\frac{\partial(f_{\uparrow k}{}^{00}-f_{\downarrow k}{}^{00})}{\partial\varepsilon}d\varepsilon$$
$$\times\left[\int_0^\infty\sqrt{\varepsilon}(f_{\uparrow k}{}^{00}+f_{\downarrow k}{}^{00})d\varepsilon\right]^{-1}\frac{[\mathbf{Es}]}{s}. \quad (28)$$

The integration with respect to ϵ can be easily carried out in the following limiting cases:

a) nondegenerate semiconductors

$$\mathbf{j}^1=\frac{35}{4\pi}\frac{\varkappa T^2\gamma}{E_gz\hbar}\frac{n}{N}\frac{1}{\Lambda^2}\alpha\frac{[\mathbf{Es}]}{s}, \quad (29)$$

b) strongly degenerate semiconductors

$$\mathbf{j}^1=\frac{\pi}{2}\frac{e^8z^3nN\gamma\tau^2(\zeta_0)}{\varkappa^3\hbar E_gm^*\zeta_0}\frac{(1+\alpha)^{7/3}-(1-\alpha)^{7/3}}{2}\frac{[\mathbf{Es}]}{s}, \quad (30)$$

where ζ_0 is the chemical potential corresponding to the total electron concentration n and to zero degree of polarization ($\alpha = 0$), γ is defined in (8), and $\tau(\zeta_0)$ $\equiv \tau(\sqrt{2m^*\zeta_0}/\hbar)$ is defined in (22).

The foregoing analysis can be easily generalized to the case when a temperature different from the lattice temperature is established in electron system by stationary illumination of the semiconductor with polarized light.

3. SPIN RELAXATION TIME IN SCATTERING BY CHARGED IMPURITIES

Using expression (7) for the scattering matrix element, we can write an equation for the density matrix in the case of scattering by charged impurities with allowance for spin-flip processes that appear "in second order in the spin-orbit interaction." To calculate the relaxation time τ_S of the z-projection of the spin (this time is designated T_1 in the review [12]) we can confine ourselves only to diagonal elements of the density matrix f_{sk}, for which the Overhauser kinetic equation holds.[13] Since the literature contains only order-of-magnitude estimates of τ_S for III-V semiconductors, and furthermore for a low degree of polarization,[14] we present the exact expressions:

a) for nondegenerate semiconductors

$$\frac{1}{\tau_s}=\sqrt{\frac{\pi}{2}}\frac{\gamma^2e^4z^2\sqrt{T}N\Lambda}{\varkappa^2E_g{}^2\sqrt{m^*}}; \quad (31)$$

b) for degenerate semiconductors

$$\frac{1}{\tau_s}=3^{-1/2}\left(\frac{3\pi}{8}\right)^{5/3}\frac{\gamma^2e^4z^2\hbar n^{1/3}N\Lambda}{\varkappa^2E_g^2m^*}\frac{(1+\alpha)^{4/3}-(1-\alpha)^{4/3}}{\alpha}. \quad (32)$$

It is interesting to compare the spin relaxation time τ_S with the time $\langle\tau\rangle$ in terms of which the electron mobility μ is determined in the case of scattering by charged impurities ($\mu = e\langle\tau\rangle/m^*$), and which can be determined from experiments on the measurement of mobility at low temperatures:

a) in the case of nondegenerate semiconductors

$$\tau_s=\frac{\pi}{4}\frac{E_g^2}{T^2\gamma^2}\langle\tau\rangle; \quad (33)$$

b) in the case of degenerate semiconductors

$$\tau_s=\frac{32}{3}\left(\frac{E_g}{\gamma\zeta_0}\right)^2\frac{\alpha}{(1+\alpha)^{4/3}-(1-\alpha)^{4/3}}\langle\tau\rangle. \quad (34)$$

4. DISCUSSION OF RESULTS

The predicted anomalous Hall effect can be observed, for example, in the following experiment: circularly polarized light is incident normally to the surface of a semiconductor in the z direction. A current of density j^0 flows in the x direction under the influence of an applied electric field E. Then an electric field $|E_1| = \sigma_S\sigma^{-2}|j^0|$ should appear in the y direction, and the sign of the field should be determined by the polarization of the electrons and by the sign of the charge of the impurity centers, so that the most suitable material for the observations are semiconductors in which impurities of definite sign predominate. We present the value of the Hall angle for such semiconductors in the case when the electron mobility is determined by scattering from charged impurities:

a) for nondegenerate semiconductors

$$\frac{\sigma_s}{\sigma}=\frac{35}{16}\sqrt{\frac{\pi}{2}}\frac{\gamma e^2z\sqrt{m^*}}{\varkappa\hbar\Lambda\sqrt{E_g}}\sqrt{\frac{T}{E_g}}\alpha\equiv\frac{35}{16}\sqrt{\frac{\pi}{2}}A\sqrt{\frac{T}{E_g}}\alpha; \quad (35)$$

b) for degenerate semiconductors

$$\frac{\sigma_s}{\sigma}=\sqrt{2}\frac{e^2z\gamma\sqrt{m^*}}{\varkappa\hbar\Lambda\sqrt{E_g}}\sqrt{\frac{\zeta_0}{E_g}}\frac{(1+\alpha)^{3/2}-(1-\alpha)^{3/2}}{2}$$
$$=\sqrt{2}A\sqrt{\frac{\zeta_0}{E_g}}\frac{(1+\alpha)^{3/2}-(1-\alpha)^{3/2}}{2}. \quad (36)$$

The values of A for InSb and GaAs are $A_{InSb}\approx 2\times10^{-2}$ and $A_{GaAs}\approx 10^{-2}$. The degree of polarization in optical excitation was estimated in [4,5] and can vary in a wide range (up to 50%). It is clear from the derivations that even when the degree of electron polarization in the sample is of the order of 10^{-2} the Hall angle is $\sigma_S/\sigma\approx10^{-4}$–$10^{-5}$.

At low temperatures, when the electron momentum scattering is from charged impurities, the spin relaxation time τ_S can be estimated from formulas (33) and (34). The obtained values are in good agreement with the experimental data.[3,15] For example, for InSb with concentration $n = 10^{15}$ cm^{-3} and $\mu\approx10^5$ cm^2/V-sec we have $\tau_S\approx 2\times10^{-9}$ sec at $T\lesssim 20$°K, when degeneracy sets in.

The effect of asymmetrical scattering can be obtained also with another experimental procedure. For example, in the case of uneven illumination of the sample, if the resultant electron-diffusion flux is not collinear with the light-incidence direction, an emf should be induced in a direction perpendicular to the plane of the light and diffusion flux. An analogous effect should appear also under conditions of uneven heating of the electrons.

All these effects are analogous in a certain sense to the well-known anomalous Hall and Nernst effects in ferro- and paramagnets. We note that the large value of the spin-orbit interaction, which must be introduced here for a quantitative description of the effects, have not been obtained theoretically as yet (see, for example, [16,17]).

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