Some Features of the Phonon Spectra of Semimetals in Magnetic Fields

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The possibility is considered that in semimetals there may occur exciton-type excitations produced by interacting electrons from different Landau bands. Interaction between these excitations and phonons should lead to anomalies in the phonon spectra of metals.

1. We consider in this article a new type of resonant absorption and renormalization of the speed of sound, connected with multiparticle effects of electron-phonon interaction. In the study of the interaction of electrons and phonons in a magnetic field, one usually investigates cases of electron scattering within the limits of one Landau band or transitions between different filled bands, which lead to giant quantum oscillations of ultrasound absorption.[1,2] The influence of such processes on the renormalization of the Green's function of phonons was investigated in detail by Blank and Kaner.[3]

We shall show that processes of phonon absorption with transition of electrons from a filled Landau band into an empty one also give rise to singularities in the velocity and in the damping of the sound, and lead to the formation of new branches of the excitation spectrum of a semimetal. The effect under consideration depends strongly on the character of the interelectron interaction.

2. Let the momentum of the phonon $\mathbf{k}_0$ be such that upon its absorption the electron goes over from the Fermi level of a filled Landau band with number n to the bottom of an empty band with number n', i.e.,

$$k_{0z} = \pm p_n,$$
$$\omega(\mathbf{k}_0) = (n' - n)\omega_c - p_n^2/2m_z, \qquad (1)$$

where p_n is the Fermi momentum of the n-th band, $\omega(\mathbf{k})$ is the frequency of the phonon, and $\omega_c = eH/m_\perp c$. The z axis is directed along the magnetic field **H**. The conditions (1) can always be satisfied simultaneously by varying the phonon-momentum component that is perpendicular to the field. We shall show that when a phonon with momentum $\mathbf{k}$ close to $\mathbf{k}_0$ is absorbed, an important role is played by the final-state interaction between the electron produced in the band n' and the hole produced in the band n.

The contribution made by the processes under consideration to the polarization operator is determined, in the zeroth approximation in the electron-electron interaction, by the diagram of Fig. 1a, where the continuous lines correspond to the Green's functions of the electrons in bands n and n':

$$G_n^0(\varepsilon, p_z) = 1/[\varepsilon - \xi_n(p_z) + i\delta \operatorname{sign} \varepsilon].$$

It is easy to see that the contribution of diagram 1a to $\Pi(\mathbf{k}, \omega)$ is proportional to

$$L = \ln \frac{p_n^2}{(k_z - p_n)^2 + 2m(\Delta - \omega)}$$

where

$$\Delta = \omega(\mathbf{k}_0) = (n' - n)\omega_c - p_n^2/2m_z,$$

i.e., it becomes large at $k_z \approx p_n$ and $\omega = \Delta$. The contribution of diagram 1b is proportional to gL^2, where $g \approx e^2/\epsilon v_n$ is the small parameter of the perturbation theory in terms of the electron-electron interaction. We see that the large value of the logarithm can offset the smallness of the interaction constant. The same pertains also to the higher-order diagrams shown in Figs. 1c and 1d. The large logarithms are accumulated upon integration, when the electron momenta in bands n and n' are respectively $p \approx -p_n$ and $p \ll p_n$, while the energies are $\epsilon \ll p_n^2/m$ and $\epsilon \approx \Delta$. Under these conditions, all diagrams of the type shown in Fig. 2a, with a momentum transfer on the order of p_n, and of the type shown in Fig. 2b, with a total momentum on the order of $-p_n$ and a total energy on the order of Δ, become important. The polarization operator is given by the sum of the diagrams 1a and 1e, where the shaded rectangle is the sum of the nonrenormalized interaction and of all the diagrams that are reducible in accordance with the type of Fig. 2a or 2b.

3. Thus, the problem reduces to a calculation of the total vertex part of the electron-electron interaction $\Gamma(p, k, k')$ shown in Fig. 3. The indices p, k, and k' denote the aggregate of the corresponding frequencies, and also of the z and x components of the momenta (the wave functions are chosen in the Landau representation).

The conditions of perturbation theory in the interelectron interaction are assumed to be satisfied. In the case of a Coulomb interaction, this means that

$$e^2/\pi\epsilon v_n \ll 1,$$

where ϵ is the dielectric constant and v_n is the Fermi velocity in band n.

The temperature smearing and the impurity broadening of the Landau levels will be neglected. For simplicity, we shall assume the electron spectrum to be quadratic and isotropic in the (x, y) plane, and the masses m_z and $m_\perp$ accordingly equal in bands n and n'. The z components of the momenta in band n will be reckoned from $-p_n$, and the frequencies in band n' from Δ. Thus, all the momenta and frequencies in the region under consideration are small quantities.

In view of the presence of two singular channels, the vertex part can be calculated only in the logarithmic approximation, i.e., by retaining in the k-th order of per-

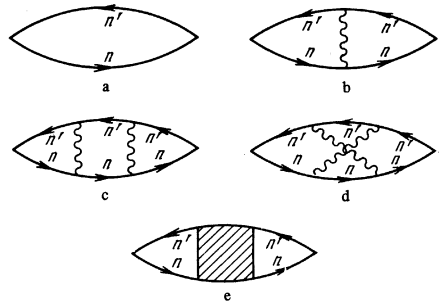

FIG. 1

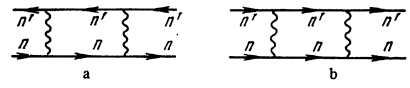

FIG. 2

turbation theory only terms of order $g(gL)^k$. Following Sudakov's method,[4] we separate in each diagram a cross section with a minimum integration momentum. We are interested in a region in which all the frequencies and squares of the z components of the momenta are of the same order. In this region, the vertex part depends (apart from the x components of the momenta) only on one logarithmic variable

$$\xi = \ln\frac{p_n^2}{\{-m\omega, k_z^2\}} = \ln\frac{p_n^2}{\{-m\omega, k_z^2\}}$$

where ω and k_Z, Ω, and q_Z are respectively the transferred and summary momenta and frequencies.

The equation for the vertex part is shown graphically in Fig. 3. The nonrenormalized vertex $E(k_Z', k_X, k_X')$ is the sum of the direct and exchange matrix elements. Here k_X is the momentum transfer from the band n to the band n', and k_X' and k_Z' are the changes of the electron momentum component in the band n. In the first matrix element we can put $k_Z' = 0$, and in the second $k_Z' = p_n$. (The case of a weakly screened Coulomb interaction that depends strongly on k_Z' will be discussed below.) Thus, $U = U(k_X, k_X')$. We emphasize the essential dependence of the nonrenormalized vertex on k_X, which is a consequence of the inhomogeneity of the system. This dependence greatly influences the character of the scattering in a channel of the type shown in Fig. 2b. Another property of the transverse motion is the fact that the total vertex, as well as the unrenormalized one, does not depend on p_X, this being the consequence of the independence of the zero Green's functions of the transverse momenta.

Taking the foregoing into account, the analytic expression for the vertex part is written in the form

$$\Gamma(k_x, k_x', \xi) = U(k_x, k_x') + \frac{1}{2\pi v_n}\int_0^{\xi} d\eta \int \frac{dk''}{2\pi}\Gamma(k_x, k_x''\eta)\Gamma(k_x, k_x' - k_x'', \eta)$$

$$-\frac{1}{2\pi v_n}\int_0^{\xi} d\eta \int \frac{dk''}{2\pi}\Gamma(k + k' - k'', k'', \eta)\Gamma(k - k'', k' - k'', \eta). \quad (2)$$

We multiply both parts of this equation by $\lambda^2 \exp\{i\lambda^2 k_y k_X'\}/2\pi$, where $\lambda^2 = \hbar c/eH$, and integrate

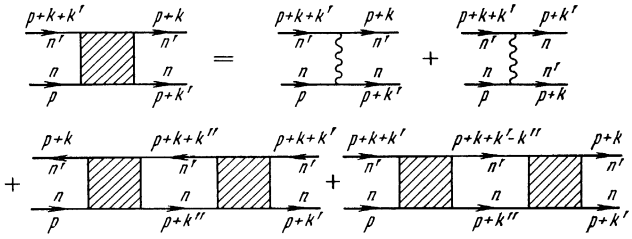

FIG. 3

with respect to k_X'. At the same time we introduce a new quantity $\Gamma(k_X, k_y, \xi)$, such that

$$\Gamma(k_x, k_x', \xi) = \int \Gamma(k_x, k_y) e^{-i\lambda^2 k_y k_x'} dk_y.$$

It is easy to verify that $U(k_X, k_y)$ depends only on $\rho = |\rho|$, where $\rho = (\lambda k_X, \lambda k_y)$. We seek the solution of (2) also in the form of a function of the variables ρ and ξ. After a number of transformations we obtain the following equation for $\Gamma(\rho, \xi) = (2\pi v_n \lambda^2)^{-1}\Gamma(k_X, k_y, \xi)$:

$$\Gamma(\rho, \xi) = U(\rho) + \int_0^{\xi} d\eta \Gamma^2(\rho, \eta)$$

$$-\int_0^{\xi} d\eta \int_0^{\infty}\int_0^{\infty} d\rho_1 d\rho_2 \Gamma(\rho_1, \eta)\Gamma(\rho_2, \eta) K(\rho, \rho_1, \rho_2) \quad (3)$$

where

$$K(\rho, \rho_1, \rho_2) = \rho_1\rho_2 \int_0^{2\pi}\int_0^{2\pi} \cos\{\rho\rho_1\cos\varphi_1 + \rho\rho_2\cos\varphi_2 + \rho_1\rho_2\sin(\varphi_1 - \varphi_2)\}\frac{d\varphi_1 d\varphi_2}{(2\pi)^2}. \quad (4)$$

The kernel $K(\rho, \rho_1, \rho_2)$ can also be represented in the form of a series in products of Bessel functions:

$$K(\rho, \rho_1, \rho_2) = \rho_1\rho_2 \sum_{k=-\infty}^{+\infty} J_{2k}(\rho\rho_1) J_{2k}(\rho\rho_2) J_{2k}(\rho_1\rho_2). \quad (5)$$

It is easy to verify that the vector $\lambda^{-1}\rho$ is an eigenvalue of the total-momentum operator $\hat{P}$ of the electron and hole (in our notation this is the momentum transferred by the electron from band n to the electron from band n'), introduced by Gor'kov and Dzyaloshinskiĭ:[5]

$$\hat{P} = -i\nabla_e - i\nabla_h + \frac{e}{c}[\mathbf{A}(\mathbf{r}_e) - \mathbf{A}(\mathbf{r}_h)] - \frac{e}{c}[\mathbf{H}, \mathbf{r}_e - \mathbf{r}_h]. \quad (6)$$

The components of this operator commute with the Hamiltonian of the interacting electrons and with one another. However, for two electrons in a magnetic field, unlike an electron and a hole, there is no total-momentum operator with commuting components even in the absence of interaction between them (as is the case also for one electron). This is the reason why the amplitude of scattering with electrons that conserve the total momentum (the third term in the right-hand side of (3) cannot be represented in the form of a product of amplitudes of consecutive scattering acts, unlike the amplitude of scattering with conserved momentum transfer. The kernel $K(\rho, \rho_1, \rho_2)$ describes in this case the interference of the eigenfunctions of the operator (6). The equation for the vertex part could be obtained immediately in the ρ representation in the general case of anisotropic dispersion laws and interaction laws and for an arbitrary gauge. To this end it would be necessary to expand all the two-particle Green's functions not in

products of single-electron wave functions, but in the eigenfunctions of the operator (6).

4. An important property of the kernel (4) is that the integrals of this kernel with respect to the variables ρ_1 or ρ_2 diverge at the upper limit. Consequently, the convergence of the integrals with respect to ρ_1 and ρ_2 in the right-hand side of (2) is ensured by the tendency of $\Gamma(\rho, \xi)$ to zero as $\rho \to \infty$ and for all ξ. Thus, for example, the nonrenormalized interaction is of the form

$$U(\rho) = gP(\rho^2)e^{-\rho^2/2}, \tag{7}$$

where P(z) is a polynomial of degree $n + n'$.

It is easy to see that Eq. (2) admits of the existence of pole solutions of the type

$$\Gamma(\rho, \xi) = f(\rho, \xi) / [\xi(\rho) - \xi],$$

where $\xi(\rho)$ is a certain function whose order is g^{-1} when $\rho \lesssim 1$. Such a solution corresponds to a bound state of an electron and a hole with transverse momentum $\lambda^{-1}\rho$, z-momentum $k_z \sim p_n$, a zero momentum along the field, and an energy

$$\omega = \Delta - C_1 \frac{p_n^2}{m} e^{-\xi(\rho)} + C_2 \frac{(k_z - p_n)^2}{m}, \tag{8}$$

where $C_1, C_2 \approx 1$. Since there are no grounds for assuming the energy of such an exciton to be independent of the transverse momentum, the function $\xi(\rho)$ is not equal to a constant and has a scale of variation on the order of unity. As a result, the pole singularities of $\Gamma(\rho, \xi)$ can be integrated with respect to ρ_1 and ρ_2 in the last term of Eq. (2). Separating the most divergent terms in Eq. (2) near the pole, we find that $a[\rho, \xi(\rho)] = 1$, i.e., as $\xi \to \xi_0$ we have

$$\Gamma(\rho, \xi) = 1 / [\xi(\rho) - \xi].$$

In addition, as $\xi \to \xi_0 = \min\{\xi(\rho)\}$ and at any value of ρ the function $\Gamma(\rho, \xi)$ has a branch point or a logarithmic singularity (the latter occurs if $\xi(\rho)$ reaches a minimum at $\rho = 0$).

We shall also calculate the polarization operator in a logarithmic approximation by the method of Larkin and Khmel'nitskiĭ.[6] To this end, we first determine the amplitude for the production of an electron-hole pair by a field $\exp\{i\mathbf{k}\cdot\mathbf{r} - i\omega t\} - \mathcal{F}(p_x, k_\perp, \xi)$. Separating the cross section with minimum z-momentum of integration, we obtain the equations shown in Figs. 4a and b. It is easy to verify that

$$\mathcal{F}(p_z, k_\perp, \xi) = \mathcal{F}(\rho, \xi) \exp\{i\lambda^2 k_y (p_x + k_x/2)\},$$

where $\rho = \lambda(k_x, k_y)$. The nonrenormalized vertex is

$$\mathcal{F}_0(\rho) = e^{-\rho^2/4} L_n^{n'-n}(\rho^2/2),$$

where $L_n^{\alpha}(z)$ is a Laguerre polynomial, $L_n^{\alpha}(0) = 1$.

In analytic notation we have

$$\mathcal{F}(\rho, \xi) = \mathcal{F}_0(\rho) + \int_0^{\xi} d\eta\, \Gamma(\rho, \eta)\mathcal{F}_0(\eta),$$

$$\Pi(\rho, \xi) = \frac{1}{2\pi\lambda^2 v_n} \int_0^{\xi} \mathcal{F}^2(\eta)\, d\eta.$$

At ξ close to $\xi(\rho)$ we use for $\Gamma(\rho, \xi)$ the expression (5) and obtain

$$\mathcal{F}(\rho, \xi) = \mathcal{F}_0(\rho) C / [\xi(\rho) - \xi], \tag{9}$$

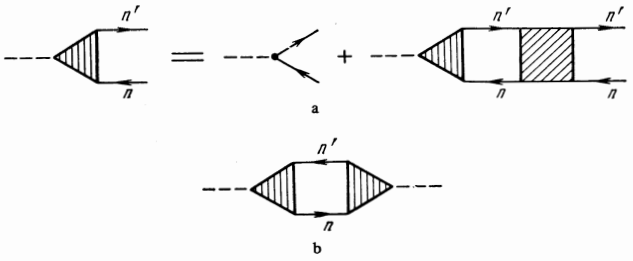

FIG. 4

where $C \approx 1$ and

$$\Pi(\rho, \xi) = C^2 \mathcal{F}_0^2(\rho) / [\xi(\rho) - \xi]. \tag{10}$$

Thus, as expected, the polarization operator has a pole as $\xi \to \xi(\rho)$.

We consider now the case of a Coulomb interaction for weak screening, i.e., when the screening radius r_0 is much larger than the magnetic length λ. If only the zeroth Landau band is filled, this condition coincides with the perturbation-theory condition, so that

$$\varkappa^2 = \lambda^2 / r_0^2 = 2e^2 / \pi\varepsilon v_0 \hbar \approx g \ll 1.$$

The direct matrix element of such an interaction is equal to

$$M(\varkappa, \rho) = g \int_0^{\infty} \frac{L_n(p^2/2) L_{n'}(p^2/2)}{p^2 + \varkappa^2 \varphi(\rho)} e^{-p^2/2} J_0(p, \rho)\, p\, dp,$$

where

$$g = \frac{e^2}{\pi\varepsilon v_n}, \quad \varkappa^2 = \lambda^2(k_z^2 + r_0^{-2}), \quad \varphi(0) = 1.$$

At small κ^2 and $\kappa\rho \ll 1$, the matrix element depends logarithmically on k_z, and can thus be included in the general scheme of logarithmic integration. As before, $\Gamma(\rho, \xi)$ satisfies Eq. (3), in which it is necessary only to replace $U(\rho)$ by $U(\rho, \xi)$ at $\xi < \delta = \ln \kappa^{-2}$.

The nonrenormalized interaction can be separated into two parts: $U(\rho, \xi) = g\varphi(\rho, \kappa) + \overline{U}(\rho)$, where $\overline{U}(\rho)$ is given by (7) and consequently has a scale of variation on the order of unity in terms of ρ, and decreases at infinity like $\exp(-\rho^2/2)$, while $g\varphi(\rho, \kappa)$ is the contribution of the scattering at large distances:

$$\varphi(\rho, \varkappa) = \int_0^{\infty} \frac{e^{-p^2/2} J_0(p\rho)}{p^2 + \varkappa^2} 2p\, dp = 2\left[K_0(\varkappa\rho) - Ei\left(-\frac{\rho^2}{2}\right)\right],$$

where $K_0(x)$ and $Ei(x)$ are respectively the Macdonald function and the integral exponential function

$$\varphi(\rho, \varkappa) \approx \begin{cases} \ln(2\gamma^3/\varkappa^2) & \rho \ll 1 \\ \ln(4\gamma^2/\varkappa^2\rho^2) & 1 \ll \rho \ll \varkappa^{-1} \\ e^{-\varkappa\rho}\sqrt{2\pi/\varkappa\rho} & \varkappa^{-1} \ll \rho \end{cases}.$$

Thus, the long-range part of the Coulomb scattering differs from the other contributions to the nonrenormalized interaction, first, in the large value at $\rho \ll 1$ [$\varphi(0, \kappa) \approx g \ln \kappa^{-2} \approx g \ln g^{-1}$], and second, in the slow decrease as $\rho \to \infty$ [$\exp(-\rho\kappa)$ as against $\exp(-\rho^2/2)$]. We shall show presently, however, that this contribution to the scattering is exactly cancelled out in all orders of perturbation theory above the first. In fact, separating one of the $n - 1$ integrals with respect to ρ_1 and ρ_2 in the aggregate of diagrams of n-th

order, we see that the squares of the Coulomb nonrenormalized terms always enter in the combination

$$\varphi^2(\rho,\varkappa)-\int_0^\infty\int_0^\infty\varphi(\rho_1,\varkappa)\varphi(\rho_2,\varkappa)K(\rho,\rho_1,\rho_2)\,d\rho_1\,d\rho_2, \qquad (11a)$$

and the first powers in the combination

$$\varphi(\rho)f(\rho)+\int_0^\infty\int_0^\infty f(\rho_1)\varphi(\rho_2)K(\rho,\rho_1,\rho_2)\,d\rho_1\,d\rho_2, \qquad (11b)$$

where $f(\rho)$ is a function of the type (7).

Since the integrals of the kernel diverge logarithmically at large ρ_i, and the functions $\varphi(\rho,\kappa)$ cut them off at $\rho_i\sim\kappa^{-1}\gg 1$, the second terms of (11a) and (11b) are proportional to δ^2 and δ, respectively. By exact calculation of the integrals it can be verified that in both expressions the terms with $\varphi^2(\rho)$ and $\varphi(\rho)$ cancel each other exactly for all ρ, and the remainders, in which we can put $\kappa=0$, have the same properties as $\bar{U}(\rho)$. To this end, it is convenient to represent $\varphi(\rho,\kappa)$ in the form

$$\varphi(\rho,\varkappa)=\int_1^\infty\exp\left\{-\alpha\frac{\varkappa^2}{2}-\frac{\rho^2}{2\alpha}\right\}\frac{d\alpha}{\alpha},$$

and the kernel in the form (5).

In expression (11a), the integration with respect to ρ_i and the summation over k are carried out in elementary fashion, and expression (11b) after integration with respect to ρ_2 and α and summation over k reduces to the form

$$\varphi(\rho,\varkappa)f(\rho)-\int_0^\infty f(\rho_1)K_\varkappa(\rho,\rho_1)\,d\rho_1,$$
$$K_\varkappa(\rho,\rho_1)=2\rho_1 z^{-1/4}\exp(-z^{1/2}/2), \qquad (12)$$

where

$$z=(\rho^2-\rho_1^2)^2+2\varkappa^2(\rho^2+\rho_1^2)+\varkappa^4.$$

We can easily see that in the integral (12) an appreciable contribution is made by the region $\rho_1\approx\rho$. The slowly varying function $f(\rho)$ can be taken outside the integral sign, and the remaining integral yields (accurate to within the remainder indicated above) the function $\varphi(\rho,\kappa)$.

Thus we see that the long-range part of the Coulomb interaction does not take part in the formation of the bound state considered by us. It does, however, exert a strong influence on the behavior of the amplitude $\mathcal{T}(\rho,\xi)$ and of the polarization operator. In fact, $\Gamma(\rho,\xi)$ can now be represented in the form

$$\Gamma(\rho,\xi)=g\varphi(\rho,\varkappa)+\Gamma_1(\rho,\xi),$$

where the first term is of the order of $g\delta_0$ and can noticeably exceed g, and $\Gamma_1(\rho,\xi)$ is a solution of the equation with a nonrenormalized term of the form $\bar{U}(\rho)$, i.e., it is expanded in powers of $g\xi$, and as $\xi\to\xi(\rho)\approx g^{-1}$ it has a pole with a residue equal to -1.

Further, in the calculation of $\mathcal{T}(\rho,\xi)$ by means of the formula

$$\mathcal{T}(\rho,\xi)=\mathcal{T}_0(\rho)\exp\left\{\int_0^\xi\Gamma(\rho,\eta)\,d\eta\right\}$$

we are permitted to retain in $\Gamma(\rho,\xi)$, besides the pole terms, also the term $g\varphi(\rho,\kappa)$, and $\mathcal{T}(\rho,\xi)$ acquires an additional large factor (when $\xi\to\xi(\rho)$)

$$e^{g\delta_0\xi}=\varkappa^{-2g\xi}. \qquad (13)$$

The pole term of the polarization operator is now equal to

$$C^2\mathcal{T}_0^2(\rho)\varkappa_0^{-4g\xi(\rho)}/[\xi(\rho)-\xi], \qquad (14)$$

i.e., the residue $\Pi(\rho,\xi)$ can be very large.

Notice should be taken of the connection between this result and the work of Nozieres et al.,[7] who solved the problem with complete compensation of the logarithms in $\Gamma(\xi)$, which led to exponentially increasing solutions for $\mathcal{T}(\xi)$ and $\Pi(\xi)$. In our case, in the presence of two types of interaction, one of which is relatively large, but is cancelled out in the higher orders of perturbation theory, we observe a combination of solutions of the Nozieres type (13) and of pole solutions due to the existence of interactions of the second type.

Unfortunately, the logarithmic accuracy of the calculations does not make it possible to determine the imaginary parts of the investigated two-particle Green's functions, and consequently we cannot determine whether their poles correspond to excitations of the exciton type or to quasistationary states.

5. Greatest interest attaches to a situation in which the excitation spectrum (8) intersects the phonon spectrum, i.e., when the phonon frequency $\omega(p_n,\lambda^{-1}\rho)$ is close to Δ. In this case the vertex part $\Gamma(p,k,k')$ depends strongly on k_z and ω, owing to the exchange interaction via the phonon. We can, however, eliminate diagrams that are reducible with respect to the phonon line, representing $\Gamma(p,k,k')$ in the form

$$\Gamma(p,k,k')=g_\phi^2(\rho)D(\mathbf{k},\omega)+\tilde{\Gamma}(p,k,k'),$$

where $g_{ph}(\rho)=g_{ph}\mathcal{T}_0(\rho)/2\pi v_n\lambda^2$, $D(\mathbf{k},\omega)$ is the total Green's function of the phonon, and the last term depends only on the logarithmic variables, since the functions $D(\mathbf{k},\omega)$, inserted as vertices into diagrams of the type shown in Fig. 2b, can be integrated with respect to the transverse momenta $\lambda^{-1}\rho$ and consequently depend little on ω and k_z. $\tilde{\Gamma}(\rho,\xi)$ is determined by an equation of the type (3), to the right-hand side of which it is necessary to add the term

$$\int_0^\xi d\eta\int_0^\infty\int_0^\infty d\rho_1\,d\rho_2\,g_\phi^2(\rho_1)D(\lambda^{-1}\rho_1)[\tilde{\Gamma}(\rho_2\xi)+g_\phi^2(\rho_2)D(\lambda^{-1}\rho_2)].$$

Obviously, this term does not influence the character of the singularity of $\tilde{\Gamma}(\rho,\xi)$ and formulas (9), (10), or (14) remain in force ($\Pi(\rho,\xi)$ and $\mathcal{T}(\rho,\xi)$ are also irreducible with respect to phonon lines).

The phonon Green's function satisfies the Dyson equation

$$D(\mathbf{k},\omega)=D_0(\mathbf{k},\omega)+g_\phi^2D_0(\mathbf{k},\omega)\Pi(\rho,\xi)D(\mathbf{k},\omega).$$

The polarization operator $\Pi(\rho,\xi)$ near the pole can be written in the form

$$\Pi(\rho,\xi)=a(\rho)/[\xi(\rho)-\xi],$$

where the function $a(\rho)$ is proportional to the small electron-phonon coupling constant g_{ph}^2. In the case of a Coulomb interaction, this small quantity can be compensated by the large factor $\kappa\exp-4g\xi(\rho)$. In addition, $a(\rho)\to 0$ as $\rho\to 0$ and as $\rho\to\infty$, owing to the factor $\mathcal{T}_0(\rho)$.

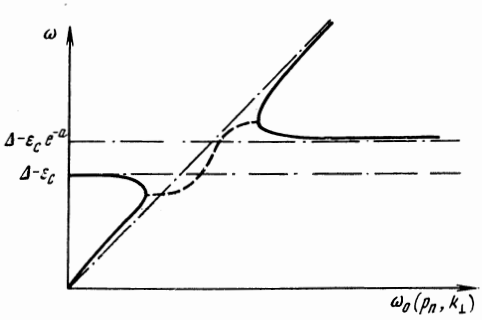

FIG. 5

For the phonon spectrum near the intersection point we obtain the equation

$$\frac{\omega^2}{\omega_0^2(k)} = 1 + a(\rho) \Big/ \left[\xi(\rho) - \ln \frac{p_n v_n}{\{\Delta - \omega, (k_z \pm p_n)^2/m\}} \right]. \quad (15)$$

We see from this equation that at $k_z \approx \pm p_n$ and for arbitrary $k_\perp$, a new undamped branch of the spectrum appears, corresponding to excitation of an exciton by a phonon. It consists of two parts, (A – A′) and (B – B′), which are located respectively at $\omega \leq \Delta - \epsilon_c$ and at $\omega \geq \Delta - \epsilon_c e^a$, where $\epsilon_c \approx p_n v_n \exp - \xi(\rho)$ is the binding energy of the exciton. The dashed line in Fig. 5 shows a plot of Re ω against $k_\perp$ for the complex solution of (15). In constructing this solution, we used the assumption that the variable $\xi(\omega) = \ln [\epsilon_n/(\Delta - \omega)]$ can be continued analytically into the complex ω plane with a cut along a line $\omega < \Delta$, which, generally speaking, is outside the framework of the logarithmic approximation. In such a method of analytic continuation, all the solutions of (15) lie on the physical sheet in the lower half-plane of the complex ω plane, i.e., they correspond to real damped excitations.

Near the branch points of the spectrum (points A and B in Fig. 5), Im ω increases in square-root fashion on the damped branch, starting with zero. Consequently, at fixed points one should observe a threshold singularity in the absorption of sound. The undamped excitations in the vicinities of these points are characterized by large transverse velocity. At the branch points themselves, the real solutions of (15) correspond to an infinitely large velocity of the excitations, as can be seen from Fig. 5. However, when account is taken of the damping of the excitations as a result of collisions and decay, their velocity should become finite (but greatly exceeding the nonrenormalized velocity of sound), and should change jumpwise on going from one branch to the other. Apparently, measurement of the transverse group velocity of sound can serve as the least complicated experimental verification of the existence of the excitations in question.

We recall that by virtue of the condition $a(0) = 0$, the effects under consideration vanish for sound considered along the magnetic field. The anomalies in the acoustic spectrum are most noticeable when $a(k_\perp)$ reaches the maximum $a_{max} \sim \epsilon g_{ph}^2 \nu_n / e^2$, i.e., when $k_\perp \sim \lambda^{-1}$. Then the width $\Delta\omega_0$ of the strong-damping region (the distance between the points A and B) is of the order of $\sqrt{a \epsilon_c u p_n}$, where u is the speed of sound.

In conclusion, let us consider the region of existence of such excitations. Allowance for the nonzero temperature and for the finite lifetime of the electrons leads to an additional cutoff of all the logarithmic integrations at momenta on the order of T/v_n and h/l, where l is the mean free path of the electron, whereas for the occurrence of a singularity in the vertex $\Gamma(x)$ it is necessary to carry out integration up to momenta on the order of ϵ_c / v_n. Consequently, the bound state exists when $T < \epsilon_c$ and $l > v_n \hbar / \epsilon_c$.

To estimate ϵ_c, we note that in the magnetic-field region of interest to us, $H \sim 10^3$–10^4 Oe, it follows from the conditions (1) that $\Delta \ll \hbar\omega_c$, i.e., these conditions are satisfied before the start of filling of the band n′. Consequently, $p_n \sim \hbar\lambda^{-1}$, $g \approx \lambda/R$, and $\epsilon_c \sim \hbar^2 \times \exp\{-R\lambda^{-1}\}/m_\perp \lambda^2$, where R is the effective Bohr radius $R = \epsilon\hbar^2/m_\parallel e^2$. Thus, for example, in bismuth, where according to [8] $m_\perp \sim m_\parallel \sim 0.01m$, $\epsilon \sim 100$, i.e., $R \sim 10^{-4}$ cm and at $g = 1/5$ we have $\epsilon_c \approx 10^{-4}$ eV, and consequently the temperature should be $T \lesssim 1°$ and the mean free path $l \gtrsim 10^{-3}$ c. Here $H \sim 10^3$ Oe and the sound frequency is $\omega \sim 10^9$–10^{10} Hz. In other semimetals, for example in antimony,[9] $m_\perp \sim 0.1m$ and $\epsilon \approx 10$, i.e., $R \sim 10^{-5}$ cm, and at the same value $g = 1/5$ we have $\epsilon_c \approx 10^{-3}$ eV, $H \sim 10^4$ Oe, $\omega \sim 10^{10}$–10^{11} Hz, and we should have $T \lesssim 10°$ and $l \gtrsim 10^{-4}$ cm.

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252