QUANTUM MAGNETO-ACOUSTIC OSCILLATIONS IN METALLIC FILMS

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Quantum oscillations of ultrasound moving along a metallic film in a parallel magnetic field H are investigated. It is shown that at field strengths $H \lesssim H_c$, for which the Larmor radius is of the order of or greater than the film thickness, characteristic features appear in the ultrasonic absorption in the films. Measurements of the period and amplitude of the quantum magneto-acoustic oscillations yield certain information on the electron energy structure of the metallic films.

1. INTRODUCTION

At low temperatures, when the length of the electronic free path l is much greater than the acoustic wavelength $\lambdabar$, the ultrasonic damping in the metal can be regarded as the direct absorption of phonons by the conduction electrons. The principal role in the absorption is played by electrons whose velocity v_κ in the direction of the acoustic wave vector κ is equal to the sound speed s.[1] Quantization of the energy of the electron in the magnetic field H (the Larmor radius $r_H < \lambdabar$) leads, as is well known,[2] to gigantic quantum oscillations of the sound absorption coefficient Γ in the bulk metal. The period of oscillation of Γ(H) ($H \parallel \kappa$) is determined by the cross-section area $S(\zeta)$ of the Fermi surface intersection with the plane p_H = const, close to the extremal area $S_e(\zeta)$,

$$\Delta(1/H) = 2\pi e\hbar / cS(\zeta), \tag{1}$$

which allows us to obtain definite information on the electron energy spectrum of metals.

Additional information on the energy spectrum of the conduction electrons can be obtained in the study of films that are thin in comparison with the free path length of the electrons. In a magnetic field $H \lesssim H_c$, when r_H becomes of the order of or greater than the film thickness L, the specific features of the energy spectrum of the electrons in the film appear essentially in the quantum oscillations of thermodynamic quantities.[3] In this region of fields, as will be shown below, characteristic changes occur in the amplitude and period of the quantum oscillations of the acoustic absorption coefficient Γ in the film, in comparison with the bulk material.

In weak magnetic fields $H \ll H_c$, quantum magneto-acoustic oscillations of a new type are possible in the films, essentially connected with the dimensional or size quantization of the energy of the conduction electron. The magnetic field shifts the film energy levels without changing the spacing $\Delta\epsilon$ between them,[4] which also brings about the oscillations of Γ(H) in the given range of fields.

In the present research, the acoustic absorption quantum oscillations for sound propagating along the film are analyzed as functions of H and L for various orientations of H in the plane of the film. The study is carried out under the condition $\Delta\epsilon > \hbar\kappa v_F$ (v_F is the Fermi velocity of the electron), when the absorption of a phonon by an electron takes place without change in the magneto-film quantum number n, which determines the energy levels ϵ_n of the electrons in the film.[1)] In the region of weak fields $H \ll H_c$, this condition is equivalent to the condition $L < \lambdabar$, while in strong fields, $r_H < \lambdabar$. The scattering of the conduction electrons by the surface of the same has been assumed to be close to specular.[2)]

2. THE SOUND ABSORPTION COEFFICIENT IN A FILM

Following [1,2], we can write down the sound absorption coefficient in the following form:

$$\Gamma = \frac{\pi}{V\rho u_0^2 s}\sum_{\alpha,\alpha'}\frac{\partial F(\varepsilon_\alpha - \zeta)}{\partial\zeta}|\langle\alpha|U|\alpha'\rangle|^2\delta\left(\frac{\varepsilon_{\alpha'}-\varepsilon_\alpha}{\hbar}-\omega\right). \tag{2}$$

Here V is the volume of the sample, $\rho u_0^2 s$ the energy flux density in the sound wave with frequency ω, F the Fermi function, α the set of quantum numbers n, p, σ, which determine the state of the electron in the film in a parallel magnetic field, where n and σ are respectively the magneto-film and spin quantum numbers, and p is the component of the quasimomentum along the film. The energy spectrum $\epsilon_n(p)$ of the conduction electron is determined by the condition of quasiclassical quantization[3]

$$S(\varepsilon, p) = 2\pi e\hbar H(n+\gamma)/c, \tag{3}$$

(see Fig. 1), since in the given problem, the $n \gg 1$ are important. With account of spin splitting $\epsilon_{n\sigma} = \epsilon_n(p) + (-1)^\sigma \mu_0 H$, where $\mu_0 = e\hbar/2m_s c$, σ = 1, 2.

Recognizing that the Fermi wavelength of the electron $\lambdabar_F \ll \lambdabar$, the diagonal matrix elements of the energy interaction operator of the electron with the sound wave propagating along the film can be written in the form

$$|\langle\alpha|U|\alpha'\rangle|^2 = |\overline{\Lambda_{ij}u_{ij}}|^2\delta_{\sigma\sigma'}\delta_{nn'}\delta(\mathbf{p}+\hbar\kappa-\mathbf{p}'). \tag{4}$$

Here u_{ij} are the amplitude values of the deformation tensor in the sound wave, and Λ_{ij} the tensor calculated in

1) Sound absorption in transitions of electrons between magnetic surface levels was considered in [5].

2) A similar problem was considered in [6] under the assumption of a parabolic approximation to the surface potential, neglecting in this situation the strong screening in metals, which brings about a sharp change in the surface potential near the boundary of the sample.

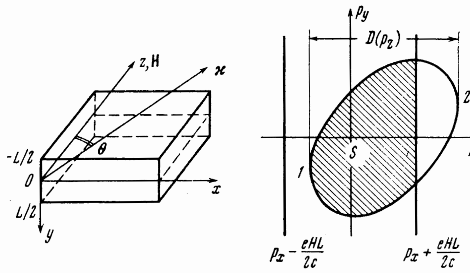

FIG. 1

[1]; averaging is carried out over the thickness of the film:

$$\bar{f} = L^{-1}\int_0^L f(y)\,dy.$$

The elements of the operator U that are nondiagonal in n and σ are unimportant in (2), since the spacing between the quantum levels is $\Delta\epsilon > \hbar\kappa v_F$ and consequently there are no transitions between levels.

Substituting (4) in (2) and summing according to the Poisson formula, we obtain the following expression for the oscillating part $\tilde{\Gamma}$ of the sound absorption coefficient:

$$\tilde{\Gamma} = \frac{c}{4\pi^2\rho u_0^2 s\hbar^3 eHL}\sum_{\sigma=1}^{2}\sum_{k=1}^{\infty}\iint d^2\mathbf{p}\int_{s>0} d\varepsilon\,\overline{|\Lambda_{ij}u_{ij}|^2}$$
$$\times\frac{\partial F(\varepsilon-\zeta_\sigma)}{\partial\zeta_\sigma}\cos\left[\frac{kc}{eH\hbar}S(\varepsilon,\mathbf{p}) - 2\pi k\gamma\right]\delta\left(\frac{\varkappa}{\partial S/\partial\varepsilon}\frac{\partial S}{\partial\mathbf{p}} + \omega\right) \qquad (5)$$

where $\zeta_0 = \zeta - (-1)^\sigma\mu_0 H$, and ζ is the chemical potential.

In the general case, the quantity $\overline{|\Lambda_{ij}u_{ij}|^2}$ depends on p, but this dependence is smooth in comparison with the rapidly oscillating factor in (5). Further calculations are similar to the calculation of the oscillations of the thermodynamic quantities.[3]

3. STRONG MAGNETIC FIELDS

In the range of fields $H > H_c \equiv cD_{max}/eL$, where D_{max} is the maximal dimension of the intersection of the Fermi surface with the plane p_z = const in the p_z direction (see Fig. 1), we get from (5)

$$\tilde{\Gamma} = \frac{4\pi\Gamma}{\cos\theta}\left(1 - \frac{H_c}{H}\right)\left|\frac{\partial^2 S_0}{\partial p_z^2}\right|^{-1}\sum_{k=1}^{\infty}\Psi\left(k\frac{2\pi^2 T}{\Delta\varepsilon_H}\right)\cos\left(k\pi\frac{m^*}{m}\right)$$
$$\times\cos\left[k\frac{cS_0(\zeta)}{eH\hbar} - 2\pi k\gamma\right], \qquad (6)$$

where $\Psi(x) = x/\sinh x$, $\Delta\epsilon_H = eH\hbar/m^*c$ is the spacing between the Landau levels, $m^* = (2\pi)^{-1}\partial S_0/\partial\zeta$, θ the angle between κ and **H** $(0 \le \theta < \pi/2)$. The quantity

$$\Gamma_0 = (m^*)^2\overline{|\Lambda_{ij}u_{ij}|^2}/2\pi\hbar^3\rho u_0^2\varkappa s \qquad (6')$$

is of the order of the sound absorption coefficient in the film for H = 0.

All the quantities in (6) are taken for the value $p_z = p_{z0}$, which satisfies the condition

$$\partial S/\partial p_{z0} + 2\pi m^* s/\cos\theta = 0. \qquad (7)$$

In the range of angles $\pi/2 - \theta \gg s/v_0$, where v_0 is the velocity of the electron at a reference point of the Fermi surface in the H direction, the second component in (7) can be neglected so that $S_0(\zeta)$ is identical with the area of the extremal cross section $S_e(\zeta)$. For angles $\pi/2 - \theta \gtrsim s/v_0$, the value of $S_0(\zeta)$ can differ considerably from S_e. Measurements of the period of oscillation $\Delta(1/H) = 2\pi e\hbar/cS_0(\zeta)$ in this range of angles permits us to determine nonextremal cross sections of the Fermi surface.

We obtain the following estimate for the oscillation amplitude A of the sound absorption coefficient from (6):

$$A \sim \begin{cases}\left(1 - \frac{H_c}{H}\right)\frac{\Gamma_0}{\cos\theta}\frac{\Delta\varepsilon_H}{T}, & T \ll \frac{\Delta\varepsilon_H}{2\pi^2}\\ \left(1 - \frac{H_c}{H}\right)\frac{\Gamma_0}{\cos\theta}\left(\frac{2\pi^2 T}{\Delta\varepsilon_H}\right)\exp\left(-\frac{2\pi^2 T}{\Delta\varepsilon_H}\right), & T \gg \frac{\Delta\varepsilon_H}{2\pi^2}.\end{cases} \qquad (8)$$

In the range of angles θ that is sufficiently close to $\pi/2$, $(0 \le \pi/2 - \theta < s/v_0)$, Eq. (6) becomes inapplicable; in this case, we have for Γ $(H \ge H_c)$

$$\tilde{\Gamma} = \Gamma_0\frac{\beta}{L}\sqrt{\frac{2\pi\hbar c}{eH}}\sum_{k=1}\frac{1}{\sqrt{k}}\Psi\left(k\frac{2\pi^2 T}{\Delta\varepsilon_H}\right)$$
$$\times\cos\left(k\pi\frac{m^*}{m_0}\right)\cos\left[k\frac{cS_e(\zeta)}{eH\hbar} - 2\pi k\gamma - \frac{\pi}{4}\right] \qquad (9)$$

where

$$\beta = 2\pi^2 m^* s(R_{1e}^{-1} + R_{2e}^{-1})\,|\partial^2 S_e/\partial p_z^2|^{-1}$$

is a number of the order of s/v_0, R_{ie} is the radius of curvature of the extremal cross section of the Fermi surface at the i-th point, i = 1, 2 (see Fig. 1).

The amplitude of the oscillations of (9) is equal to

$$A \sim \begin{cases}\Gamma_0\frac{s}{v_0}\sqrt{\frac{\lambda_F H_c}{LH}}\left(\frac{\Delta\varepsilon_H}{2\pi^2 T}\right)^{1/2}, & T \ll \frac{\Delta\varepsilon_H}{2\pi^2}\\ \Gamma_0\frac{s}{v_0}\sqrt{\frac{\lambda_F H_c}{LH}}\left(\frac{2\pi^2 T}{\Delta\varepsilon_H}\right)\exp\left(-\frac{2\pi^2 T}{\Delta\varepsilon_H}\right), & T \gg \frac{\Delta\varepsilon_H}{2\pi^2}\end{cases} \qquad (10)$$

in order of magnitude.

The presence of oscillations for $\theta = \pi/2$ is essentially a film effect. For the bulk material, as is well known, oscillations are absent in this case. The analysis in the range of fields $H > H_c$ given above reveals an interesting dependence of the amplitude A of the quantum oscillations of the sound absorption coefficient on the angle θ between H and κ, shown in Fig. 3: $A(\theta)$ increases as $1/\cos\theta$, while for $\pi/2 - \theta \lesssim s/v_0$ it falls off sharply.

Near the critical field H_c, the amplitude of oscillations described by Eq. (6) $(\pi/2 - \theta > s/v_0)$ decreases appreciably and in a narrow range of fields is equal to

$$A \sim \begin{cases}\frac{\Gamma_0}{\cos\theta}\left(\frac{\lambda_F^2}{L^2}\frac{\Delta\varepsilon_H}{2\pi^2 T}\right)^{1/3}, & T \ll \frac{\Delta\varepsilon_H}{2\pi^2}, \quad \left|1 - \frac{H_c}{H}\right| \ll \left(\frac{\lambda_F}{L}\frac{2\pi^2 T}{\Delta\varepsilon_H}\right)^{2/3}\\ \frac{\Gamma_0}{\cos\theta}\left(\frac{\lambda_F}{L}\right)^{2/3}\left(\frac{2\pi^2 T}{\Delta\varepsilon_H}\right)\exp\left(-\frac{2\pi^2 T}{\Delta\varepsilon_H}\right), & T \gg \frac{\Delta\varepsilon_H}{2\pi^2},\\ & \left|1 - \frac{H_c}{H}\right| \ll (\lambda_F/L)^{2/3}\end{cases} \qquad (11)$$

for low and high temperatures, respectively.

With decrease in the field $H < H_c$, the amplitude of the oscillations of Γ increases. In the given range of fields, the oscillations depend weakly on the angle θ. For $H_c/H - 1 \gg (\lambda_F/L)^{1/2}$ and $0 \le \theta \le \pi/2$, the oscillating part of the sound absorption coefficient is determined by the values of the averaged area S_e^L that are extremal over p_x and p_z (see Fig. 1):

$$\tilde{\Gamma} = \frac{\Gamma_0}{\pi m^{*2}}\left(\frac{\partial S_e^L}{\partial\zeta}\right)^2 \frac{1}{L}\sqrt{\frac{2\pi\hbar c}{eH}}\,|I|_e^{-1/2}\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}\Psi\left(k\frac{2\pi^2T}{\Delta\varepsilon_L}\right) \times \cos\left(\frac{k}{2m_*}\frac{\partial S_e^L}{\partial\zeta}\right)\cos\left[k\frac{cS_e^L(\zeta,H)}{e\hbar H} - 2\pi k\gamma - \frac{\pi}{4}\right], \quad (12)$$

where

$$I = \frac{\partial^2 S_e^L}{\partial p_\kappa^2}\left[\frac{\partial^2 S_e^L}{\partial p_x^2}\frac{\partial^2 S_e^L}{\partial p_z^2} - \left(\frac{\partial^2 S_e^L}{\partial p_x \partial p_z}\right)^2\right],$$

p_κ is the projection of the quasimomentum on the κ direction, $\Delta\epsilon_L$ the spacing between neighboring magnetofilm levels, determined by Eq. (3). The amplitude of the oscillations of (12) is v_0/s times the estimate (10) obtained for A in the case $H = H_c$ and $\theta = \pi/2$.

In the given range of fields, the period of $\Delta(1/H)$ of the oscillations of the sound absorption coefficient $\Gamma(H)$ depends on the magnetic field H and the thickness of the film L, as also in the case of oscillations of thermodynamic quantities.[3]

4. WEAK MAGNETIC FIELDS

In the region of weak magnetic fields $H \ll H_c$, under conditions of the quantum size effect, the oscillations of the sound absorption coefficient, as also the oscillations of thermodynamic quantities,[4] develop as a result of the displacement of the size-quantum energy levels of the electron in the film with the magnetic field. For $\tilde{\Gamma}$ in this case, we obtain

$$\tilde{\Gamma} = \frac{\Gamma_0}{\pi m^{*2}}\left(\frac{1}{|v_{y1}|} + \frac{1}{|v_{y2}|}\right)^2\sqrt{\frac{2\pi\hbar}{L}}\,|j|_e^{-1/2}\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}\Psi\left(k\frac{2\pi^2T}{\Delta\varepsilon_L}\right) \cdot \cos\left[\frac{kL}{\hbar}\left(d_e(\zeta) - \frac{b}{24}\left(\frac{eHL}{c}\right)^2\right) \pm \frac{\pi}{4}\right], \quad (13)$$

where

$$b = \left(1 + \frac{v_x^2}{v_y^2}\right)^{3/2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \quad \Delta\varepsilon_L = \frac{2\pi\hbar}{L|\partial d_e/\partial\zeta|}$$

is the distance between the size-quantized energy levels of the electron. Here v_x and v_y are the projections of the velocity of the electron at the points of intersection of the Fermi surface with the extremal chord $d_e(\zeta)$ which is perpendicular to the film; R_1 and R_2 are the radii of curvature of the Fermi surface at these points in the plane perpendicular to H:

$$j = J(\gamma_0 + \gamma_1\cos 2(\varphi - \varphi_0) + \gamma_2 \sin 2(\varphi - \varphi_0)). \quad (14)$$

The values of J and γ_i are of the order $|J| \sim (\lambda_F/\hbar)^2$ and $|\gamma_i| \sim \lambda_F/\hbar$, and are expressed in terms of the principal radii of curvature of the Fermi surface in the formulas given in [4]; the angle φ determines the direction of propagation of the sound wave in the plane of the film. The plus sign in the argument of the cosine is taken for $j < 0$, the minus for $j > 0$.

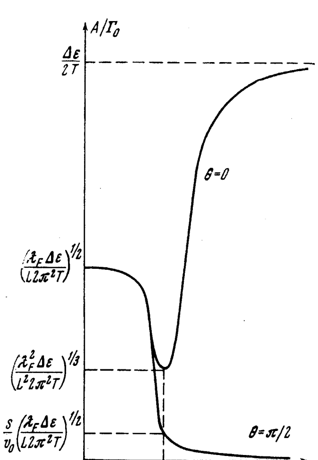

FIG. 2. Dependence of the amplitude of quantum magneto-acoustic oscillations on the magnetic field H.

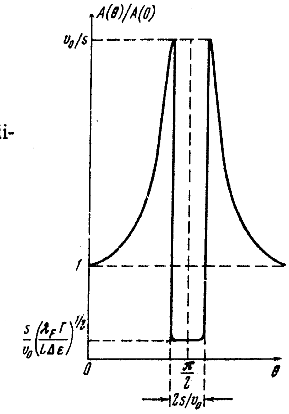

FIG. 3. Dependence of the amplitude of quantum magneto-acoustic oscillations on the angle θ between the magnetic field H and the direction of propagation of the sound wave κ for $H > H_C$.

It follows from (13) that the sound absorption coefficient oscillates upon variation of the magnetic field, with period

$$\Delta(H^2) = 48\pi\hbar c^2/e^2L^3b. \quad (15)$$

Moreover, oscillations of Γ with change in the film thickness are also possible. The period of the quantum size oscillations $\Gamma(L)$ is equal to

$$\Delta(L) = 2\pi\hbar/d_e(\zeta). \quad (16)$$

In the case of $J < 0$ (according to [4], this corresponds to logarithmic singularities of the density of electron states), the quantity j changes sign for certain values of the angle φ, which leads to a jumpwise change in the phase of the oscillations in the change of the direction of propagation of the sound wave. Thus, if $J < 0$, then the quantity $\partial\Gamma/\partial\varphi$ has a δ-like singularity.

In the region of not too low temperatures ($2\pi^2T \gg \Delta\epsilon_L$), only the fundamental (k = 1) is left in the sum (13), with amplitude of the order of

$$A \sim \Gamma_0\sqrt{\frac{\lambda_F}{L}}\left(\frac{2\pi^2T}{\Delta\varepsilon_L}\right)\exp\left(-\frac{2\pi^2T}{\Delta\varepsilon_L}\right). \quad (17)$$

For sufficiently low temperatures ($2\pi^2T \ll \Delta\epsilon_L$) we have

$$A \approx \Gamma_0\sqrt{\frac{\lambda_F}{L}}\left(\frac{2\pi^2T}{\Delta\varepsilon_L}\right)^{-1/2}, \quad (17')$$

i.e., the amplitude of oscillations can become equal to the value of the smooth part as $T \to 0$. But in this range of temperatures, the broadening of the energy levels, due to scattering of the conduction electrons by impurities and by the surface of the film, becomes very significant, which can be taken into account by the introduction of the Dingle factor.[8] Then, for T = 0, we have

$$A \sim \Gamma_0 \sqrt{\frac{\lambda_F}{L}} \left(\frac{\pi^2 \hbar}{\tau_0 \Delta \varepsilon_L} \right)^{-1/2}, \qquad (18)$$

assuming $\hbar/\tau_0 \ll \Delta\epsilon_L$, where τ_0 is the time of free flight of the conduction electrons.

A similar estimate of the amplitude of oscillations in the region of strong magnetic fields as $T \to 0$ can be obtained by the substitution $T \to \hbar/2\tau_0$ in Eqs. (8), (10), (11).

5. CONCLUSION

It follows from the analysis given above that in the range of fields $H \sim H_c$ (for films of thickness $L \sim 10^{-3}$–10^{-4} cm, $H_c \sim 10^4$–10^5 Oe), the sound absorption in the films possesses a number of characteristic features. Changes in the period with amplitude of the quantum magneto-acoustic oscillations for different orientations of the magnetic field **H** and direction of propagation of the sound wave allow us to determine the area ($S(\zeta)$) and dimensions ($D_{max}(\zeta)$) of both extremal and nonextremal cross sections of the Fermi surface, the cyclotron mass ($m^* = (2\pi)^{-1} \partial S/\partial\zeta$) and also the value of the velocity of the electron (v_0) at the reference points of the Fermi surface. In weak magnetic fields $H \ll H_c$, the value of the radii of curvature of the Fermi surface can be found from experimental investigations of the oscillations of Γ, at the points of the intersection of the surface with the extremal chord and the separation $\Delta\epsilon_L$ between the film levels.

For observation of the indicated effects, specular scattering is required for the surface of the film for those electrons which are responsible for the oscillations of Γ. In fields $H \sim H_c$, these are grazing electrons, for which the specular condition is effectively satisfied for many metals. In the case $H \ll H_c$, these are electrons which collide with the surface of the film at large angles, so that the oscillations of $\Gamma(H)$ in a weak magnetic field should evidently be expected only in semimetals.

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