Theory of Cyclotron Echo in Semiconductors

YA. YA. ASADULLIN AND U. KH. KOPVILLEM

Kazan' Physico-technical Institute, USSR Academy of Sciences

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The theory of cyclotron echo in semiconductors is considered. In contrast to a gaseous plasma for which a classical treatment is valid, quantization of electron motion in a magnetic field is important in solids. It is found that the dipole interaction of the electrons with the alternating-field pulses leads to the formation of an echo signal only if nonlinear processes entailing doubling of the variable field frequency occur in the matter or two-quantum transition are involved in the excitation process. A combination of dipole and quadrupole interactions or pure quadrupole interaction also leads to echo formation. The instants of appearance and the intensities of the signals are found. The effect of electron-impurity interaction on the echo signals is taken into account. In the adiabatic approximation, the interaction between pulses II and III is switched off on formation of a stimulated echo and hence the signal may be comparable in order of magnitude with the usual two-pulse echo.

CYCLOTRON echo was first observed in a weakly-ionized plasma of inert gases and nitrogen.[1] The signal intensities of both the two-pulse and three-pulse echos admitted of wide-range variation of both the parameters of the microwave excitation pulses (pulse duration Δt_γ, where $\gamma = 2$ and $\gamma = 3$ for two- and three-pulse echoes respectively; the intervals between pulses namely τ between the first and second pulses and τ_1 between the first and third pulses, $\tau_1 \gg \tau$; the pulse intensities) and the plasma parameters (temperature, electron concentration n_0, pressure, etc.). The main mechanisms of echo production, depending on the experimental conditions, can be[2] (i) the velocity-dependent electron relaxation due to collisions with ions or atoms (molecules), (ii) nonlinear interaction with pulses, and (iii) dependence of the cyclotron frequency in the amplitude. This phenomenon was treated quantum-mechanically earlier in [3].

We investigate here the possibility of observing cyclotron induction and echo in semiconductors. It is obvious that the experimental conditions in gases and semiconductors are essentially different. In particular, even in a cold plasma the average electron energy is of the order of an electron volt, and consequently in a magnetic field corresponding to $\omega_c = eH/mc \approx 10^{10}$ Hz[1] the Landau levels with $N \approx 10^5$ are populated. In semiconductors, on the other hand, it is easy to attain the ultraquantum conditions $\hbar\omega_c \gg k_Bt$ (for nondegenerate electrons) or $\hbar\omega_c \gg \zeta$ (for a degenerate semiconductor, where ζ is the Fermi level), when only the lower Landau level with $N = 0$ is populated. It is clear that whereas for gases one can treat the electron motion classically, in solids the Landau quantization is essential. Furthermore, owing to the non-parabolicity of the conduction band, the Landau levels may become non-equidistant.

Cyclotron induction and echo signals can yield considerable information concerning electron-impurity, electron-phonon, and other interactions.

The system Hamiltonian in a magnetic field $H \parallel z$ is given by

$$\mathcal{H} = \int \psi^+(\mathbf{r})(\mathcal{H}_e + U(\mathbf{r}))\psi(\mathbf{r})\,d\mathbf{r},$$

$$\mathcal{H}_e = \pi^2/2m, \quad \boldsymbol{\pi} = \mathbf{p} - e\mathbf{A}/c. \tag{1}$$

The function $U(\mathbf{r})$ is the potential for scattering by impurities:

$$U(\mathbf{r}) = \sum_{k=1}^{N_I} u(\mathbf{r} - \mathbf{R}_k) \tag{2}$$

($\mathbf{R}_k$ is the radius-vector of the k-th impurity, and N_I is the number of impurities in the sample). We assume a temperature low enough to neglect electron-phonon interaction.

We choose the vector-potential of the external magnetic field in the form $\mathbf{A} = (0, Hx, 0)$. Then the eigenvalues and the wave functions of the unperturbed Hamiltonian

$$\mathcal{H}_e = \pi^2/2m = \hbar\omega_c(a^+a + 1/2) + \pi_z^2/2m \tag{3}$$

are respectively

$$\varepsilon_\mu = \hbar\omega_c(N + 1/2) + p_z^2/2m, \tag{3a}$$

$$\psi_\mu(x, y, z) = L^{-1}\varphi_N(x - X)\exp\{i(p_z z\hbar^{-1} - Xyl^{-2})\} \tag{3b}$$

In these formulas a^+ and a are the Bose creation and annihilation operators, $\varphi_N(x)$ is the harmonic-oscillator wave function, $l = (\hbar c/eH)^{1/2}$ is the radius of the zeroth Landau orbit, X is the coordinate of the center of the electron cyclotron orbit, $\mu = (N, p_z, X)$ is the set of quantum numbers characterizing the state, and L is the dimension of the sample along the x axis. (The symbols + and * will henceforth denote Hermitian and complex conjugation, respectively.)

We express the electron coordinates in the (x, y) plane perpendicular to H in terms of the coordinates (X, Y) of the cyclotron orbit and the relative coordinates (ξ, η) of the cyclotron motion:[4]

$$X = x - \xi, \quad Y = y - \eta; \quad \xi = \frac{c\pi_y}{eH} = \frac{i}{\sqrt{2}}l(a - a^+),$$

$$\eta = -\frac{c\pi_x}{eH} = -\frac{1}{\sqrt{2}}l(a + a^+). \tag{4}$$

Further, following Miyake,[5] we expand $U(\mathbf{r})$ in powers of the relative coordinates and take into account terms up to second order inclusive. The Hamiltonian of the system is then written in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}', \quad \mathcal{H}_0 = \hbar\omega_c(a^+a + 1/2) + p_z^2/2m + U(X, Y, z),$$

$$\mathscr{H}' = \mathscr{H}'_{\text{ad}} + \mathscr{H}'_{\text{nonad}} \quad \mathscr{H}'_{\text{ad}} = (l^2/2)\,U_{+-}(a^+a + {}^1/_2),$$
$$\mathscr{H}'_{\text{nonad}} = (il/\sqrt{2})\,(U_+a - U_-a^+) - (l^2/4)\,(U_{--}a^{+2} + U_{++}a^2). \qquad (5)$$

We have introduced here the following abbreviated notation:

$$U_\pm = U_x \pm iU_y, \quad U_{+-} = U_{xx} + U_{yy}, \quad U_{\pm\pm} = U_{xx} \pm 2iU_{xy} - U_{yy}.$$

We proceed to consider the interaction of the system with an alternating field. Assume that the system is acted upon by two coherent pulses of an alternating electric field of durations Δt_1 and Δt_2 with an interval τ between pulses. The operator of the interaction of the electric dipole (p) and quadrupole (Q) moments of the system, induced by the external magnetic field, with the field $E = E_0 \cos(\omega t - ky)$, is given (in the occupation number representation) by

$$\mathscr{H}_1^{\text{dip}} = E_0 p_0 \sum_\mu \sqrt{N+1}\,(\nu_\mu{}^* c_N{}^+ c_{N+1} e^{i\omega t} + \nu_\mu c_{N+1}^+ c_N e^{-i\omega t}), \qquad (6a)$$

$$\mathscr{H}_1^{\text{dip}} = |\operatorname{grad} E|\,Q_0 \sum_\mu \sqrt{(N+1)\,(N+2)}\,(\alpha_\mu{}^* c_N{}^+ c_{N+2} e^{i\omega t} + \alpha_\mu c_{N+2}^+ c_N e^{-i\omega t}). \qquad (6b)$$

The quantities p_0 and Q_0 depend on the magnetic field intensity H; ν and α are complex numbers with unity modulus, and ω is the frequency of the alternating field. As a result of the action of the first pulse, the system of multipoles goes over into the superradiating state. Owing to the inhomogeneity of the magnetic or the local internal fields, the coherent multipoles go rapidly out of phase (it is assumed that the time of this reversible relaxation is much shorter than the time of relaxation on the impurities). The field inhomogeneity is characterized by the parameter $\Delta\omega = \omega_c - \omega_{c0}$, where ω_{c0} is the average cyclotron frequency for the system. Consequently, the Hamiltonian of the reversible relaxation is

$$\mathscr{H}_2 = \sum_\mu \hbar\Delta\omega_\mu (N + {}^1/_2)\,c_N{}^+ c_N, \quad \Delta\omega_\mu = \Delta\omega(X, Y, z). \qquad (7)$$

If the system is subjected to a second pulse, then at a time τ after this pulse the system of multipoles will radiate an echo signal. We note that the influence of the relaxation during the time of action of the pulses is neglected in the calculations.

The calculations show that if the interaction of the pulses with the sample is only of the dipole type, then signals of the induction type are possible and there are no echo signals. On the other hand, if the interaction is of the quadrupole type or if one pulse is dipole and the other is quadrupole, then signal echoes are also produced. Cases are possible, however, when an echo signal can appear after application of two pulses of an alternating field with equal carrier frequency ω_c. This may occur, first, in substances which are effectively capable of doubling the pulse carrier frequency. Second, this can happen in two-quantum excitations of the system by an interaction of the dipole type. The operator of such an interaction is of the form (6b), where $|\operatorname{grad} E|\,Q_0$ has been replaced by E_0p_0. The value of p_{0e} in two-quantum excitation is of the order of $p_0(p_0E_0/\hbar\omega_c)$ or $p_0(p_0E_0/\hbar\Delta\omega_c)$, where $\Delta\omega_c$ is the homogeneous line width. If k_1 is the wave vector of the field, then α_μ depends on the effective wave vector $k_0 = 2k_1$. On the other hand, if the excitation is by two alternating fields with $\mathbf{k}_1$ and $(-\mathbf{k}_1)$, we get $\mathbf{k}_e = 0$. Possible variants of the occurrence of induction and echo signals for the case of three pulses are listed in the table.

The first column indicates the types of interaction of the coherent external field with matter. The first group of lines corresponds to the case when the external-field pulses interact only with the electric quadrupole moment of the system. The second, third, and fourth groups of

Coherent Responses

Type of interaction	For quadrupole moment			For dipole moment		
	Instant of appearance	Wave vector	Response amplitude	Instant of appearance	Wave vector	Response amplitude
1. All pulses quadrupole.	0	$\mathbf{k}_1$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1$			
	τ	$\mathbf{k}_2$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_2\,\text{ch}\,4\theta_1$			
	2τ	$2\mathbf{k}_2 - \mathbf{k}_1$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1\,\text{sh}^2\,2\theta_2$			
	τ_1	$\mathbf{k}_3$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_3\,\text{ch}\,4\theta_1\,\text{ch}\,4\theta_2$			
	$\tau_1 + \tau$	$\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_1$	$(i/2)(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1\,\text{sh}\,4\theta_2\,\text{sh}\,4\theta_3$			
	$2\tau_1 - 2\tau$	$\mathbf{k}_1 - 2\mathbf{k}_2 - 2\mathbf{k}_3$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1 \cdot \text{sh}^2\,2\theta_2\,\text{sh}^2\,2\theta_3$			
	$2\tau_1 - \tau$	$2\mathbf{k}_3 - \mathbf{k}_2$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_2\,\text{sh}^2\,2\theta_3\,\text{ch}\,4\theta_1$			
	$2\tau_1$	$2\mathbf{k}_3 - \mathbf{k}_1$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1\,\text{sh}^2\,2\theta_3\,\text{ch}^2\,2\theta_2$			
2. First pulse dipole, second and third quadrupole.	0	$2\mathbf{k}_1$	$-\theta_1^2$	0	$\mathbf{k}_1$	$i\theta_1$
	τ	$\mathbf{k}_2$	$i[(a^+a + {}^1/_2)\,\text{sh}\,4\theta_2 + \theta_1^2\,\text{sh}\,4\theta_2]$	2τ	$\mathbf{k}_2 - \mathbf{k}_1$	$\theta_1\,\text{sh}\,2\theta_2$
	2τ	$2\mathbf{k}_2 - 2\mathbf{k}_1$	$\theta_1^2\,\text{sh}^2\,2\theta_2$	$2\tau_1 - 2\tau$	$\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_3$	$i\theta_1\,\text{sh}\,2\theta_2\,\text{sh}\,2\theta_3$
	τ_1	$\mathbf{k}_3$	$i[(a^+a + {}^1/_2)\,\text{ch}\,4\theta_2 + \theta_1^2\,\text{ch}^2\,2\theta_2]\,\text{sh}\,4\theta_3$	$2\tau_1$	$\mathbf{k}_3 - \mathbf{k}_1$	$\theta_1\,\text{sh}\,2\theta_3\,\text{ch}\,2\theta_2$
	$\tau_1 + \tau$	$\mathbf{k}_2 + \mathbf{k}_3 - 2\mathbf{k}_1$	$({}^1/_2)\,\theta_1^2\,\text{sh}\,4\theta_2\,\text{sh}\,4\theta_3$			
	$2\tau_1 - 2\tau$	$2\mathbf{k}_1 - 2\mathbf{k}_2 + 2\mathbf{k}_3$	$-\theta_1^2\,\text{sh}^2\,2\theta_2\,\text{sh}^2\,2\theta_3$			
	$2\tau_1 - \tau$	$2\mathbf{k}_3 - \mathbf{k}_2$	$i[(a^+a + {}^1/_2) + \theta_1^2]\,\text{sh}\,4\theta_2\,\text{sh}^2\,2\theta_3$			
	$2\tau_1 \{$	$2\mathbf{k}_3 - 2\mathbf{k}_1$	$\theta_1^2\,\text{sh}^2\,2\theta_3\,\text{ch}^2\,2\theta_2$			
		$\mathbf{k}_3$	$i\theta_1^2\,\text{sh}^2\,2\theta_2\,\text{sh}\,4\theta_3$			
3. First pulse dipole, first and third quadrupole.	0	$\mathbf{k}_1$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1$	τ	$\mathbf{k}_2$	$i\theta_2$
	τ	$2\mathbf{k}_2$	$-\theta_2^2$	$2\tau_1 - \tau$	$\mathbf{k}_3 - \mathbf{k}_2$	$\theta_2\,\text{sh}\,2\theta_3$
	τ_1	$\mathbf{k}_3$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_3\,\text{ch}\,4\theta_1$			
	$2\tau_1 - \tau$	$2\mathbf{k}_3 - 2\mathbf{k}_2$	$\theta_2^2\,\text{sh}^2\,2\theta_3$			
	$2\tau_1 \{$	$2\mathbf{k}_3 - \mathbf{k}_1$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1\,\text{sh}^2\,2\theta_3$			
		$\mathbf{k}_3$	$i\theta_2^2\,\text{sh}\,4\theta_3$			
4. Third pulse dipole, first and second quadrupole.	0	$\mathbf{k}_1$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1$	τ_1	$\mathbf{k}_3$	$i\theta_3$
	τ	$\mathbf{k}_2$	$i(a^+a + {}^1/_2)\,\text{sh}\,4\theta_2\,\text{ch}\,4\theta_1$			
	2τ	$2\mathbf{k}_2 - \mathbf{k}_1$	$(a^+a + {}^1/_2)\,\text{sh}\,4\theta_1\,\text{sh}^2\,2\theta_2$			
	τ_1	$2\mathbf{k}_3$	$-\theta_3^2$			

lines correspond to excitation of the system via different combinations of the dipole and quadrupole action. The second, third, and fourth columns contain parameters that pertain to excitation of the superradiating state of a system of electric quadrupole moments. The last columns indicate the corresponding parameters for a system of electric dipole moments. The third and sixth columns indicate the wave vectors relative to which the system turned out to be in the superradiating state as a result of the action of the external-field pulses. The last column (response amplitude) contains the diagonal terms in the expressions for the dipole moment $p^+(t) = a^+(t)$ and quadrupole moment $\mathbf{Q}^+(t) = [a^+(t)]^2$ (see below). The following notation is used: if an alternating-field pulse interacts with a dipole moment, then $\theta_\gamma = \hbar^{-1} E_0 p_0 \Delta t_\gamma$; for action on a quadrupole moment we have $\theta_\gamma = \hbar^{-1} |\mathrm{grad}\, E|\, Q_0 \Delta t_\gamma$.

Let us make a few remarks concerning the dependence of the echo signals on the direction of the wave vector of the exciting pulses. When all the pulses are of the quadrupole type, a response at the instant of time 2τ is possible only if $\mathbf{k}_1 \parallel \mathbf{k}_2$, since the lengths of the vectors $\mathbf{k}$, $\mathbf{k}_1$, and $\mathbf{k}_2$ are equal. If the first pulses of the dipole type, an echo from either the dipole or the quadrupole moment is likewise possible after two pulses, provided that $\mathbf{k}_1 \parallel \mathbf{k}_2$ ($|\mathbf{k}_2| = 2|\mathbf{k}_1|$). As seen from the table, when the first or second pulse is of the quadrupole type, then two quadrupole-echo signals are produced at the instant of time $2\tau_1$. One of them is possible when $\mathbf{k}_3 \parallel \mathbf{k}_1$, the other at any orientation of $\mathbf{k}_3$, and both responses merge when $\mathbf{k}_3 \parallel \mathbf{k}_1$.

We note that from the point of view of the theory the presence or absence of echo signals is connected with the fact[3] that the dipole-interaction operator gives rise to a resolvable Lie algebra, and the quadrupole-interaction operator to a nonresolvable algebra. As seen from the table, some induction and echo signals due to certain alternating field signals depend strongly on the values of the preceding pulses which do not generate the signals directly. For example, the intensity of the induction signal from the quadrupole moment of the system at $t = \tau$, when all the pulses are of the quadrupole type, depend on the value of the first pulse like $\cosh^2 4\theta_1$. The echo signal from the quadrupole moment at $t = 2\tau_1$, due to the first and third pulses of the quadrupole type, depends on the second pulse like $\cosh^4 2\theta_2$, etc. Thus, the echo-signal intensities can be increased by the action of the preceding pulses. We note that in case 1, when all the pulses are of the quadrupole type, the times of appearance of the signals and the corresponding wave vectors coincide exactly with the analogous results obtained for the spin operators.[6] As to the response amplitudes, they are obtained from the corresponding results for the spins by replacing the trigonometric functions by hyperbolic. This is the consequence of the fact that the operators a^{+2}, a^2, and a^+a form a Lie algebra of three terms, as do the spin operators S_+, S_-, and S_z. The commutation rules for these algebras differ only in the signs of the corresponding commutators, which is indeed the reason for the appearance of the hyperbolic functions in place of the trigonometric ones. In the cases 2–4, when the system is excited by a combination of pulses of the dipole and quadrupole type, the response of the system no longer duplicates the results of this type for the spin operators.

It is seen from the table that certain responses do not depend on the initial population of the energy levels. They recall the buildup of a classical oscillator under the influence of a driving force, and can therefore be called macroscopic quantum signals. Other signals, on the other hand, contain also parts which depend on the initial degree of excitation of the system, as is indeed the case upon excitation of ordinary spin-echo signals.

The intensity of the response signal is determined by the formula

$$I(t) = I_0(\mathbf{k})\,\mathrm{Sp}\{F^+(t)F(t)\rho_0\}, \qquad F(t) = [\mathscr{L}(t)]^{-1}F\mathscr{L}(t), \tag{8}$$

where F is the operator of the observable quantity; for the dipole and quadrupole moments of the system we have respectively

$$F = p = \sum_\mu \gamma_\mu^* \sqrt{N+1}\, c_N^+ c_{N+1}, \tag{9a}$$

$$F = Q = \sum_\mu \beta_\mu^* \sqrt{(N+1)(N+2)}\, c_N^+ c_{N+2}. \tag{9b}$$

The evolution operator $L = L(t)$ is determined by formulas (5)–(7), and the equilibrium density matrix is

$$\rho_0 = [\mathrm{Sp}\exp\{-\beta(\mathscr{H}_e - \zeta N_1)\}]^{-1} \exp\{-\beta(\mathscr{H}_e - \zeta N_1)\},$$
$$\beta = 1/k_B T, \tag{10}$$

where N_1 is the particle-number operator and where we neglect the electron-impurity interaction. $I_0(\mathbf{k})$ in (8) is the intensity of radiation of one multipole in the direction of the vector $\mathbf{k}$.

Let us examine in greater detail the quadrupole-moment response for the case when all pulses are of the quadrupole type. The evolution operator for a two-pulse echo is

$$\mathscr{L}(t) = \exp(-i\hbar^{-1}\mathscr{H}_0 t)\exp\left\{-i\hbar^{-1}\int_\tau^t [\mathscr{H}'(t') + \mathscr{H}_2(t')]\,dt'\right\}$$
$$\times \exp(-i\hbar^{-1}\Delta t_2 \mathscr{H}_1(t))\exp\left\{-i\hbar^{-1}\int_0^\tau [\mathscr{H}'(t') + \mathscr{H}_2(t')]\,dt'\right\}$$
$$\times \exp(-i\hbar^{-1}\Delta t_1 \mathscr{H}_1(t)),$$
$$\mathscr{H}(t) = \exp(i\hbar^{-1}\mathscr{H}_0 t)\,\mathscr{H}\exp(-i\hbar^{-1}\mathscr{H}_0 t). \tag{11}$$

Writing all the operators in the occupation-number representation and performing all the bounding for (9b), we obtain for the echo signal

$$I(t) = I_0(\mathbf{k})\,\mathrm{sh}^4 2\theta_2 \Big\{ \mathrm{sh}^4 2\theta_1 \sum_\mu (N+2)(N+1) f_{N+2}(1-f_N)$$
$$+ \mathrm{ch}^4 2\theta_1 \sum_\mu (N+2)(N+1) f_N (1-f_{N+2})$$
$$+ \mathrm{sh}^2 4\theta_1 \sum_\mu (N+{}^1/_2) f_N \beta_\mu \alpha_{1\mu} \alpha_{2\mu}^{*2} \langle \exp\{-i\Phi_{1\mu}\}\rangle \langle \exp\{i\Phi_{2\mu}\}\rangle$$
$$\times \exp\{i2(t-2\tau)\Delta\omega_\mu\} \sum_{\mu'} (N'+{}^1/_2) f_{N'} \beta_{\mu'}^* \alpha_{1\mu'}^* \alpha_{2\mu'}^2$$
$$\times \langle \exp\{i\Phi_{1\mu'}\}\rangle \langle \exp\{-i\Phi_{2\mu'}\}\rangle \exp\{-i2(t-2\tau)\Delta\omega_{\mu'}\}. \tag{12}$$

Here $\theta_\gamma = \hbar^{-1}|\mathrm{grad}\, E|\, Q_0 \Delta t_\gamma$, f_N is the Fermi distribution for the state (N, p_z, X), and the coefficients α_γ and β are given by

$$\beta_\mu = \beta_0 e^{i\mathbf{k}\mathbf{R}_\mu}, \quad \alpha_{\gamma\mu} = \alpha_{0\gamma} e^{i\mathbf{k}_\gamma \mathbf{R}_\mu}, \quad |\alpha_{0\gamma}| = |\beta_0| = 1,$$
$$\mathbf{R}_\mu = \mathbf{r} - \mathbf{r}_{\text{осн}} = (X, Y, z). \tag{13}$$

The quantities $\langle \exp\{\pm i\Phi_{12}\}\rangle$ are correlation functions that characterize the attenuation of the signal as a result of relaxation by the impurities; the angle brackets denote averaging over the impurities. As shown by Miyake,[5] for the case $r_c \ll a$ ($r_c \sim l$ is the dimension of the cyclotron orbit, a is the action radius of the impurity), the influence of H'_{nonad} in H' can be neglected compared with H'_{ad}. We then have

$$\langle \exp\{\pm i\Phi_{1\mu}\}\rangle = \left\langle \exp\left\{\pm i\int_0 \delta_t(X,Y,z)\,dt\right\}\right\rangle,$$

$$\langle \exp\{\pm i\Phi_{2\mu}\}\rangle = \left\langle \exp\left\{\pm i\int_\tau^{2\tau} \delta_t(X,Y,z)\,dt\right\}\right\rangle. \qquad (14)$$

Here

$$\delta_t(X,Y,z) = \sum_{j=1}^{N_I} \delta_{tj}(X,Y,z) = l^2 U_{t+-},$$

the index t denotes the time dependence resulting from the noncommutitivity of H_0 and H'_{ad}. Assuming that the modulation of the cyclotron frequency by the impurities is a Poisson process, we obtain in the approximation of the impact theory ($\tau \gg t_d$, where t_d is the characteristic time of modulation or the collision time)

$$\langle e^{i\Phi_{1\mu}}\rangle = \exp\left[n_I|v_z|\tau\int_0^\infty 2\pi b\,db\{\exp i\alpha(b,v_z)-1\}\right]$$

$$= \exp\{-n_I\tau|v_z|\sigma_1(v_z) + in_I\tau\sigma_2(v_z)\}, \qquad (15)$$

$$\sigma_1(v_z) = \int_0^\infty 2\pi b\,db[1-\cos\alpha(b,v_z)], \qquad (16)$$

$$\sigma_2(v_z) = \int_0^\infty 2\pi b\,db|v_z|\sin\alpha(b,v_z), \qquad \alpha(b,v_z) = \int_{-\infty} \delta_t(t)\,dt.$$

n_I is the impurity concentration, b is the impact parameter, and v_z is the electron velocity along the magnetic field.

It is seen from (12) that the intensity of the signal is maximal in the direction $\mathbf{k} = 2\mathbf{k}_2 - \mathbf{k}_1$. To calculate this intensity, let us consider the case of the ultraquantum limit $\hbar\omega_c \gg \zeta$ (or k_BT), when only the level $N = 0$ is populated:

$$I(t=2\tau) = I_0(\mathbf{k})n\,\mathrm{sh}^4 2\theta_2[2\mathrm{ch}^4 2\theta_1 + {}^1\!/_4\mathrm{sh}^2 4\theta_1 + {}^1\!/_4(n-1)\mathrm{sh}^2 4\theta_1\exp\{-4n_I\tau|v_z|\sigma_1(v_z)\}]. \qquad (17)$$

If we introduce the time of electron-impurity relaxation T_{2I}, then the expression for the coherent response becomes

$$I_{\text{coh}}(t=2\tau) = {}^1\!/_4 I_0(\mathbf{k})n(n-1)\,\mathrm{sh}^2 4\theta_1\mathrm{sh}^4 2\theta_2 e^{-4\tau/T_{2I}},$$
$$T_{2I} = 1/n_I|v_z|\sigma_1. \qquad (18)$$

We have calculated also the intensities of the echo signals after three pulses. Without giving the concrete calculations, we note only the following singularity of the stimulated-echo signal that appears at the instant of $\tau_1 + \tau$ ($\tau_1 \gg \tau$). It turns out that the impurity relaxation in the adiabatic approximation act effectively only during the time from zero to τ and from τ_1 to $\tau_1 + \tau$, and does not act in the interval between the second and third pulses (during the time $\tau_1 - \tau$). One can therefore expect the intensity of the stimulated echo to be of the same order as that of a two-pulse echo.

An analogous calculation of the two-pulse echo of the dipole moment of the system, when the first pulse is of the dipole type, yields

$$I(t) = I_0(\mathbf{k})\,\mathrm{sh}^2 2\theta_2\Big\{\sum_\mu (N+1)f_N(1-f_{N+1}) + \theta_1^2\sum_\mu f_N$$
$$+ \theta_1^2\sum_{\mu\neq\nu} f_\mu f_\nu \exp[i(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2)(\mathbf{R}_\mu-\mathbf{R}_\nu)]\exp[i(\Delta\omega_\mu-\Delta\omega_\nu)(t-2\tau)]$$
$$\times\langle e^{-i\Phi_{1\mu}}\rangle\langle e^{i\Phi_{2\mu}}\rangle\langle e^{i\Phi_{1\nu}}\rangle\langle e^{-i\Phi_{2\nu}}\rangle\Big\}. \qquad (19)$$

The correlation functions have the same form as before, with the exception of the fact that the quantities Φ are now half as large. As seen from (19), the coherent signal does not depend on the temperature and for $\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$ ($|\mathbf{k}_2| = 2|\mathbf{k}_1|$) it is equal to

$$I_{\text{coh}}(t=2\tau) = I_0(\mathbf{k})\,\mathrm{sh}^2 2\theta_2\theta_1^2 n(n-1)\exp\{-4\tau/T'_{2I}\}. \qquad (20)$$

Let us discuss the possibility of experimentally observing the phenomenon in question. To detect the signal it is necessary to satisfy the following conditions: $\Delta t_\gamma \ll T_2^* < \tau$, $\tau_1 < T_2$; T_2^* and T_2 are the times of reversible and irreversible relaxation, respectively. The main difficulty is raised by the smallness of the relaxation time T_2 of the electron pulse. To increase it is necessary to work at sufficiently low temperatures, so as to suppress the electron-phonon relaxation and to attain high purity of the samples. The use of strong magnetic fields should also lead to a lengthening of the relaxation time, since the nonadiabatic relaxation becomes ineffective in this case. With increasing magnetic fields, the localization of the carriers increases and the drift (diffusion) of the carriers decreases in both the longitudinal[4] and the transverse direction. As a result of the drift, the electrons fall into other local fields, which also leads to a loss of the phase memory and to a shortening of T_2. It was found in a recent paper[7] that in the infrared region of the spectrum of cyclotron resonance in n-InSb, the line becomes much narrower with increasing magnetic field intensity.

In conclusion we present estimates of the signals of dipole and quadrupole echo, using n-InSb as a sample. For a magnetic field $H = 2 \times 10^4$ G and an effective mass $m = 0.015m_0$ we have $\omega_c = 2 \times 10^{13}$ sec^{-1}. Let the pulse duration $\Delta t_\gamma \approx 10^{-11}$ sec.

1) Response of the quadrupole moment of the system. The quadrupole moment is $Q_0 = \hbar c/H \approx 1.5 \times 10^{-21}$ cgs esu. In order to have $\theta_\gamma = \hbar^{-1}|\text{grad } E|Q_0\Delta t \approx 1$ at a wavelength $\lambda_1 = \lambda/2 = 39.2 \times 10^{-4}$ cm, we need a field with amplitude $E_0 = 50$ cgs esu ($\lambda = 78.4 \times 10^{-4}$ cm is the emission wavelength of an H_2O laser). The radiation intensity of one quadrupole is $I_0 \approx \hbar^2e^2\omega^4/m^2c^5 \approx 10^{-13}$ erg/sec, and the total radiation intensity of the sample with volume $3 \times 3 \times 3$ mm^3 at a carrier density $n_0 = 10^{14}$ cm^{-3} is $I \approx n^2 I_0\lambda_1^2/S \approx 10^8$ erg/sec. Here S is the area of the end face of the sample.

2) Response of dipole moment of the system. The dipole moment is $p_0 = el = 8 \times 10^{-16}$ cgs esu. To have $\theta_1 = \hbar^{-1}E_0p_0\,\Delta t \sim 1$, we need a field with amplitude $E_0 \approx 0.1$ cgs esu. The radiation intensity of one dipole is $I_0 = \hbar e^2\omega_c^3/4\pi mc^3 \approx 5 \times 10^{-10}$ erg/sec, and then the emission intensity of a sample with the parameters given above is $I \approx n^2 I_0\lambda^2/S \approx 10^{12}$ erg/sec.

The foregoing estimates show that cyclotron echo at infrared frequencies can be observed experimentally in semiconductors. A study of this phenomenon can yield additional information on the interaction of the band carriers with one another and with lattice inhomogeneities. The theory proposed here can be generalized to describe cyclotron-echo signals of holes in p-semiconductors.

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