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Influence of Impurities on Combined Resonance in Semiconductors

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A theory of combined resonance on band carriers in semiconductors is developed for conditions when the spin relaxation time τ_S of the carriers appreciably exceeds their momentum relaxation time τ. It is shown that under these conditions the electric dipole absorption spectrum, corresponding to pure spin transitions, generally must consist of two bands which are superimposed on each other. One of these bands has a width τ^{-1}, and its intensity is determined by the average value of the square of the matrix element of the velocity operator, which causes the spin transitions. The second band has a width τ_S^{-1}, and its intensity is determined by the square of the average value of this matrix element. The value of this average essentially depends on the specific form of the spin-orbit terms in the dispersion law of the current carriers. Examples are indicated when the integrated intensity of the narrow band must be several orders of magnitude smaller than the integrated intensity of the wide band.

From an analysis of the experimental data it is concluded that the effect of the impurities on the combined resonance does not reduce to a change of its spectral distribution, and impurities may introduce a substantial contribution to the integrated intensity of the resonance. In this connection we investigate the mechanism, due to the influence of random Coulomb fields on the donor centers, of combined resonance with impurity electrons in a many-valley compensated semiconductor. According to the obtained estimates, the combined resonance should dominate over the paramagnetic resonance for the parameters of germanium, starting with concentrations on the order of 10^{15} cm^{-3}.

1. INTRODUCTION

ALL of the articles on combined resonance of band carriers in semiconductors (that is, electric dipole transitions accompanied by a change of the spin state[1]), in which the authors achieved a clear understanding of the mechanism for excitation of the resonance and carried out successfully a comparison of theory with experiment, pertain to the case $\omega\tau \gg 1$, where ω is the absorption frequency and τ is the usual relaxation time of the carriers.

However, a strong inequality $\tau_S \gg \tau$ exists in many semiconductors, where τ_S denotes the spin relaxation time. Therefore, the situation when $\omega\tau_S \gg 1$ whereas $\omega\tau \lesssim 1$ is quite possible. Bell's experiments[2] in n-InSb may serve as an example; so far there is no explanation of his experiments.

It is obvious that when $\omega\tau \lesssim 1$ all of the bands corresponding to a simultaneous change of the orbital and spin quantum numbers possess a large width $\sim\tau^{-1}$. Therefore, they can scarcely be distinguished against the background of the considerably more powerful cyclotron absorption. The situation is more complicated for the band associated with a pure spin transition. In the present article it is shown that for $\tau_S \gg \tau$ this band must consist of a narrow band with a width $\sim\tau_S^{-1}$ and broad wings of width $\sim\tau^{-1}$. For the reasons indicated above, apparently only the narrow part of the spin band can be observed when $\omega\tau \lesssim 1$; we shall henceforth call it the spin line. The total intensity of the spin band for crystals of different symmetry was calculated earlier in a number of articles; a formula is derived below for the intensity of the spin line.

An analysis of the experimental data has led us to the conclusion that in certain conditions the influence of the impurity centers on the combined resonance does not reduce to a scattering of the carriers; by changing the energy spectrum of the system, the impurity centers lead to the emergence of new mechanisms for combined resonance. As a specific example, a new mechanism is investigated for the excitation of combined resonance with large-radius donor centers in many-valley semiconductors, this mechanism being due to the Coulomb interaction of the impurities. One can propose that the combined resonance in compensated n-Ge, which was observed in[3], is caused by this mechanism.

2. INFLUENCE OF CARRIER SCATTERING ON THE INTENSITY OF COMBINED RESONANCE

We start with a very large difference between the two relaxation times, $\tau_S \gg \tau$. Since the contour of the spin line is formed during a time of the order of its reciprocal half-width, that is, of the order of τ_S^{-1}, it is clear that during the time of a spin transition the electron is able to undergo a large number of collisions ($\tau_S/\tau \gg 1$), which change its momentum but not its spin. In what follows we shall call such collisions momentum collisions. In the theory of paramagnetic resonance, momentum collisions lead to the well-known effect of kinematic narrowing.[4] However, under the conditions for combined resonance the electron momentum $\mathbf{k}$ influences not only the electron's g-factor but also generally the matrix element of the spin transition. Therefore, momentum scattering may substantially change the intensity of the spin line.

We consider nondegenerate bands. We write the Hamiltonian of the system without scattering in the form

$$\mathscr{H} = \mathscr{H}_0(\hat{\mathbf{k}}) + g(\mathbf{SH}) - \frac{e}{c}(\mathbf{v}(\hat{\mathbf{k}})\mathbf{A}(t)), \qquad (1)$$

where $\hat{\mathbf{k}}$ is the momentum operator in the magnetic field **H**, **A**(t) is the vector potential of the electromagnetic wave, and

$$\mathbf{v}=\sum_i S_i \mathbf{v}_i(\hat{\mathbf{k}})\left(\begin{matrix} S_i=S_+,\ S_-,\ S_z; \quad \mathbf{v}_i=\mathbf{v}_-,\ \mathbf{v}_+,\ \mathbf{v}_z; \\ S_\pm=2^{-1/2}(S_x\pm iS_y)\end{matrix}\right) \qquad (2)$$

is the velocity operator; the z axis is directed along **H**. For simplicity we have assumed the g-factor to be isotropic and to be independent of $\hat{\mathbf{k}}$, because this hardly changes the results. The eigenstates of the Hamiltonian (1) for **A** = 0 are classified according to the coordinate and spin quantum numbers, and so long as $\tau_S \gg \tau$ it is appropriate to divide the collision integral into two parts corresponding to momentum and spin collisions (W and W_S). We represent the density-matrix correction ρ_1, which is linear in **A**, in the form

$$\rho_1=\rho_1{}^0+\boldsymbol{\rho}_1\mathbf{S};\quad \boldsymbol{\rho}_1=(\rho_-,\ \rho_+,\ \rho_z).$$

Then in the interaction representation with respect to $\mathscr{H}_0$, the equation for ρ_1 has the form

$$\frac{\partial\rho_1}{\partial t}+ig[(\mathbf{S}\mathbf{H}),\rho_1]+\tilde{W}(\rho_1)+\tilde{W}_S(\rho_1)=\frac{ie}{c}[\mathbf{v}(t)\mathbf{A}(t),\rho_0]. \qquad (3)$$

As usual, it is assumed that the electron has a characteristic energy $\epsilon_{\text{char}} \gg \tau^{-1}$; $\tilde{W}$ and $\tilde{W}_S$ denote the linearized collision integrals.

Inside the spin line for the component ρ_- (for g > 0) the sum of the first two terms in (3) is of the order of $(\omega_S-\omega)\rho_- \sim \rho_-\tau_S^{-1}$, where $\omega_S = gH$; the last term is of the same order of magnitude. The third term for ρ_- of general form is of the order of $\rho_-\tau^{-1}$ and exceeds the remaining terms by a factor $\tau_S/\tau \gg 1$. Therefore, in the lowest-order approximation one should seek ρ_1 from the condition $\tilde{W}(\rho_1) = 0$.

It is not difficult to find the general form of the solution of the equation $W(\rho) = 0$. First, ρ should be diagonal with respect to the configurational quantum numbers, since the momentum collisions lead the system into equilibrium with respect to these degrees of freedom. Since the momentum collisions still do not change the spin, it follows that in a basis in which ρ is diagonal with respect to the spin quantum numbers, its components are Fermi functions f with arbitrary chemical potentials. If ρ is close to ρ_0, then the bases in which they are diagonal are similar. Therefore ρ can be transformed into a basis in which ρ_0 is diagonal by making an infinitesimal rotation. Confining ourselves to the lowest-order terms in $\rho - \rho_0$ (that is, to the terms linear in the angle of rotation and in the changes of the chemical potentials), we obtain

$$\rho(\varepsilon)-\rho_0(\varepsilon)=\begin{pmatrix}(\eta_1-\eta)\dfrac{\partial f_1(\varepsilon_1-\eta)}{\partial\eta} & \gamma(f(\varepsilon_2-\eta)-f(\varepsilon_1-\eta)) \\ \gamma^*(f(\varepsilon_2-\eta)-f(\varepsilon_1-\eta)) & (\eta_2-\eta)\dfrac{\partial f(\varepsilon_2-\eta)}{\partial n}\end{pmatrix} \qquad (4)$$

Here ϵ is the kinetic energy of the electrons, $\epsilon_{1,2}(\epsilon) = \epsilon \pm \omega_S/2$ denote the total energies of electrons with different spin orientations, $\eta_{1,2}$ denote their Fermi quasilevels, η is the equilibrium chemical potential, and γ is a complex parameter characterizing the rotation of the basis. From conservation of the number of electrons it follows that $\eta_1 + \eta_2 = 2\eta$. This same matrix can be written in the form

$$(\rho-\rho_0)_{ij}=\mathscr{A}_{ij}f(\varepsilon_i-\eta)f(\varepsilon_j-\eta)\exp\{(\varepsilon_i+\varepsilon_j)/2\},\quad \mathscr{A}_{ij}=\mathscr{A}_{ji}{}^*. \qquad (5)$$

Since both functions ρ and ρ_0 cancel W, the difference $\rho - \rho_0 = \rho_1$ calculated here cancels the linearized integral $\tilde{W}(\rho_1) = 0$.

The matrix element ρ_- is needed in order to calculate the intensity of transitions with spin reorientation. The energy dependence of this matrix element is determined by formula (4); it remains to calculate the magnitude of the coefficient γ. For this purpose we calculate the trace of Eq. (3) with respect to all configurational quantum numbers. In this connection the term W, as a term which preserves the number of particles with a given spin, is cancelled irrespective of the form of ρ_1, and in the remaining terms, having the same order of magnitude, it is necessary to substitute the approximate expression for ρ_- from (4) or (5). After evaluating the trace, the term $\tilde{W}_S$ reduces to a constant which is multiplied by γ; thus the spin relaxation time τ_S arises in a natural way. Finally we obtain

$$\frac{\partial\gamma}{\partial t}+i\omega_S\gamma+\frac{\gamma}{\tau_S}=\frac{ie}{c\sqrt{2}}\langle\mathbf{v}_-\rangle\mathbf{A}(t);$$
$$\langle\mathbf{v}_-\rangle=\frac{\operatorname{Sp}\{\mathbf{v}_-[f(\varepsilon_2-\eta)-f(\varepsilon_1-\eta)]\}}{\operatorname{Sp}\{f(\varepsilon_2-\eta)-f(\varepsilon_1-\eta)\}}. \qquad (6)$$

In Eq. (6) the quantity $\langle \mathbf{v}_- \rangle$ does not depend on t since it only contains elements diagonal in the configurational quantum numbers. Calculating the current with the aid of the result found for the density matrix, we find the following expression for the conductivity tensor:

$$\sigma_{\alpha\beta}=\frac{e^2}{\omega}\frac{\langle v_+\rangle_\alpha\langle v_-\rangle_\beta(n_2-n_1)}{i(\omega_S-\omega)+\tau_S{}^{-1}}, \qquad (7)$$

where $n_{1,2}$ denote the concentrations of electrons with different spins. Formula (7) represents a Lorentz curve with a small width $\sim\tau_S^{-1}$. The fundamental characteristics of this formula is that it contains the averages of the matrix elements of the velocity (but not the averages of their squares, as happens in the case of weak scattering). It is natural that in many cases such averages must vanish, and then there should be no narrow band in the combined resonance spectrum. At this point a basic difference appears in the effect of kinematic averaging on paramagnetic resonance and on combined resonance. In paramagnetic absorption the matrix element of the transition essentially does not depend on the configurational quantum numbers, and therefore kinematic averaging leads only to a narrowing of the band. Conversely, in combined resonance the transition matrix element as a rule depends significantly on the configurational quantum numbers, and therefore kinematic averaging may abruptly reduce the intensity of the spin line. Several examples will be considered in the following section.

Let us emphasize that the scattering does not change the total intensity of the spin transitions; it is always determined by the mean-squared velocity-matrix elements. However, this absorption consists of two mutually superimposed bands with widths $\sim\tau^{-1}$ and τ_S^{-1}. When $\omega\tau \gg 1$ both the broad band and the narrow line in its background can be observed. When $\omega\tau \lesssim 1$ the broad band should be lost in the background of cyclotron absorption, and only the narrow line, whose intensity is determined by formula (7) for arbitrary values of $\omega\tau$, can be observed. Formula (7) was derived under the usual limitation $\epsilon_{\text{char}}\tau \gg 1$. However, there is no reason to doubt that in a model in which the influence of the impurities reduces to scattering of the carriers the basic result obtained above should remain in force even under more general conditions, namely, the matrix elements of the velocity operator, averaged over k-space in the appropriate manner, must enter in the intensity of the combined resonance.

3. RESONANCE IN CRYSTALS OF DIFFERENT SYMMETRIES. DISCUSSION OF THE EXPERIMENTAL DATA

As is clear from formula (7), a spin line should be present in the combined resonance spectrum if $\langle v_\pm \rangle \neq 0$. The dependence of v on k is determined by the symmetry of the crystal and by the mechanism for the excitation of the combined resonance.

Two cases are possible when v does not depend on k at all. First, the spin-orbit coupling may be realized by dispersion-law terms linear in $\hat{k}$, as happens, for example, in crystals of the wurtzite type.[5] Second, it may be accomplished by similar terms in the g-factor, as for example in n-Si.[6] It is obvious that here the entire intensity is concentrated in the narrow line; this case was considered in[7].

For the Hamiltonians of the spin-orbit interaction of higher-order in $\hat{k}$, two cases are possible. For the Hamiltonians of even order, the velocity is odd in $\hat{k}$, and therefore averaging yields $\langle v_\pm \rangle = 0$; consequently the spin line is absent from the spectrum. As an example, terms $\sim\hat{k}^4$ due to nonparabolicity may appear in the Hamiltonian of the electrons in InSb. They should only lead to absorption with a width $\sim\tau^{-1}$; its magnitude was estimated in[8,1,9]. For the Hamiltonians of odd order, the velocity is even in $\hat{k}$, and the result of its averaging depends on the specific symmetry and on the magnitude of the magnetic field.

Let us consider, as an example, the effect of the cubic terms arising in the Hamiltonian of an electron in InSb because of the absence of an inversion center. Using the matrix elements of the velocity for a pure spin transition (see[10], formulas (31) and (32)), it is not difficult to verify that

$$\langle v_\pm \rangle \sim \langle \hat{k}_x{}^2 + \hat{k}_y{}^2 - 2\hat{k}_z{}^2 + \lambda^{-2} \rangle, \tag{8}$$

where λ is the magnetic length. In the quasiclassical limit when $\epsilon_{\text{char}} \gg \omega_c$ (ω_c denotes the cyclotron frequency), the leading terms in (8), which are of the order of ϵ_{char}, cancel out. As a result the matrix element is reduced by a factor $\omega_c/\epsilon_{\text{char}} \ll 1$. Therefore, almost all of the absorption is concentrated in the broad band. For $\epsilon_{\text{char}} \sim \omega_c$ the absorption intensities in the spin band and in the line are comparable.

According to[10] the angular dependence of the velocity matrix elements does not depend on the configurational quantum numbers. Therefore the averaging associated with the scattering should not have any influence on the angular dependence of the intensity of the resonance (the line of reasoning cited above is valid for $\epsilon_{\text{char}}\tau \gg 1$[1]). We recall that strong angular dependences of the intensity were obtained in[10].

The spin transition in the combined resonance spectrum has been experimentally observed in the UHF[2] and infrared[11] bands. In the second case the condition $\omega\tau \gg 1$ was satisfied. Below we shall be interested in the results on UHF absorption, when this criterion was roughly violated, as is evident from the curve of the cyclotron resonance presented in[2].

Bell[2] reported the observation of a spin electric dipole line, in which the absorption had the following properties: 1) it was isotropic and 2) the same for right-hand and left-hand polarizations of the UHF field. Since under the conditions of his experiments the Fermi energy is $\eta \sim \omega_c$, the very presence of a spin line due to the cubic terms in the dispersion law does not contradict the results obtained above. However, in the remainder of the picture, the absorption differs from the theoretical prediction, in that the intensity should be anisotropic and different in the cases of right-hand and left-hand polarizations.

In discussing the possible reasons for this disagreement, it is necessary first to emphasize that Bell's experiments were performed under the conditions $\eta\tau \lesssim 1$; formula (8) therefore is not valid. Its extrapolation into this region can be carried out only in the sense of taking account of the purely "scattering" effect of the impurities. In actual fact, however, the rearrangement of the energy spectrum, which was not taken into account above,[2] becomes important when $\eta\tau \sim 1$.

Estimates of the absolute intensity of the resonance also lead to difficulties. From the absorption curve given in[2] it follows that the oscillation strength of the spin line is large and amounts to 10^{-4} to 10^{-3} of the oscillator strength of the cyclotron resonance. The theoretical value of the intensity is determined by the coefficient δ_0 associated with the cubic terms.[10] A rough estimate based on Kane's model gives $\delta_0 \approx 200$ atomic units;[10] according to experimental data $\delta_0 \approx 50$ atomic units.[11] Neither value explains such a high intensity of the combined resonance.

The above difficulties in explaining the absolute magnitude of the intensity and its angular dependence force us to assume that in strongly doped crystals the impurities play an essentially different role in the combined resonance. Their influence does not reduce to carrier scattering that leads to a broadening of the levels and an averaging of the transition matrix ele-

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[1] It was proposed earlier[2,1] that strong scattering may smooth out the angular dependence of the resonance. From what has been said above, it follows that this does not occur when $\epsilon_{\text{char}}\tau \gg 1$. When $\epsilon_{\text{char}}\tau \sim 1$ it is already impossible to neglect the influence of the scatterers on the energy spectrum, and therefore the formulation of the problem itself must be changed (see below).

[2] In this connection let us emphasize that it is important to carry out new, more detailed experiments, in particular under the conditions $\omega_c\tau \lesssim 1$, but with $\epsilon_{\text{char}}\tau \gg 1$.

ments. By causing a rearrangement of the energy spectrum of the system, they lead to the appearance of new mechanisms for the combined resonance and, consequently, to a new contribution to the oscillator strength of the combined resonance, which in certain cases turns out to be predominant.

It is not difficult to indicate in principle several mechanisms whereby the impurities contribute to the intensity of the combined resonance. For example, according to Blount,[12] the impurity potential $V(\mathbf{r})$ generates an additional term in the velocity (the anomalous velocity); in n-InSb it has the structure $\mathbf{v} = \boldsymbol{\sigma} \times \operatorname{grad} V$. Further, the impurity centers create a deformation which in turn leads to the appearance of terms linear in $\hat{\mathbf{k}}$ in the dispersion law of the electrons in InSb[13], and so forth. However, we shall not discuss this question in more detail.

Interesting experimental data was recently obtained about the influence of the impurity concentration on the intensity of the combined resonance with impurities. Gershenzon, Pevin, and Fogel'son[3] showed that in compensated samples of Ge doped with As and P, a combined resonance band appears in which the integrated intensity considerably exceeds the intensity of the paramagnetic resonance.

The very observation of the combined resonance with donors in Ge is of interest for the following reasons. Combined resonance with band electrons in Ge should be caused by the dependence of the g-factor on $\hat{\mathbf{k}}$, and according to estimates,[6] its intensity should be small (on the order of the intensity of the paramagnetic resonance). According to[14] the binding of the carriers to the donors should sharply reduce the intensity of the combined resonance, provided the mechanism for its excitation remains unchanged. The contribution to the intensity of the combined resonance with large-radius donors in Ge was estimated in[15], this contribution being due to the absence of an inversion center in the point group; this intensity turned out to be lower than the intensity of the paramagnetic resonance. Therefore, according to the theoretical data, one would expect a low intensity of combined resonance with donors in Ge, and the results of[3] were a definite surprise.

At the same time the experimental data of[3] clearly indicate that the high intensity of the combined resonance is a consequence of the interaction of the impurity centers, most likely Coulomb interaction. We shall demonstrate below that such an effect actually should arise in many-valley crystals.[3)] According to the models which have been developed thus far, a multivalley structure does not have any influence on the intensity of the combined resonance of the band electrons and the electrons of isolated donor centers. We therefore encounter here a definite example of the switching-on of a new mechanism for combined resonance with increased degree of doping. The corresponding calculations are contained in the following section.

4. INFLUENCE OF ELECTRIC FIELDS ON THE COMBINED RESONANCE OF THE DONOR ELECTRONS

In partially compensated samples the neutral donor centers are located in the field of the positive (ionized) donors and negative acceptors. In the simplest approximation, neglecting the correlations in the arrangement of the centers and assuming that the average distance between them appreciably exceeds the radius a_B of the donor center, one can assume the field to be homogeneous within the limits of the center.

Neglecting the interaction of the valleys the Hamiltonian of an electron belonging to the n-th valley is given by

$$\mathcal{H}^{(n)}_{l\sigma,\, l'\sigma'} = \varepsilon_l \delta_{ll'} \delta_{\sigma\sigma'} + \delta_{ll'} (\mathbf{H}\hat{g}^n \mathbf{S})_{\sigma\sigma'} + \delta_{\sigma\sigma'} e\mathbf{r}^n_{ll'} \mathbf{E} \tag{9}$$

Here l and σ denote the Coulomb and spin quantum numbers. Assuming in what follows that the valley-orbital splitting Δ is small in comparison with the Coulomb energy E_B, we take it into account only in the lowest level $l = 0$. Changing from valley functions to functions that diagonalize the total Hamiltonian at $\mathbf{H} = \mathbf{E} = 0$ (the corresponding quantum number is N, and the transformation coefficients are C_{Nn}), we obtain

$$\begin{aligned}\mathcal{H}_{Nl\sigma,\, N'l'\sigma'} &= (\varepsilon_l - \Delta\delta_{N1}\delta_{l1})\delta_{ll'}\delta_{NN'}\delta_{\sigma\sigma'} \\ &+ \delta_{ll'} \sum_n C_{Nn}{}^* C_{N'n} (\mathbf{H}\hat{g}^n\mathbf{S})_{\sigma\sigma'} + \delta_{\sigma\sigma'} \sum_n C_{Nn}{}^* C_{N'n} e\mathbf{r}^n_{ll'}\mathbf{E}.\end{aligned} \tag{10}$$

The lowest level--an orbital singlet--is denoted by $N = 1$. For what follows it is convenient, assuming the last two terms in (10) to be a small perturbation, to construct the spin Hamiltonian $\mathcal{H}_S$ for this level. Carrying out the calculations in third-order perturbation theory and selecting the leading terms in the parameter Δ/E_B, we obtain

$$\begin{aligned}\mathcal{H}_s = g(\mathbf{SH}) + \frac{2}{\Delta}\Big[\frac{1}{\nu}\sum_{n=1}^{\nu}(\mathbf{H}\hat{g}^n\mathbf{S})(\mathbf{E}\hat{p}^n\mathbf{E}) \\ - \frac{1}{\nu}\sum_{n=1}^{\nu}(\mathbf{H}\hat{g}^n\mathbf{S}) \cdot \frac{1}{\nu}\sum_{m=1}^{\nu}(\mathbf{E}\hat{p}^m\mathbf{E})\Big],\end{aligned} \tag{11}$$

where ν denotes the number of valleys,

$$p_{\alpha\beta}{}^n = \sum_{l \neq 0} \frac{(d_\alpha)_{1l}{}^n (d_\beta)_{l1}{}^n}{\varepsilon_l - (\varepsilon_1 - \Delta)}, \qquad g = \frac{1}{3}(g_\parallel + 2g_\perp), \tag{12}$$

$g_\parallel$ and $g_\perp$ denote the principal values of the tensor $\hat{g}^n$, and $(d_\alpha)^n_{ll'}$ are the matrix elements of the dipole moment.

The summations over the valleys appearing in Eq. (11) can be calculated for specific band structures. For example, for Ge

$$\mathcal{H}_s = g(\mathbf{SH}) + \frac{4}{9\Delta}(g_\parallel - g_\perp)(p_\parallel - p_\perp)\Big[(\mathbf{HE})(\mathbf{ES}) - \sum_\alpha H_\alpha E_\alpha{}^2 S_\alpha\Big], \tag{13}$$

where the subscript α labels the Cartesian coordinates in the principal axes of the crystal; $p_\parallel$ and $p_\perp$ denote the principal values of the tensor $\hat{p}^n$.

The dependence of the spin frequency on $\mathbf{E}$ follows immediately from (13):

$$\omega_s(\mathbf{E}) = \omega_s\Big\{1 + \frac{4}{9\Delta}\frac{g_\parallel - g_\perp}{g}(p_\parallel - p_\perp)\frac{1}{H^2}\Big[(\mathbf{HE})^2 - \sum_\alpha H_\alpha{}^2 E_\alpha{}^2\Big]\Big\}. \tag{14}$$

[3)] There is a certain analogy between the role of a many-valley structure in the problem under consideration and in the theory of spin relaxation.[16]

The operator $\mathbf{d}\cdot\tilde{\mathbf{E}}$ describing the interaction of the electron with the high-frequency field and responsible for the combined resonance is immediately obtained from (13):

$$\mathscr{H}_{s\,int}=\tilde{\mathbf{E}}d\mathscr{H}_s/d\mathbf{E}=\tilde{\mathbf{E}}\hat{K}\mathbf{S}=\frac{4}{9\Delta}(g_\parallel-g_\perp)(p_\parallel-p_\perp)\Big[(\tilde{\mathbf{E}}\mathbf{H})(\mathbf{E}\mathbf{S})+(\mathbf{H}\mathbf{E})(\tilde{\mathbf{E}}\mathbf{S})-2\sum_\alpha \tilde{E}_\alpha E_\alpha H_\alpha S_\alpha\Big]. \tag{15}$$

The conductivity tensor calculated for a single center is expressed in terms of the tensor $\hat{K}$ introduced here:

$$\sigma_{\alpha\beta}(\omega,E)=\frac{\pi}{4}K_{\alpha+}K_{\beta-}\omega\,\mathrm{th}\left(\frac{\omega}{2T}\right)\delta(\omega-\omega_S(E)). \tag{16}$$

$K_{\alpha\pm}=2^{-1/2}(K_{\alpha x}\pm iK_{\alpha y})$, where x and y denote the coordinates in the system in which the direction of **H** is chosen as the z axis.

Expression (16) must be averaged over all positions of the impurities. It is convenient to find first the distribution function for the fields **E** acting on the spin

$$\psi(\mathbf{E})=\int\prod_i\left(\frac{d^3\mathbf{R}_i}{\Omega}\right)\delta\left(\mathbf{E}-\sum_i\mathbf{E}(\mathbf{R}_i)\right);\quad \mathbf{E}(\mathbf{R}_i)=-\frac{e\mathbf{R}_i}{R_i^3}. \tag{17}$$

Here Ω is the volume of the sample, and $\mathbf{E}(\mathbf{R}_i)$ is the field created by the i-th charged impurity. Representing the δ-function in the form

$$\int\exp\left\{i\mathbf{k}\left(\mathbf{E}-\sum_i\mathbf{E}(\mathbf{R}_i)\right)\right\}d^3\mathbf{k}/(2\pi)^3$$

and noting that the factors pertaining to individual impurities are averaged independently, we obtain

$$\psi(\mathbf{E})=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\exp\left\{i\mathbf{k}\mathbf{E}-n\int d^3\mathbf{r}\left(1-e^{i\mathbf{k}\mathbf{E}(\mathbf{r})}\right)\right\}=\frac{1}{2\pi^2E_0^2E}\varphi\left(\frac{E}{E_0}\right), \tag{18}$$

where n denotes the concentration of the charged impurities. The function φ of dimensionless argument is defined by the formula

$$\varphi(x)=\int_0^\infty k\sin(kx)\exp(-k^{3/2})\,dk, \tag{19}$$

and the characteristic field E_0, determined by the impurity concentration, is given by

$$E_0=4\pi e(n/30)^{2/3}. \tag{20}$$

For $x \ll 1$ we have $\varphi(x)\sim x$, and for $x \gg 1$

$$\varphi(x)\approx 15\sqrt{2\pi}/16x^{7/2}.$$

Thus, averaging formula (16) over the positions of the impurities reduces to multiplication by $\psi(\mathbf{E})$ and integration with respect to $d^3\mathbf{E}$. Let us introduce the unit vectors e, $\tilde{e}$, and h directed respectively along **E**, $\tilde{\mathbf{E}}$, and **H**, and let us express the angular dependences in formula (16) with their aid. Changing to the dimensionless integration variable $x = E/E_0$, with (18) taken into account, we obtain from (16) the following formula for the conductivity tensor:

$$\sigma_{\alpha\beta}(\omega)=\sigma_0\int_0^\infty x^3\varphi(x)\,dx\int d\Omega_e(M_{\alpha-}M_{\beta+}). \tag{21}$$

$$\times\left(\omega-\omega_S-\omega_S a x^2\Big[(\mathbf{e}\mathbf{h})^2-\sum_\alpha e_\alpha^2h_\alpha^2\Big]\right),$$

where the following notation has been introduced:

$$\sigma_0=\frac{2E_0^2H^2}{81\pi}(g_\parallel-g_\perp)^2(p_\parallel-p_\perp)^2\omega\,\mathrm{th}\frac{\omega}{2T}; \tag{22}$$

$$a=\frac{4}{9\Delta}\frac{g_\parallel-g_\perp}{g}(p_\parallel-p_\perp);\qquad M_{\alpha\beta}=h_\alpha e_\beta+(\mathbf{h}\mathbf{e}-2h_\alpha e_\beta)\delta_{\alpha\beta}.$$

As $\omega\to\omega_S$ the conductivity $\sigma_{\alpha\beta}$ diverges logarithmically; here the decisive contribution arises from these orientations of the field **E** for which the frequency shift cancels out (see formula (14)). For large values of $|\omega-\omega_S|$ it decreases like $|\omega-\omega_S|^{-3/4}$, that is, the curve has slowly decaying wings, and the area under these wings diverges. The characteristic curve width at which the logarithmic law is replaced by a power law is of the order of $|\omega-\omega_S|\sim a\omega_S$; the curve width estimated from experiments should be of this order of magnitude. According to formula (14), the frequency shift $a\omega$ corresponds to a perturbing field $E\sim E_0$. The criterion for the applicability of perturbation theory with respect to the field E_0,

$$(ea_BE_0)^2/\Delta E_B\ll 1$$

is equivalent to the condition $a \ll 1$. Therefore $|\omega-\omega_S \ll \omega_S$.

The absorption curve depends significantly on the orientation of **H**. When **H** is directed along a fourfold axis, the expression inside the square bracket in (14) cancels out and the absorption curve becomes a δ-function. Since upon averaging over the orientations of **E** this bracket cancels out for any arbitrary orientation of **H**, there is no reason to expect any appreciable asymmetry of the band relative to the unbiased frequency ω_S.

A formula analogous to (21) can also be written down for the paramagnetic resonance spectrum, that is, the absorption spectrum under the conditions of magnetic excitation.[4] It differs from (21) primarily in that $\varphi(x)$ is preceded by x rather than x^3. As a result the decrease of the absorption is much faster, like $|\omega-\omega_S|^{-7/4}$, that is, the spectrum is narrowed down.

Rough estimates of the intensity, carried out as applied to the parameters of Ge according to formula (21) at $E\sim E_0$, indicate that at 10^{10} Hz the combined resonance must dominate over the paramagnetic resonance beginning with charged-impurity concentrations $\sim 10^{15}\ \mathrm{cm}^{-3}$. This estimate, and also the general shape of the spectrum, do not contradict the experimental data of[3]. With regard to the narrow paramagnetic resonance bands which are preserved in[3] against the background of the combined resonance band, they must correspond to centers which are located at points where **E** is small.

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[4] The broadening of the paramagnetic resonance band of small-radius centers, due to the Coulomb interaction of the impurities, was investigated in[17].

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