Resonances in a Nucleon-Antinucleon System

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Regge trajectories, masses, and elastic and total resonance widths are calculated for a nonrelativistic $N\bar{N}$ system. The results indicate two distinctive quasinuclear characteristics of heavy mesons: (a) large partial widths (of the order 10–30 MeV) for the $N\bar{N}$ decay channel, and (b) an upper limit of the resonance spectrum at masses of the order 2.0–2.3 GeV due to rapid increase of the annihilation probability at small distances.

1. INTRODUCTION

We have previously[1-4] considered the nonrelativistic (quasinuclear) bound states of a $N\bar{N}$ system. The energy spectrum of such states, calculated with the Bryan-Phillips (BP) potential, which describes satisfactorily the data on $N\bar{N}$ interactions,[5] is given in[3,4]. This spectrum contains 17 states with binding energies $\epsilon = 2m - M$ [m and M are the masses of the nucleon and the boson $(N\bar{N})$] lying in the range 25–590 MeV (which corresponds to M = 1289–1855 MeV), and with annihilation widths varying from 60 to 150 MeV. We have the boson spins $J \leq 3$; the orbital angular momentum of relative $N\bar{N}$ motion is $l \leq 4$.

In the present article we study the resonance spectrum in the $N\bar{N}$ system $(M > 2m)$. We again limit ourselves to nonrelativistic states, which alone admit the potential approximation. We emphasize that only within this framework is it meaningful to speak of a boson constituted by $N\bar{N}$.

The system will be regarded as nonrelativistic if it satisfies two conditions: (a) radius $R > 1/m$ (assuming $\hbar = c = 1$), and (b) $|M - 2m| \ll 2m$. It will be seen subsequently that these conditions are realized in both bound and resonance $N\bar{N}$ states.

In the present work the energy spectrum of $N\bar{N}$ resonances is obtained by calculating Regge trajectories J(M) for the potential problem. The $N\bar{N}$ system possesses a total of 8 trajectories, corresponding to nodeless radial wave functions (which alone are of interest here, because all bound states possess no radial nodes).

Each trajectory corresponds to a definite isospin (I = 0, 1), a combined particle spin (S = 0, 1) and a given vector difference between the total and orbital angular momenta $(S' = 0, \pm 1)$ for S = 1. The mass of a resonance was obtained from the condition Re J(M) = n, where n is an integer. The imaginary part of the angular momentum determines the resonance width $\Gamma_{N\bar{N}}$ for decay in the elastic channel (into N and $\bar{N}$):

$$\Gamma_{N\bar{N}} = 2\,\mathrm{Im}\,J \Big/ \frac{dJ}{dM}. \tag{1}$$

We have, in order of magnitude,

$$\Gamma_{N\bar{N}} \approx 1 / mR^2, \tag{2}$$

which for $R \approx 1/\mu$ (μ is the pion mass) gives $\Gamma_{N\bar{N}} = 10$ MeV. This comprises about 10% of the annihilation width of the level. This large partial width for decay in the $N\bar{N}$ channel is a distinctive characteristic of quasinuclear bosons. In the case of a boson with mass $M > 2m$, for which the $N\bar{N}$ channel is not at all prominent, $\Gamma_{N\bar{N}}$ should be of the order 0.1–1%.[1)]

The annihilation width Γ was calculated for resonance states, just as for the bound states, from the formula

$$\Gamma = v\sigma_a\overline{|\psi(0)|^2}, \tag{3}$$

where v is the relative velocity of the particles, σ_a is the annihilation cross section, and $\overline{\psi(0)^2}$ is the mean density in the annihilation region. The linear size of this region, $r_0 \approx 1/m$ (see[6]), i.e., according to condition (a), is much smaller than the radius of the system. This last circumstance makes it possible to use (3), which is valid with the accuracy $m|f_a|(r_0/R)^2$, where f_a is the scattering amplitude derived from the annihilation diagrams. For $f_a \approx 1/\mu$ and the given values of r_0 and R, the correction of (3) will be, in order of magnitude, $\mu/m \approx 15\%$. When (3) was used in the case of the resonance states, the ψ function was normalized in a volume of finite radius:

$$\int_0 r^2 dr \int |\psi|^2 d\Omega = 1.$$

The radius R was based on the condition of equality between the elastic widths $\Gamma_{N\bar{N}}$ as calculated from the Regge formula (1) and also directly from the particle current passing through a sphere about the coordinate origin:

$$\Gamma_{N\bar{N}}(R) = \oint \mathbf{j}\,ds \Big/ \int |\psi|^2 dV = 2\,\mathrm{Im}\,J \Big/ \frac{dJ}{dM}, \tag{4}$$

where j is the current density.

The results of the calculations are presented below.

2. RESONANCE SPECTRUM

In our version of the BF potential, both the potential and the centrifugal barrier were cut off at distances under 0.6 F. In this way two facts were reconciled—the small radius of the annihilation region and the equal contributions to the annihilation cross section from waves with different orbital momentum $(l \leq 3)$. The cutoff of the centrifugal barrier at small distances is equivalent to introducing l-dependent attractive forces.

1) This estimate is based on a comparison of the phase volumes for different decay channels.

For these potentials the growth of Re J(M) with M is unbounded, but Im J(M), beginning at some value M_{max}, diminishes and approaches zero as $M \to \infty$. The point M_{max}, where Im J(M) attains its maximum, corresponds to a ψ function concentrated mainly in a region lacking centrifugal repulsion, i.e. at small distances. It is therefore clear that the behavior of the trajectories and also the level spectrum for $M \geq M_{max}$ are of hardly any physical interest because of the strong dependence on the form of the potential at small distances. In actuality the upper limit of the resonance spectrum is under M_{max}. Even for a relatively small (absolute) value of $\overline{|\psi(0)|^2}$ the annihilation width is too large; we recall that it reaches about 100 MeV for bound states.[3,4] Therefore resonances in the $N\overline{N}$ system disappear through annihilation even before its state becomes relativistic because condition (a) is violated.

As already mentioned, the wave functions of $N\overline{N}$ bound states possess no radial nodes. Therefore in calculating the Regge trajectories it was permissible for us to solve the Schrödinger equation for the logarithmic derivative of the wave function rather than for the wave function itself. We thus avoided, to a considerable degree, the instability of a numerical calculation that is associated with an exponential boundary condition for the wave function at infinity.[2)] The boundary conditions for the logarithmic derivative are

$$\left.\frac{\chi_l'(r)}{\chi_l(r)}\right|_{r\to\infty} = ik, \quad \left.\frac{\chi_l'(r)}{\chi_l(r)}\right|_{r=a} = k \operatorname{ctg} ka \qquad (5)$$

where a = 0.6 F is the potential radius cutoff and k is the wave vector. The equations were integrated by the Runge-Kutta method. The desired function J(M) was obtained by setting the boundary condition on the right side and minimizing the solution with respect to the complex parameter l (to satisfy the left-side boundary condition). The results are shown in Figs. 1 and 2 and in the table. Figure 1 shows one of the Regge trajectories (I = 1, S = 0). We observe that the trajectory differs markedly from a straight line; the behavior of the Re J(M) and Im J(M) curves is in accordance with the foregoing general considerations. We note that the P and G parities are not conserved along the Regge trajectories for the potential approximation. When relativistic corrections corresponding to crossing symmetry are introduced, each trajectory of the type shown in Fig. 1 splits into two trajectories with unchanged P and G.

Figure 2 contains graphs of $|\chi(r)|^2$, where $\chi(r)$ is the radial part of $r\psi(r)$. The solid curve corresponds to the $^1p_1$ bound state with 1815-MeV mass (63 MeV binding energy); the dashed curve corresponds to a resonance in the $^1d_2$ state with 1955-MeV mass; the annihilation region is shaded. This figure shows that, despite the 140-MeV difference between the masses, there is little difference between the particle densities for the two states within the range of the forces. This indicates that uncertainties in the potential as a result of insufficient experimental data on the $N\overline{N}$ interaction can hardly change the basic result, which is the conclusion that resonances clearly exist in the $N\overline{N}$ system.

The spectrum of resonances having total widths $\Gamma_{tot} \leq 400$ MeV is given in the table. We observe herein

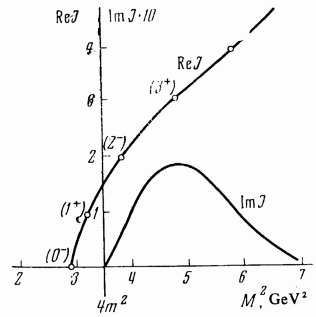

FIG. 1

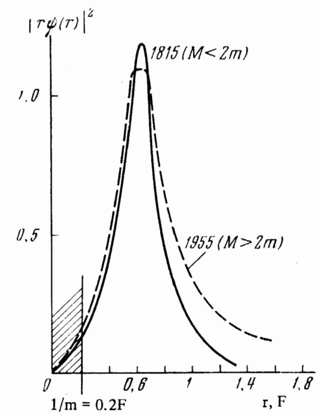

FIG. 2

FIG. 1. Nonrelativistic Regge trajectories with I = 1 and S = 0.

FIG. 2. Wave functions for the $^1p_1$ bound state (1815) (solid curve) and the $^1d_2$ resonance state (1955) (dashed curve) in the $N\overline{N}$ system. The functions are normalized to $|r\psi(r)|^2 = 1$ for r = 0.6F. The annihilation region is shaded.

that the model predicts at least three resonances having relatively large partial widths $\Gamma_{N\overline{N}}$, in agreement with the previous qualitative estimate. We note that the annihilation widths Γ for the resonance states are larger, on the average, than for the bound states[3,4] this results from the increase of $\overline{|\psi(0)|^2}$. It must also be emphasized here that the table does not give final values of Γ_{tot} even within the framework of the given quasinuclear model. Indeed, $\overline{|\psi(0)|^2}$ is strongly influenced by the height of the centrifugal barrier at the boundary of the annihilation region. The given calculations are like the earlier calculations of bound states in[3,4], which were based on the hypothesis that the centrifugal forces are totally compensated by attraction at sufficiently small distances. Because of the velocity-dependent terms in the Hamiltonian [such as $\mathbf{p}^2V(r) + V(r)\mathbf{p}^2$, where $\mathbf{p}$ is the momentum operator] this compensation may not be maintained in the transition from bound to resonance states. Calculations show that a centrifugal barrier at small distances induces a relatively small (30—50 MeV) shift of the levels, but strongly influences the magnitude of $\overline{|\psi(0)|^2}$, changing the annihilation widths by an order of magnitude. Therefore the appearance of even a small effective repulsion at small distances can change Γ appreciably.[3)]

At the present time the meagerness of the available data on $N\overline{N}$ annihilation prevents us from determining the energy dependence of the cross section for partial waves with $l \neq 0$. Consequently, the values of Γ given in the table should be regarded as only the possible upper limits of the annihilation widths of quasinuclear resonances in the $N\overline{N}$ system.

3. CONCLUDING REMARKS

We shall now summarize the most important qualitative predictions of the quasinuclear model that are derived from the foregoing results.

1) Boson resonances in the mass region 1880 < M

[2)]This is the method of N. N. Meĭman, who used it in[8] to investigate the zeros of Bessel functions.

[3)]It should also be remembered that the experimental value of the annihilation cross section which was used to evaluate Γ from (3) is clearly larger than the value determined from pure annihilation diagrams.

Spectrum of resonance states in the $N\bar{N}$ system

Spectroscopic	$I^G(J^P)$	M, MeV	$\Gamma_{N\bar{N}}$, MeV	Γ, MeV	Γ_{tot}, MeV	Spectroscopic	$I^G(J^P)$	M, MeV	$\Gamma_{N\bar{N}}$, MeV	Γ, MeV	Γ_{tot}, MeV
$^1d_2$	$1^-(2^-)$	1955	28	168	196	$^3d_3$	$1^+(3^-)$	2025	122	>300	>400
	$0^+(2^-)$	1930	15	136	151		$0^-(3^-)$	1880	0.0	203	203
$^3d_2$	$1^+(2^-)$	1925	10	148	158	$^3f_3$	$1^-(3^+)$	2165	76	>300	>400
	$0^-(2^-)$	$<2m$					$0^+(3^+)$	1880	0.0	311	311

< 2300 MeV should possess isospins $I \leq 1$ and spins $J \leq 3$.

2) The tabular data indicate that the mass region near twice the nucleon mass should contain at least three boson resonances with large partial widths $\Gamma_{N\bar{N}}$. These resonances should be revealed in elastic backward scattering of $\bar{N}$ by nucleons, and possibly in the total cross section for the interaction. The available experimental data, although not precise ($\Gamma_{N\bar{N}}$ has not been measured) do not conflict with this conclusion. In two instances (1925 and 1945 MeV) within the mass range 1925–2380 MeV, structure has been observed in the energy dependence of the backward elastic scattering cross section; in three instances the total cross section reveals structure.[7]

3) The increase of $\overline{|\psi(0)|^2}$ with the resonance mass, despite the aforementioned uncertainties in the potential of the interaction between N and $\bar{N}$, prevents the existence of resonances in $N\bar{N}$ systems having masses exceeding 2200–2300 MeV. Heavy mesons in the so-called X region (2500–3500 MeV) can hardly be constituted by $N\bar{N}$, and thus should have a small ($\ll 10$ MeV) partial width for decay in the $N\bar{N}$ channel. Several considerations indicate that bound quasinuclear states of a $2N2\bar{N}$ system probably exist in the X region.

4) The cross section for the production of quasinuclear mesons in $\pi + N \rightarrow X + N$ processes should decrease rapidly with increase in the energy of the colliding particles, because of a reduced fraction of the phase volume corresponding to the formation of a few correlated particles with small (nonrelativistic) relative velocities.

We emphasize, in conclusion, that, as in the case of bound states, the error in calculating resonance masses can be of the order of Γ when annihilation effects are neglected.[3,4] Variations in the form of the potential can also change the locations of some levels. At the same time there would be hardly any change in the basic qualitative results of the calculations—the number of resonances, their region of localization on the energy axis, and the orders of magnitude of elastic and annihilation widths.

The weakest point of the entire model is the assumption of a small annihilation radius. Although this assumption is derived necessarily from contemporary theoretical ideas and is confirmed by the experimental ratio of annihilation and elastic scattering cross sections, we must still not entirely exclude the existence of an annihilation "tail" encompassing a spatial region that is larger than has been assumed. However, this question can be answered only experimentally. Through experimental tests of predictions based on the quasinuclear model of boson resonances considerable additional information would be obtained about the characteristics of nucleon-antinucleon interactions at nonrelativistic energies.

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