EFFECT OF IMPURITIES ON THE TOPOLOGY OF THE INDIUM FERMI SURFACE. II

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The dependence of $\partial T_c/\partial P$ and ΔT_c on C is studied in a broad range of concentrations in the In-Tl and In-Hg systems, in both the tetragonal and cubic phases. Peculiarities of the dependence of $\partial T_c/\partial P$ on C are observed in the tetragonal phase for concentrations between 0 and 3 at.% Hg in the In-Hg system and between 0 and 6 at.% Tl in the In-Tl system. At high concentrations, peculiarities of the dependence of $\partial T_c/\partial P$ on C are observed in the 6–9% concentration range of Hg for the In-Hg system and in the 22–30% range of Tl in the In-Tl system. A singularity of $\partial T_c/\partial P$ is observed in the cubic phase of the In-Tl system at Tl concentrations between 32 and 50 at.% and also in the ternary system In-Tl(35%)-Hg(x at.%) ($0 < x < 6$ at.%). The singularities are ascribed to alteration of the Fermi-surface topology, induced by the impurity. The magnitude of the singularities of $\partial T_c/\partial P$ does not depend on the residual resistance of the alloys, which is proportional to the lifetime of the quasiparticles. It is suggested that in alloys there exist singularities in the density of the electron states, which form a coherent state, i.e., Cooper pairs. On the other hand, from the viewpoint of modern theory of normal metals, it is meaningless to speak of singularities of the electron state density in these alloys.

I N our preceding study[1] we investigated solid solutions of indium with cadmium, mercury, lead, and tin impurities in the tetragonal phase. It was found that nonlinear dependences of $\partial T_c/\partial P$ on the impurity concentration C are observed under the influence of impurities whose valence is both larger and smaller than that of indium.

Such a behavior of $\partial T_c/\partial P$ as a function of C was connected with the change of topology of the Fermi surface under the influence of the impurity[1-4]. From the experimental data on the dependence of $\partial T_c/\partial P$ on C for binary and ternary systems it follows that the magnitude of the observed effect in $\partial T_c/\partial P$ is not sensitive to the mean free path of the conduction electrons[1,5,6].

The present paper is devoted to a further investigation of the changes of the electronic spectrum of indium under the influence of impurities. We investigated the dependence of $\partial T_c/\partial P$ on C in a wide interval of concentrations both in the face-centered tetragonal phase and in the face-centered cubic phase, in the systems In-Tl and In-Hg.

SAMPLES AND MEASUREMENT METHOD

We investigated binary solid solutions of indium with mercury and thallium impurities, and also the ternary solutions In-Tl35%-Hg, in which the mercury concentration ranged from 0 to 6 at.%[1)].

Indium with impurity batches was placed in an ampoule of Pyrex glass. The ampoule was evacuated to $(1-2) \times 10^{-5}$ mm Hg and sealed off in the same vacuum. It was then placed in an oven, where the melt was soaked for several weeks at a temperature slightly exceeding the melting temperature, and was continuously stirred. The melt was then abruptly cooled. Small-concentration alloys were prepared by diluting solutions having higher concentrations. The samples were prepared in the form of wire of 0.4 mm diameter and length 10–15 mm by extruding through a die, after which they were annealed for a week, and in some cases for several months, at a temperature 20–30°C below the melting temperature of the alloy. Sufficiently good samples were obtained with a homogeneous impurity distribution, as can be judged by the width of the superconducting transition (Fig. 1).

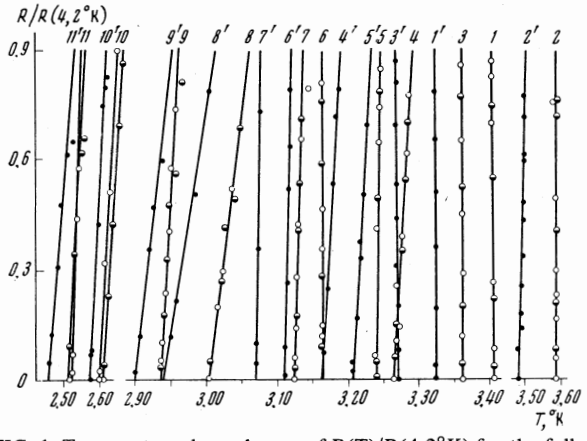

FIG. 1. Temperature dependences of R(T)/R(4.2°K) for the following systems: 1, 1′–pure In; 2.2′–In-Hg (8%); 3.3′–In–Hg (0.8%); 4.4′–In–Tl (26%); 5.5′–In–Hg (10%); 6.6′–In–Tl (3%); 7.7′–In-Tl (35%); 8.8′–In-Tl (35%)–Hg (4%); 9.9′–In–Tl (35%)–Hg (2%); 10.10′–In-Tl (45%); 11.11′–In-Tl (50%). Curves 1–11 were plotted at P = 0 and 1′–11′ at P = 1730 atm.

The ternary solutions In-Tl35%-Hg were prepared in the following manner. The maximum-concentration alloys In-Tl35%-Hg6% and In-Tl35% were prepared first. Ternary solutions with lower mercury concentration was obtained by diluting the ternary solution with the maximum concentration with the binary

[1)]The concentrations are given in atomic percent throughout.

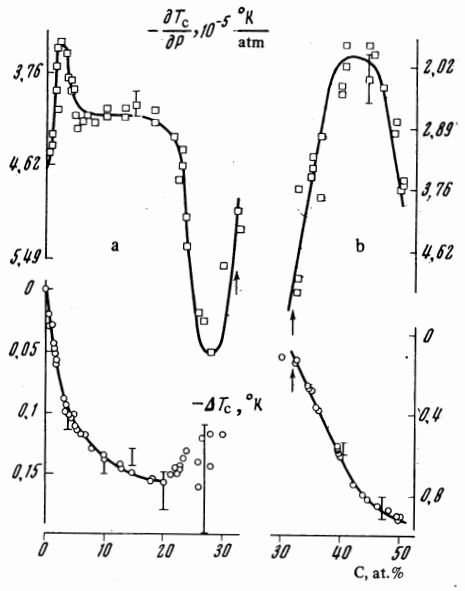

FIG. 2. Plots of $\partial T_c/\partial P$ and ΔT_c of In-Tl alloys against the Tl impurity concentration: a—face-centered tetragonal phase; the arrow corresponds to the phase transition; b—face-centered cubic phase. (The data of [7] are given together with the limits of the errors).

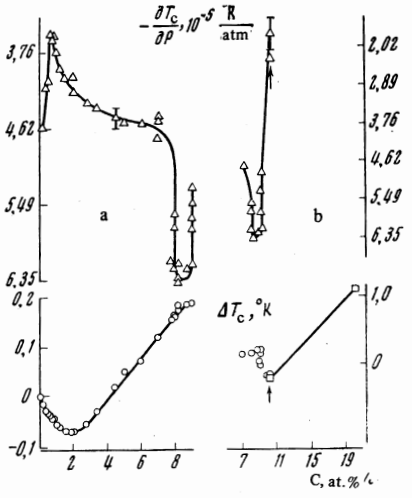

FIG. 3

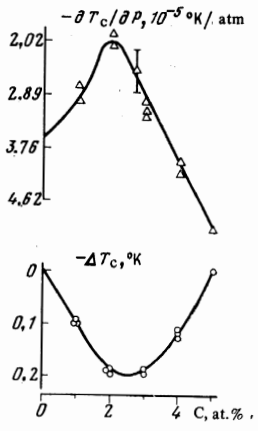

FIG. 4

FIG. 3. Dependence of $\partial T_c/\partial P$ and ΔT_c of In-Hg alloys on the mercury impurity concentration: a—face-centered tetragonal phase, the arrow corresponds to the phase transition; b—face-centered cubic phase (□—data of [8]).

FIG. 4. Dependence of $\partial T_c/\partial P$ and ΔT_c of In-Tl (35%)-Hg alloys on the mercury impurity concentration.

In-Tl35% solution. From x-ray diffraction investigations and from the width of the superconducting transition (see Fig. 1) it can be concluded that the ternary solutions are single-phase[2)] and have large crystals at the investigated mercury-impurity concentrations (up to 6%). According to x-ray diffraction investigations, the ternary solutions In-Tl35%-Hg with Hg concentration up to 6%, as well as the In-Tl and In-Hg solutions in the concentration intervals 31—50% Tl[7] and 9—20% Hg[8] have face-centered cubic lattices.

The temperature of the superconducting transition was determined from the change of the resistance ratio R(T)/R(4.2°K) with changing temperature (Fig. 1). The derivative of the superconducting-transition temperature with respect to the pressure, $\partial T_c/\partial P$, was determined by a method described earlier[1]. The accuracy with which $\partial T_c/\partial P$ was measured in the In-Tl and In-Hg solutions in the concentration intervals 0—20% Tl and 0—8% Hg was $\pm 1.1 \times 10^{-6}$°K/atm. With further increase of the thallium and mercury concentrations in the binary systems, and also of the mercury concentration in the ternary In-Tl35%-Hg systems, the accuracy was $\pm 3.3 \times 10^{-6}$°K/atm.

MEASUREMENT RESULTS

1. In-Tl solutions. For indium with a thallium impurity we observe a nonlinear dependence of $\partial T_c/\partial P$ on the impurity concentration in both the tetragonal (Fig. 2a) and the cubic phases (Fig. 2b). When the thallium impurity is added to pure indium, the absolute value of $\partial T_c/\partial P$ decreases and reaches at 2% Tl a value 3.5×10^{-5} deg/atm, which remains unchanged up to 3.2% Tl. Further increase of the impurity concentration increases the absolute value of $\partial T_c/\partial P$, which reaches 4.16×10^{-5} deg/atm at 6% Tl. In the concentration interval 6—20%, $\partial T_c/\partial P$ remains practically unchanged. However, a further increase of the thallium impurity concentration leads to an appreciable increase of the absolute magnitude of $\partial T_c/\partial P$, which reaches a value 6.35×10^{-5} deg/atm at 28% Tl. Indium alloys with up to 30% Tl impurity are single-phase and have a tetragonal lattice. In the interval 30—31% Tl, the indium lattice goes over at 3.3°K from face-centered tetragonal to face-centered cubic[7]. It should be noted that $\partial T_c/\partial P$ is continuous in the entire concentration region and is not very sensitive to the scatter of ΔT_c. The phase transition corresponds to a sharp change in the dependence of T_c on the impurity concentration (the arrow in Fig. 2b corresponds to the phase-transition point). In the cubic phase, in the concentration interval 32—50%, a nonlinear change of $\partial T_c/\partial P$ is observed. The value of $\partial T_c/\partial P$ changes in absolute magnitude from 5.2×10^{-5} to 1.7×10^{-5} deg/atm, i.e., by a factor of 3 (Fig. 2b).

The presented experimental plots of ΔT_c against the thallium impurity concentration (Figs. 2a and 2b) agree with previously published data by others[7].

2. In-Hg solutions. In these solutions, the dependence of $\partial T_c/\partial P$ on the mercury impurity concentration is nonlinear (Fig. 3), just as in the In-Tl system. The dependence of $\partial T_c/\partial P$ on the mercury impurity concentration, up to 4.5%, was investigated in detail earlier[5]. In the concentration interval 4.5—6%, $\partial T_c/\partial P$ remains unchanged. With further increase of the mercury impurity concentration, an appreciable increase of the absolute value of $\partial T_c/\partial P$ is observed, and reaches, just as in the In-Tl system, the value 6.35×10^{-5} deg/atm at 8.2% Hg. In the interval 9—10% Hg, the indium lattice is transformed from face-centered tetragonal into face-centered cubic[8]. This phase transition corresponds to a jump in the dependence of ΔT_c on the impurity concentration

[2)]The authors are grateful to L. F. Belyak for x-ray diffraction investigations of the ternary solutions.

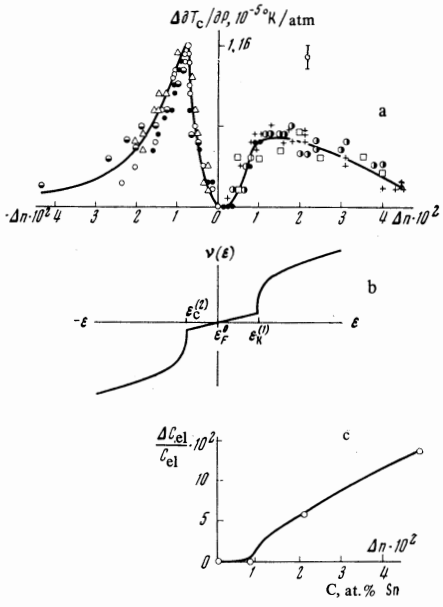

FIG. 5. Dependence of $\Delta\partial T_c/\partial P$ of indium on the electron density change Δn ($\Delta n = \alpha C^*_{eff}$ at.%; (○–In-Cd; ◒–In–Hg; +–In-Sn; ●–In–Pb (2%)–Cd, ◑–In–Pb [1]; △–In-Tl). b) Density of electronic states of indium near the Fermi energy ϵ_F^0 ($\epsilon_c^{(1)}$ and $\epsilon_c^{(2)}$ are the critical energies at which the topology of the Fermi surface changes). c) Change of electronic specific heat of indium following introduction of Sn impurity [9].

(Fig. 3b, the arrow corresponds to the transition into the face-centered cubic phase). In this system, the $\partial T_c/\partial P$ dependence in the region of the structural transformation is similar to that in the In-Tl system. The presented experimental dependences of ΔT_c on the mercury impurity concentration (Fig. 3) are in good agreement with data previously published by others[8].

3. Ternary solutions In-Tl35%-Hg. The dependence of $\partial T_c/\partial P$ on the mercury impurity concentration in these solutions is similar to the analogous dependence of $\partial T_c/\partial P$ on C in the In-Tl system in the concentration interval 35–50% Tl (Fig. 4). The temperature of the superconducting transition of the initial In-Tl35% solution decreases when 2% Hg is added. With further increase of the mercury impurity concentration, a rise in the temperature of the superconduction transition is observed. Thus, the shift of $\Delta T_c(C)$ for the In-Tl35%-Hg solution differs greatly from the shift of $\Delta T_c(C)$ of the In-Tl solutions in the concentration interval 35–50% (Fig. 4), in spite of the fact that these solutions have similar dependences of $\partial T_c/\partial P$ on C.

DISCUSSION OF RESULTS

1. Tetragonal phase. In our preceding communication, by a joint study of the binary and ternary systems In-Hg, In-Cd, In-Pb, In-Sn, In-Pb2%-Cd, we have shown that the nonlinear dependence of $\partial T_c/\partial P$ of indium on the impurity concentration is due to the change in the electron density (the Fermi energy) (Fig. 5a). Such a behavior of $\partial T_c/\partial P$ was attributed to the presence, near the Fermi energy of indium, of two critical energies $\epsilon_c^{(1)}$ and $\epsilon_c^{(2)}$, at which a change takes place in the topology of the Fermi surface and causes the change of the density of the electron states $\nu(\epsilon)$, shown in Fig. 5b. The hypothesis was advanced that these singularities should also become manifest in the electronic properties of the investigated solutions in the normal state, for example, in the electronic specific heat. Recently, in a study of the influence of a tin impurity on the electronic specific heat of indium, White and McCollum[9] observed a sharp change in the density of the electronic states at an impurity concentration larger than 0.8 at.% (Fig. 5c). Such a behavior of the electronic part of the specific heat of indium was also attributed by the authors of[9] to the change of $\nu(\epsilon)$ upon formation of α tubes in the third Brillouin zone.

Thus, the assumption that a connection exists between the change of the topology of the Fermi surface[1] and the observed nonlinear dependence of $\partial T_c/\partial P$ of indium following the addition of an impurity has been confirmed, at least qualitatively, in a more conventional experiment characterizing the behavior of the density of the electronic states of the metal. In our subsequent discussion, following[2], we shall associate the results of the observation of the extrema of $\partial T_c/\partial P$ as functions of the impurity concentration with the singularities in the density of the electronic states[3)].

On comparison of the experimental data (Fig. 5b and 5c) it is natural to assume that upon the decrease of the Fermi energy of indium, when the β tubes in the third Brillouin zone are broken[4], the electronic specific heat should decrease nonlinearly under the influence of the mercury and cadmium impurities with increasing impurity concentration.

The maximum change of $\partial T_c/\partial P$ of indium under the influence of a thallium impurity is the same as in the In-Hg system (Fig. 5a). We can therefore conclude that an admixture of thallium, which has a valence equal to the valence of indium, just like a mercury impurity, decreases the Fermi energy of indium. The Tl impurity acts on the density of the electronic states of In less effectively than the Hg impurity, which has a lower valence than In. Indeed, if we assume as before that the effectiveness of mercury is equal to unity ($C_{eff} = C_{Hg}$[1]) and that of thallium is equal to 0.3 ($C_{eff} = 3.3\ C_{Tl}$) in the In-Hg and In-Tl solutions, then the dependences of $\partial T_c/\partial P$ of these solutions on the effective impurity concentration are described by a single curve (Fig. 6b).

Thus, these results, in conjunction with the data of Fig. 5a, show that the nonlinear change of $\partial T_c/\partial P$ as a function of C of all the investigated systems, connected with the change of the topology of the Fermi surface, is described by a single curve if an appropriate choice is made of the effectiveness of the influence of the impurities on the Fermi energy of indium. The effectiveness of the action of impurities on the Fermi energy is connected with many factors, such as the valence, the potential of the impurity-atom ions, and the change of the parameters of the unit-cell lattice of indium following introduction of the impurity.

[3)] Strictly speaking [28], the nonlinear dependence of $\partial T_c/\partial P$ on ϵ is due to the singularities of $\nu(\epsilon)$, if we assume that the electron-phonon interaction constant does not contain a singularity at $\epsilon_F = \epsilon_c$. This assumption does not contradict the experimental data on the tunnel effect in superconductors [27].

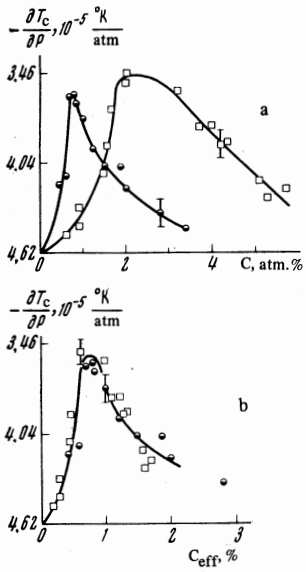

FIG. 6. Dependence of $\partial T_c/\partial P$ of indium on the impurity concentration (a) and on the effective impurity concentration ($C_{eff} = C_{Hg} = 3.3\ C_{Tl}$ (b); ◒–In-Hg system; □–In-Tl system.

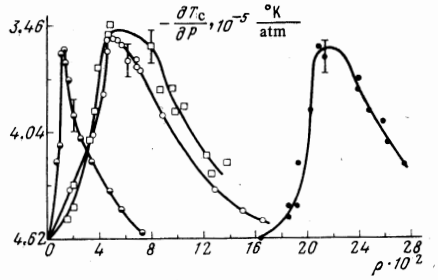

FIG. 7. Dependence of $\partial T_c/\partial P$ on the relative residual resistivity ρ on the following systems: ○–In-Hg; ○–In-Cd; □–In–Tl; ○–In–$Pb_{2\%}$–Cd.

Introduction of impurities into a metal, besides changing the Fermi energy, also destroys the periodicity of the lattice and consequently leads to electron scattering processes and to damping of single-particle electronic states, a measure of which may be the relative residual resistivity $\rho = R(4.2°K)/[R(273°K) - R(4.2°K)]$. In the investigated systems, the change of the residual resistivity differed greatly for different impurities. Consequently, the damping of the single-particle excited states of the electrons was different in the systems In-Hg, In-Tl, In-Cd[1], In-$Pb_{2\%}$-Cd[1]. It should be noted that the maxima of $\partial T_c/\partial T$, connected with the change in the Fermi-surface topology, did not depend on the electron scattering processes (on the residual resistivity), within the limits of experimental accuracy (Fig. 7).

These experimental data point to significant differences in the manifestation of scattering processes in the thermodynamics of metals in the superconducting and normal states.

The density of states in a metal is determined by the expression

$$\nu(\varepsilon) = \nu_0(\varepsilon) + \delta\nu(\varepsilon), \tag{1}$$

where $\nu(\epsilon)$ is a smooth function of the energy and $\delta\nu(\epsilon)$ has a "root" singularity and is due to the change of the Fermi-surface topology with changing energy energy[10,21].

As is well known[11], scattering processes do not influence the value of $\nu_0(\epsilon)$ if $\Gamma/\epsilon_F \ll 1$, where $\Gamma = \hbar v_F/kl$, v_F is the average electron velocity of the Fermi surface, and $\hbar$ and k are the Planck and Boltzmann constants. At the same time, the quantity $\delta\nu(\epsilon)$ depends strongly on the scattering processes. In this case the quantities $\delta\nu(\epsilon_F)$ and $(\partial/\partial P)\delta\nu(\epsilon_F)$ take the form[11]

$$\delta\nu(\varepsilon_F) = \frac{(m_1m_2m_3)^{1/2}}{\sqrt{2}\pi^2}\{[(\varepsilon_F-\varepsilon_c)^2+\Gamma^2]^{1/2} \pm (\varepsilon_F-\varepsilon_c)\}^{1/2},$$
$$\frac{\partial}{\partial P}\delta\nu(\varepsilon_F) = \pm\frac{1}{2}\frac{\partial(\varepsilon_F-\varepsilon_c)}{\partial P}\frac{\delta\nu(\varepsilon_F)}{[(\varepsilon_F-\varepsilon_c)^2+\Gamma^2]^{1/2}} \tag{2}$$

The plus and minus signs in (2) pertain to the cases of the points of maximum and minimum energy, while m_1, m_2, and m_3 are the principal values of the effective-mass tensor $m^{-1}_{ik} = \partial^2\epsilon/\partial P_i \partial P_c$ at the point $\mathbf{P} = \mathbf{P}_c$.

The thermodynamic characteristics of a metal in the superconducting state (the energy gap Δ_0, the critical magnetic field H_c, and the temperature of the superconducting transition) are determined by the following parameters; the density of the electronic states and the amplitude of the effective electron-electron attraction $I(\epsilon, \epsilon')$, where ϵ and ϵ' are the electron energies. If it is assumed that the scattering processes do not influence the amplitude of the effective electron-electron attraction, and the density of the electronic states is a smooth function of the energy, then the scattering processes do not influence the thermodynamic characteristics of the superconductor if $\Gamma/\epsilon_F \ll 1$[12]. At the same time, using the formalism developed by Abrikosov and Gor'kov[12], it was shown that allowance for the change of the Fermi-surface topology under the influence of the impurity leads to a nonlinear dependence of T_c on C[13]. In this case, the nonlinear variation of T_c and $\partial T_c/\partial P$ as functions of C is due to two causes: the change of the Fermi energy and scattering processes.

The nonlinear dependence of $\partial T_c/\partial P$ on $\epsilon_F - \epsilon_c$ is determined by the expression[13]

$$\frac{\partial T_c}{\partial P} = \lambda\gamma\int_0^\infty \frac{d\omega}{2\pi}\frac{\operatorname{th}\omega}{\omega}\int_{-\beta}^{\infty}\left|I\left(0,\frac{T_c^0 x}{k_F s}\right)\right|^2(\beta+x)^{-1/2}$$
$$\times\left[\frac{1}{(\omega+x)^2+\gamma^2}+\frac{1}{(\omega-x)^2+\gamma^2}\right]dx, \tag{3}$$

where

$$\gamma = \frac{\Gamma}{4T_c^0},\quad \beta = \frac{(\varepsilon_F-\varepsilon_c)}{2T_c^0},\quad \lambda = \frac{(2m_1m_2m_3T_c)^{1/2}}{2\pi^2\nu_0(\varepsilon_F)}\frac{\partial(\varepsilon_F-\varepsilon_c)}{\partial P},$$

s is the speed of sound in the metal and k_F is the Fermi wave vector. In the Froehlich-Debye model[14] we have

$$I(0,x) = 1 - 2|x| + 2x^2\ln|1+1/x|.$$

It is seen from Fig. 8 that $\partial T_c/\partial P$ depends significantly on γ. This is connected with the "smearing" of the quantity $\delta\nu(\epsilon_F)$ as a result of the scattering processes.

Thus, the singularities in the density of the electronic states should become "smeared out" by the scattering processes, regardless of whether the metal is in the normal or in the superconducting state[11,13]. The increments of the thermodynamic characteristics of the metal due to the change in the topology of the Fermi surface (for example, the increments of the

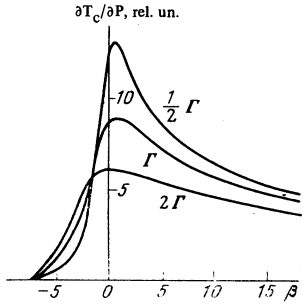

FIG. 8. Dependence of $\partial T_c/\partial P$ on $\beta = (\epsilon_F - \epsilon_c)/2T_c^0$ [15].

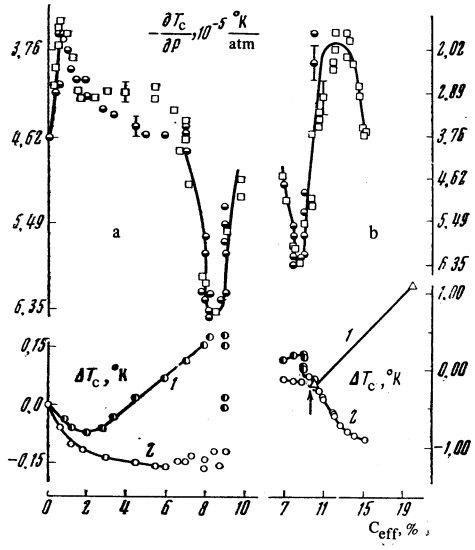

FIG. 9. $\partial T_c/\partial P$ and ΔT_c of the alloys In-Hg and In-Tl vs. the effective impurity concentration ($C_{eff} = C_{Hg} = 3.3\,C_{Tl}$). a—face centered tetragonal phase; the arrow corresponds to the phase transition, b—Face-centered cubic phase.

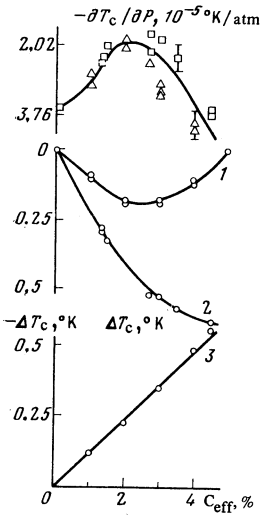

FIG. 10. Dependence of $\partial T_c/\partial P$ and ΔT_c of the alloys In-$Tl_{35\%}$—Hg (curve 1) and In-Tl (curve 2) and of the difference of T_c of the systems In-$Tl_{35\%}$-Hg and In-Tl (line 3) on the effective impurity concentration ($C_{eff} = C_{Hg} = 3.3C_{Tl}$).

electronic component of the thermal expansion coefficient, to the temperature of the superconducting transition, to the derivative of T_c with respect to pressure, etc.) should depend on the scattering processes[11].

The experimental data (Fig. 7) show that the existing theories[12,13,15], which take into account the influence of the scattering processes on the thermodynamics of a superconductor, contradict the experimental data: $\partial T_c/\partial P$ for the system In-Pb2%--Cd should decrease at the maximum in comparison with the same quantity for the In-Hg system by a factor of 3, in accordance with expression (3)[4]. Apparently, in the superconducting state, the impurity changes only the energy difference $\epsilon_F - \epsilon_c$, and does not lead to a smearing of the density of the coherent states. At the same time, as already noted, in the normal state of the metal the impurity not only changes the energy difference $\epsilon_F - \epsilon_c$, but leads also to a damping of the elementary excitations and to a smearing of their density of states. This apparently is also the reason why the correlation existing at low concentrations between the singularities in the dependences of the magnetic susceptibility χ on C and of $\partial T_c/\partial P$ on C in indium, vanishes completely on going over to larger impurity concentrations.

Indeed, the scattering processes cause the singularity of $\chi(C, T)$ in the In-Tl system[18] to become less pronounced than in the In-Cd system[19], since $\Gamma_{In\text{-}Tl} > \Gamma_{In\text{-}Cd}$ at the maximum of $\chi(C)$. With further increase of the Tl impurity concentration in the In-Tl system, no singularities of $\chi(C, T)$ were observed in the tetragonal phase[18]. At the same time, in accordance with the superconducting characteristics [$\partial T_c/\partial P(C)$], singularities in the dependence of $\partial T_c/\partial P$ on C are observed at concentrations 20—31% Tl in the In-Tl system and 6—9% Hg in the In-Hg system.

The dependences of $\partial T_c/\partial P$ on C for the systems In-Tl and In-Hg in the entire interval of the investigated concentrations are described by a single curve if the effectiveness of the action of the Tl and Hg impurities on the density of the electronic states of indium at high impurity concentrations is assumed to be the same as at low concentrations ($C_{eff} = C_{Hg} = 3.3C_{Tl}$) (Fig. 9).

A dependence of $\partial T_c/\partial P$ on C_{eff} (near $C_{eff} = 8.2\%$) with a decrease of ϵ_F by the impurity is possible in two very simple topological cases[2]: formation of a bridge between the cavities of the Fermi surface or formation of new cavities of the Fermi surface. In the concrete case, apparently, a more complicated topological transition is observed. According to the experimental data of[20], a Cd impurity decreases the maximum γ-section of the electronic surface of indium in the third Brillouin zone by 7.7%/(1 at.% Cd). Using linear extrapolation, it can be assumed that at 13 at.% Cd the electronic surface in the third Brillouin zone vanishes completely. Taking into account the different effectivenesses of the Cd, Hg, and Tl impurities ($C_{eff} = C_{Hg} = 1.67C_{Cd} = 3.3C_{Tl}$), we can assume that the vanishing of the electron group will be observed at 7.8% Hg or 26% Tl. This should lead to a nonlinear decrease of the absolute value of $\partial T_c/\partial P$ as a function of C[2].

The observed increase of the absolute value of $\partial T_c/\partial P$ at 8.2% Hg or 28% Tl should apparently be related with the fact that simultaneously with the vanishing of the electronic surface in the third Brillouin zone there are produced open sections in the hole surface of

[4] The value of Γ at the maximum is 7.8°K for the In-Hg system and 136°K for the In-$Pb_{2\%}$-Cd system. For indium $v_F \approx 1 \times 10^8$ cm/sec, $\sigma/l = 9 \times 10^{10}$ ohm^{-1} cm^{-2} [16], and $\sigma = 11.4 \times 10^4$ ohm^{-1} cm^{-1} [17], hence $1/l = 8 \times 10^5\,\rho$.

the second zone or "holes" in the first Brillouin zone.

The different behavior of $\partial T_c/\partial P$ in the systems In-Hg and In-Tl in the Hg and Tl impurity concentration intervals between the two singularities (Fig. 9) can be attributed to the different variations of the smooth part of the density of the electron states $\nu_0(\epsilon_F)$[2], which is not connected with the change of the topology of the Fermi surface.

Following the topological transition in both systems, In-Tl and In-Hg, a phase transition takes place, namely, the tetragonal face-centered lattice becomes cubic face-centered (see Figs. 2 and 3). It should be noted that the phase transitions in these systems occur at identical effective electron concentrations (Fig. 9). One cannot exclude the possibility that the phase transition in these systems is stimulated by changes in the topology of the Fermi surface[21].

2. Face-centered cubic phase. As already noted, in the cubic phases of the systems In-Tl and In-Tl35%-Hg, a nonlinear dependence of $\partial T_c/\partial P$ and ΔT_c on C is observed (Figs. 2b and 4). Comparing the plots of $\partial T_c/\partial P$ against C for both systems, we can determine the relative effectiveness of the Hg and Tl impurities in the face-centered cubic phase. As seen from Fig. 10, the relative effectivenesses of Hg and Tl in this phase are practically the same as in the tetragonal phase ($C_{eff} = C_{Hg} = 3.3C_{Tl}$). The maximum of $\partial T_c/\partial P$ in the In-Tl35%-Hg system is observed at a relative residual resistivity $\rho = 58.6 \times 10^{-2}$ ($\Gamma = 358°K$), and in the In-Tl system at $\rho = 66 \times 10^{-2}$ ($\Gamma = 403°K$).

A study of the magnetic susceptibility of the In-Tl system in this phase has revealed only an insignificant linear variation of $\chi(C)$, which was attributed to the absence of a singularity in the density of the electronic states[18]. This, too, is not surprising, since such values of Γ smear out practically completely the singularity in $\nu(\epsilon_F)$ when the metal is in the normal state, in accord with formula (2).

In the cubic phase we observe a large nonlinear change of $\partial T_c/\partial P(C)$ at 32–50% Tl and 0–6% Hg in the systems In-Tl and In-Tl35%-Hg, respectively. This change is connected with the change of the conduction-electron concentration. In analogy with the singularities of $\partial T_c/\partial P$ as functions of C in the tetragonal phase, due to singularities in $\nu(\epsilon)$, such a behavior of $\partial T_c/\partial P$ (see Figs. 2b and 4) can be naturally connected with the singularities in the density of the electronic states forming coherent states—"Cooper pairs."

It is seen from Fig. 10 that the course of $\Delta T_c(C_{eff})$ for these systems is nonlinear and is different. We assume that the observed dependences of ΔT_c on C are the result of the action of two mechanisms, a nonlinear and a linear one. Since the difference $\Delta T_c(C_{eff})$ in Fig. 10 (curve 3) is linear, this means that the nonlinear mechanisms in $T_c(C)$ are the same for the systems In-Tl and In-Tl35%-Hg, while the linear ones differ from each other. Within the framework of the existing theories[22,23], at such high impurity concentrations and at such a residual resistivity, there are no nonlinear mechanisms in $T_c(C)$. It is natural to assume that the nature of this mechanism in $T_c(C)$ is the same as in the nonlinear dependence of $\partial T_c/\partial P$ on C, i.e., it is due to the singularity of the density of the coherent electronic states. The differences in the linear mechanism of $\Delta T_c(C)$ in the systems In-Tl and In-Tl35%-Hg at the same topological structure of the Fermi surface may apparently be due to the large difference between the potentials of the internal shells of the Tl and Hg impurity ions and that of the indium ion[24].

Thus, it can be concluded from the entire aggregate of the experimental data that singularities exist in the density of the electronic states forming a coherent state—"Cooper pairs." At the same time, it is completely meaningless, from the point of view of modern theory of metals in the normal state, to speak of singularities of the density of the electronic states in these alloys.

It should be noted that the singularities in the density of the phonon states become smeared out under the influence of external actions (impurity concentration, lattice distortions)[25-27]. This is connected with the fact that when the metal goes from the normal to the superconducting state, the phonon system does not experience a radical realignment, unlike the electronic system, where "Cooper pairs" are produced.

Because of this circumstance, an investigation of the topological singularities of the electronic spectrum of a metal with superconducting characteristics is effective in those cases when the customarily employed methods (the de Haas-van Alphen effect, absorption of ultrasound, etc.) cannot be used.

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