IONIZATION WAVE PROPAGATING BECAUSE OF DIFFUSION OF RESONANT QUANTA AND MAINTAINED BY MICROWAVE RADIATION

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A theory is presented of an ionization wave propagating in low-density noble gases as a result of diffusion of the resonance radiation. The wave is supplied with energy by microwave radiation. The theory is developed to explain phenomena in a waveguide when the plasma front, arising at the end of the waveguide at an appreciable distance from the microwave radiation source, moves rapidly toward the source, covering the waveguide, and all this occurs at field strengths much smaller than the threshold values required for breakdown of the gas. The problem of the regime of a wave that is stationary in the reference system in which it rests is formulated and solved approximately. The wave propagation velocity and electron density in the plasma are determined as functions of the microwave power. The existence threshold of the regime is estimated. Qualitative agreement with the experiments is obtained.

1. INTRODUCTION

As shown by Bethke and Ruess,[1,2] if a localized plasma is produced by means of a shock wave or a discharge at a waveguide end far from a microwave-radiation source, then the plasma front becomes detached from its initial place and moves rapidly towards the source. The experiments were performed in a cylindrical waveguide of 2.5 cm radius and length exceeding 1 meter, at 8.35 GHz, in Xe, Kr, and Ar at pressures from 0.3 to 3 mm Hg. The effect was produced even at low microwave-radiation fluxes exceeding only 0.2–1 W/cm^2. The flux necessary to break down the gases under the same conditions is 40–200 W/cm^2. When the flux was increased from 0.2 to 50 W/cm^2, the front velocity increased from tens of meters to tens of kilometers per second. The maximum electron concentrations were $(0.7-9)\times 10^{12}$ cm^{-3} (the critical concentration is 0.86×10^{12} cm^{-3}). The gas remained stationary and the propagation of the plasma front had the character of an ionization wave. Dielectric windows transparent to the microwave radiation were placed in the path of the wave. The ionization wave was stopped by a window made of plastic with a shortwave transparency boundary $\lambda \approx 2000$ Å, but passed through a window of LiF, which transmits the ultraviolet down to approximately 1100 Å. This means that the wave propagation mechanism is the diffusion of the resonant radiation of the atoms, the wavelengths of which lie precisely in the interval 1000–1500 Å.

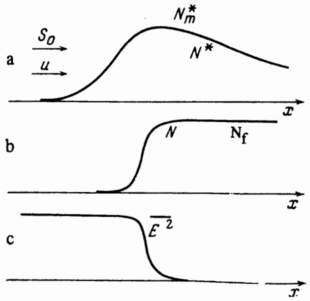
We present in this article a theory of such an ionization wave. The problem consists of determining the propagation velocity and the electron concentration behind the wave, and of estimating the minimal (threshold) power necessary to maintain the wave. The kinetic processes during the course of which the ionization develops are complicated and varied,[2] and we shall describe them very schematically. Even in the simplest physical formulation, the problem of the regime of the ionization wave turns out to be so complicated that it is necessary to employ appreciable simplifications for its solution. For this reason we claim no more than elucidation of the main physical laws governing the phenomenon and an estimate of the orders of magnitude.[1)]

2. FORMULATION OF PROBLEM AND EQUATIONS

We assume a very simple scheme of the process: the electrons acquire energy in the microwave field and excite atoms to a single resonant level, after which the excited atoms are ionized by electron impact. (Their ionization potential I_i^* is one-third the excitation potential I^*.) Since the fields are much lower than the breakdown thresholds, the electrons do not reach the ionization potential I_i of the nonexcited atoms at all. On the other hand, the excitation from the plasma is transferred to the unperturbed layers by diffusion of the resonant radiation. The primary electrons result from photoionization of the excited atoms, the photoeffect from the walls, etc. We shall disregard excitation of metastable states, impact transitions between metastable and resonant levels, quenching of the excitation, successive increases of the degree of excitation, associative ionization, etc., since they do not play the principal role and can only influence numerically the total excitation and ionization rates. The recombination of the electrons and their diffusion towards the walls (mainly ambipolar) is estimated to be negligible; they lead only to a decay of the plasma behind the wave.

Let us consider the one-dimensional stationary regime of the ionization wave in a coordinate system where the wave is at rest (see the figure). In the laboratory system, the gas is stationary and the density of all the atoms N_a is constant. A macroscopic gas particle passes through the wave along the x axis with constant velocity u, equal in magnitude to the wave propa-

[1)]It must be stated that at large microwave powers (kilowatts), a propagation of a discharge towards the source, having a similar character, was also observed at high (atmospheric) pressure in waveguides containing air [3]. The velocities in this case are of the order of several meters per second. A physical interpretation and theory of this process were given in [4].



Schematic distributions of the quantities in a plane stationary ionization wave: a—density of excited atoms N*, b—electron density N, c—square of microwave electric field $\overline{E^2}$ averaged over the period of the oscillations. The arrows indicate the directions of the influx of nonionized gas into the wave u and of the energy flux in the incident electromagnetic wave S_0.

gation velocity, and d/dt = ud/dx. The unknown functions of the coordinate x in the wave are the electron density N, the density N* of the excited atoms ($N^* \ll N_a$), and the microwave electric field E. The function N(x) satisfies the kinetics equation, which takes, without allowance for the electron losses, the form

$$u dN / dx = \alpha N N^*, \quad \alpha = \overline{v \sigma_i^* (v)}, \tag{1}$$

where v is the electron velocity and σ_i^* the ionization cross section of the excited atom, and the bar denotes averaging over the electron spectrum.

In the presence of diffusion of the resonant radiation, the density N* of the excited atoms is described by a well known integrodifferential equation.[5] We shall disregard the radial distribution of N* and assume the tube to be infinite and its internal walls to absorb the resonant quanta. We assume a kernel such that all the quantum absorption points can be regarded as located on the tube axis. We then have

$$u \frac{dN^*}{dx} = -\frac{N^*(x)}{\tau} + \frac{1}{\tau} \int_{-\infty}^{+\infty} N^*(\xi) K(|\xi - x|) d\xi + \alpha^* N N_a - \alpha N N^*,$$

$$K(z) = \frac{1}{2} \int_0^\infty d\nu F(\nu) k_\nu \int_0^{\operatorname{arctg} R/z} e^{-k_\nu z/\cos\theta} \operatorname{tg}\theta \, d\theta.$$

Here τ is the lifetime of the excited atom relative to emission of a quantum, R the tube radius, k_ν the coefficient for absorption of a quantum of frequency ν, $F(\nu)$ the emission-line contour normalized to unity, θ the angle between the direction of motion of the quantum and the x axis, and $\alpha^* = \overline{v\sigma^*(v)}$, where σ^* is the cross section for the excitation of the atoms by electron impact (the excitation rate is determined by the microwave field via the electron spectrum).

Let us simplify the integro-differential conversion by converting it into a differential equation of the diffusion type. We assume that the density of the excited atoms varies slowly along the x axis, expand $N^*(\xi)$ about the point x, and terminate the expansion with the term proportional to d^2N^*/dx^2. The coefficient of dN^*/dx vanishes by virtue of the symmetry of the kernel. Combining the integral with the first term of the expansion and the term $-N^*(x)/\tau$, we obtain the equation

$$u \frac{dN^*}{dx} = D \frac{d^2 N^*}{dx^2} - \frac{N^*}{T} + \alpha^* N N_a - \alpha N N^*, \tag{2}$$

where

$$D = \frac{1}{\tau} \int_0^\infty z^2 K(z) dz, \quad \frac{1}{T} = \frac{1}{\tau} \left[1 - 2 \int_0^\infty K(z) dz \right].$$

The constant D has the physical meaning of the diffusion coefficient and T has the meaning of the average time in which the excitation reaches the wall. The principal role in the transport is played by the wings of the line, since the mean free path l_0 of the quanta at the center of the line is exceedingly small. Therefore the line contour can be assumed to be of the dispersion type and the corresponding equations can be used for $F(\nu)$ and k_ν. An approximate calculation of the integrals yields

$$D = \frac{l^2}{3\tau}, \quad \frac{1}{T} = \frac{3D}{R^2}, \quad l = \frac{\pi^{1/2} R^{3/4} l_0^{1/4}}{18^{1/4} \Gamma(3/4)} = 0.7 R^{3/4} l_0^{1/4}. \tag{3}$$

l has the meaning of the mean free path of the quanta effecting the diffusion. The expression for T is typical of the diffusion process. The diffusion approximation is valid if $l |dN^*/dx| \ll N^*$, which is satisfied in our case, since $l \ll R$ as a result of the extreme smallness of l_0.

The average rate at which the electron acquires energy in the field is given by the well known formula[6]

$$\frac{d\varepsilon}{dt} = \frac{e^2 \overline{E^2}}{m(\omega^2 + \nu_m^2)} - \frac{2m}{M} \varepsilon \nu_m, \tag{4}$$

where ω is the field frequency, ν_m the frequency of the elastic collisions of the electrons with the atoms and m and M the masses of the electron and of the atom, and the bar over E^2 denotes averaging over the period of the field oscillations. It is seen from (4) that in an excessively weak field the elastic losses do not permit the electrons to reach the energy $\epsilon = I^*$ needed for excitation of the atoms, and this imposes a limit on the existence of this regime. An estimate of the threshold field from the condition $d\epsilon/dt = 0$ at $\epsilon = I^*$ makes it possible to find the corresponding threshold energy fluxes: $S = c\overline{E^2}/4\pi$. For the conditions of the experiments in [1,2] we obtain 0.4 W/cm² in Xe and 1.2 W/cm² in Ar, which agrees with the experimental thresholds in order of magnitude.

At fluxes greatly exceeding the threshold values, the elastic losses can be neglected. In addition, as shown by estimates, the energy lost by the electrons to ionization of the excited atoms is in general no larger than the loss to excitation of the nonexcited atoms. Further, an electron possessing an energy barely exceeding I*, excites an atom with a high degree of probability. Under these conditions, the average time necessary for the electron to complete the excitation act is approximately equal to the time it needs to acquire the energy I* (disregarding elastic losses, i.e., at M = ∞). Consequently, the frequency of the excitations $\alpha^* N_a$ is equal to $(d\epsilon/dt)_{M=\infty}/I^*$. We can state this differently: the dissipated field energy goes mainly into excitation of the atoms and $\sigma\overline{E^2} = I^* \alpha^* N N_a$, where $\sigma = e^2 N \nu_m / m(\omega^2 + \nu_m^2)$ is the conductivity; the two statements are equivalent.

Thus, owing to the assumptions made, we express the rate of excitation of the atoms $\alpha N N_a$ directly in terms of the Joule heating. Neglecting also, in accord with the same assumptions, the loss of excited atoms due to their ionization, we obtain in place of (2) the equation

$$u \frac{dN^*}{dx} = D \frac{d^2 N^*}{dx^2} + \frac{\sigma \overline{E^2}}{I^*} - \frac{N^*}{T}. \tag{5}$$

The microwave electric field is described by a wave equation that follows from Maxwell's equations. For a monochromatic field $E \sim e^{-i\omega t}$ we have

$$\frac{d^2E}{dx^2}+\frac{\omega^2}{c^2}\left(\varepsilon'+i\frac{4\pi\sigma}{\omega}\right)E=0, \tag{6}$$

where $\epsilon' = 1 - 4\pi e^2 N/m(\omega^2+\nu_m^2)$ is the dielectric constant.

If the electron energy distribution function $f(\epsilon)$ is assumed to be quasistationary, the elastic losses and the influence of the inelastic collisions with the excited atoms are neglected, and it is also assumed that at $\epsilon = I^*$ there is a powerful "sink," i.e., $f(I^*) = 0$, then we obtain from the kinetic equation[7] $f(\epsilon) = (3N/I^*) \times [1-(\epsilon/I^*)^{1/2}]$. Under these assumptions $f(\epsilon)$, and consequently also α, does not depend on the field (the excitation-rate constant α^* is determined by the "flux" along the energy axis and is given in this case by the formula presented above). In a weak field, such that $\overline{E^2} < E_f^2$, where E_f^2 is determined from the condition $d\epsilon/dt = 0$ at $\epsilon = I_i^*$, ionization due to elastic losses stops. Thus, we put α = const at $\overline{E^2} > E_f^2$ and $\alpha = 0$ at $E^2 > E_c^2$.

Let us establish the boundary conditions for the formulated system (1), (5), (6). Ahead of the ionization wave at $x = -\infty$ we are given the field amplitude E_0 or the energy flux S_0 in the incident electromagnetic wave, $N^* = 0$, and we must specify some small electron density N_0, without which the ionization cannot start. The results depend little on the value of N_0. Formally at $N_0 \neq 0$ the electromagnetic wave is completely absorbed even before its approach to the ionization wave. This unphysical difficulty can readily be eliminated by assuming, for example, that the conductivity is proportional not to N but to $N - N_0$. Behind the ionization wave, at $x = +\infty$, we have $E = 0$ (since $N \neq 0$), and $N^* = 0$ as a result of the drift of the excitations towards the walls. It is easily seen that one of the conditions imposed on N^* is "superfluous" for the system (1), (5), (6). This indeed enables us to determine the unknown propagation velocity u.

3. APPROXIMATE SOLUTION

As shown by calculations, the excitation diffuses from the sources to distances greatly exceeding the width of the zone in which the sources are located. We can therefore put in (5) in the zeroth approximation $\sigma\overline{E^2} = S_1\delta(x)$. This yields, with allowance for the boundary conditions

$$N^*(x)=\frac{S_1}{I^*\sqrt{u^2+u^{*2}}}\cdot\begin{cases} e^{x/\Delta_1}, & x\leqslant 0\\ e^{-x/\Delta_2}, & x\geqslant 0\end{cases}, \tag{7}$$

$$\frac{1}{\Delta_{1,2}}=\frac{u}{2D}\left[\sqrt{1+(u^*/u)^2}\pm 1\right],\quad u^*=\sqrt{4D/T}. \tag{8}$$

The quantity

$$S_1=\int_{-\infty}^{\infty}\sigma\overline{E^2}\,dx$$

is the dissipating part of the incident electromagnetic energy flux: $S_1 = S_0(1-\rho)$, where ρ is the reflection coefficient. Integrating (1) with the aid of (7), we obtain the first approximation for the electron distribution:

$$N(x)=N_0e^{\gamma(x)},\quad \gamma=\frac{\alpha}{u}\int_{-\infty}^{x}N^*dx, \tag{9}$$

$$\gamma(x)=\frac{\alpha S_1}{uI^*\sqrt{u^2+u^{*2}}}\cdot\begin{cases}\Delta_1e^{x/\Delta_1}, & x\leqslant 0\\ \Delta_1+\Delta_2(1-e^{-x/\Delta_2}), & x\geqslant 0\end{cases} \tag{10}$$

The second formula in (10) is valid only up to a certain point $x_f > 0$, at which the field, which attenuates as it penetrates into the ionization wave, drops to the value E_f. At $x > x_f$ we have $\overline{E^2} < E_c^2$, $\alpha = 0$, and $N(x)$ = const. This final (largest) value of the electron density in the plasma is equal to $N_f = N_0 \exp(\gamma_f)$, where $\gamma_f = \gamma(x_c)$.

In principle, knowing $N(x)$, we could solve the wave equation (6) and determine x_f, i.e., N_f, as well as ρ. This cannot be done in practice, and we proceed in a highly approximate manner (just as in [4]). We separate approximately the effects of wave dissipation and creation of the reflected wave, and assume that the dissipation is determined by the absorption of only the transmitted electromagnetic wave, in which the energy flux S attenuates in the same manner as when the wave propagates in a homogeneous medium. The absorption coefficient μ is then calculated in accordance with the well known formulas[6] in terms of the local values of $\epsilon'[N(x)]$, $\sigma[N(x)]$, and ρ is calculated as the reflection of a wave incident normally from vacuum on the sharp boundary of a medium with $\epsilon'(N_f)$ and $\sigma(N_f)$. In this approximation we have

$$\frac{dS}{dx}=-\mu S,\quad \sigma\overline{E^2}=\mu S, \tag{11}$$

$$S=S_1e^{-\tau(x)},\quad \tau(x)=\int_{-\infty}^{x}(\mu-\mu_0)\,dx, \tag{12}$$

where we subtracted $\mu_0 \equiv \mu(N_0)$ from μ in order to exclude the nonphysical divergent part of the optical-thickness integral. The coordinate x_f is determined from the condition

$$\tau(x_f)=\int_{-\infty}^{x_f}(\mu-\mu_0)\,dx=\ln\frac{S_1}{S_f},\quad S_f=\frac{n_fcE_f^2}{4\pi}, \tag{13}$$

where n_f is the refractive index corresponding to N_c.

To calculate the integral (13), we note that usually the $\mu(N)$ dependence ranges from $\mu \sim N$ to $\mu \sim \sqrt{N}$. We put for simplicity $\mu = bN^{1/\beta}$, where b and β are constants ($1 \le \beta \le 2$). Using (9) and (10) and noting that $\gamma_1 \equiv \gamma(0)$ is a large number, since $N_1 \equiv N(0) \gg N_0$, we obtain the optical thickness of the pre-ionization zone (prior to the start of intense dissipation):

$$\tau(0)=\int_{-\infty}^{0}(\mu-\mu_0)\,dx=\mu_0\int_{0}^{\gamma_1}\frac{e^{\gamma/\beta}-1}{d\gamma/dx}d\gamma\approx\frac{\beta}{\gamma_1}\mu_1\Delta_1, \tag{14}$$

where $\mu_1 \equiv \mu(N_1)$ (Δ_1 is the effective width of the zone). In the region of developed ionization (strong dissipation), we neglect approximately the change of N^*, putting $d\gamma/dx = \gamma_1/\Delta_1$. We obtain

$$\int_{0}^{x_f}(\mu-\mu_0)\,dx\approx\mu_0\int_{\gamma_1}^{\gamma_f}\frac{e^{\gamma/\beta}d\gamma}{d\gamma/dx}\approx\frac{\beta}{\gamma_1}(\mu_f-\mu_1)\Delta_1, \tag{15}$$

where $\mu_f \equiv \mu(N_f)$. Adding (14) and (15) we get $\tau(x_f) = \beta\mu_c\Delta_1/\gamma_1$, after which we obtain on the basis of (13), (8), and (10) an expression that relates the final electron density directly with the wave propagation velocity u:

$$N_f = \left[\frac{\alpha S_1 \ln(S_1/S_k)}{\beta\, bu I^* \sqrt{u^2+u^{*2}}}\right]^{\beta}, \quad S_1 = S_0[1-\rho(N_f)]. \tag{16}$$

Actually this equation defines N_f and not u.

To derive an equation for the unknown parameter u, which is the eigenvalue of the system (1), (5) and (11), we turn to the initial assumption that the dissipative term $\overline{\sigma E^2}$ be replaced by a δ function. This replacement is equivalent to replacing the energy flux S(x) satisfying Eq. (11) by a step function:

$$S = S_1 \text{ for } < 0, \qquad S = 0 \text{ for } x > 0. \tag{17}$$

Of course, we are free to choose the spatial point at which we place the "step," since this is essentially the question of the choice of the origin. But the electron density N_1 at this point, which effectively separates the region where the field dissipation is negligible from that where it is intense, is far from arbitrary. It is clear that the optical thickness of the entire layer of "weak" dissipation $\tau(0)$ should be of the order of unity. Indeed, if $\tau(0)$ greatly exceeds unity, then the electromagnetic wave will become too strongly absorbed even when $N < N_1$, prior to the arrival at the zone of the "main" dissipation. On the other hand, if $\tau(0)$ is much smaller than unity, the wave will propagate in some section of the "dissipation" zone without significant absorption. Either assumption contradicts the definition of the concept of "main dissipation." This reasoning leaves a certain leeway in the choice of the concrete number to which we must equate $\tau(0)$. It is easily seen, however, that the approximation (17) corresponds to $\tau(0) = 1$. In fact, (11) leads to the exact integral relation[2)]

$$S_1 = \int_{-\infty}^{\infty} \mu S dx. \tag{18}$$

Substituting the approximating step function (17) in (18), we indeed obtain $\tau(0) = 1$. Subtracting further μ_0 from μ, as before, we obtain the equation

$$\tau(0) \equiv \int_{-\infty}^{0} \{\mu[N(x)] - \mu_0\} dx = 1, \tag{19}$$

which enables us to determine u. It is physically clear that the condition (19) should regulate the velocity of the ionization wave, for in accordance with (1) it is precisely the velocity u which determines the rate of spatial growth of the ionization, and consequently also of the optical thickness, up to the point where a considerable electron density N_1 is reached.

Substituting $\mu_1 = \mu_0 \exp(\gamma_1/\beta)$ in expression (14) for $\tau(0)$ and taking the logarithm of (19), we obtain

$$\gamma_1 = \beta \ln \frac{\gamma_1}{\beta \mu_0 \Delta_1}, \quad \gamma_1 = \frac{\alpha \Delta_1 S_1}{I^* u \sqrt{u^2 + u^{*2}}}, \tag{20}$$

where the second expression determines γ_1 in terms of u in accordance with formula (10). Equation (20) describes the velocity of the ionization wave u with a dissipating energy flux S_1. All the remaining quantities in (20) are known. Having defined $u(S_1)$, we can calculate $N_f(S_1)$ from (16) and then obtain ρ and then $S_0 = S_1/(1-\rho)$, i.e., obtain as a result $u(S_0)$ and $N_f(S_0)$, which is indeed the final purpose of this solution. The electron densities on the boundaries of the main dissipation zone are connected by the relation $\tau(x_f)/\tau(0) = \mu_f/\mu_1 = \ln(S_1/S_f)$, whence $N_f/N_1 = (\ln(S_1/S_f))^{\beta}$.

As shown by the calculations, the characteristic velocity u^* is a rather large quantity, and this makes it possible, in a broad range of not too high microwave powers, when $u < u^*$, to obtain explicit expressions for the propagation velocity and the other parameters of the wave. In this case, in accordance with (18) and (13), $\Delta_1 \approx \Delta_2 \approx \sqrt{DT} = R/\sqrt{3}$, and in accordance with the first equation of (20) $\gamma_1 = \text{const} \equiv \gamma_c$. The second formula of (20) yields

$$u = \frac{\alpha T}{2\gamma_c I^*} S_1. \tag{21}$$

Also proportional to S_1 is the maximum density of the excited atoms $N^*_{max} = S_1/I^* u^*$ (see (7)). The electron density at the start of the dissipation zone N_1 is constant, and the final density depends on S_1 only logarithmically:

$$N_1 = (\sqrt{3}\gamma_c/\beta b R)^{\beta}, \qquad N_f = N_1 (\ln(S_1/S_f))^{\beta}. \tag{22}$$

4. NUMERICAL CALCULATIONS AND DISCUSSION

By way of an example, let us consider xenon at p = 3 mm Hg. Xenon has two strong resonant lines, which we combine, taking N^* to be the summary population, and D and 1/T to be mean values. According to Wilkinson's data,[9] for λ = 1470 Å we have $\tau = 3.74 \times 10^{-9}$ sec and the oscillator strength is f = 0.26. When account is taken of the excitation exchange in the collisions of atoms of one gas,[10] we get $l_0 = 2.6 \times 10^{-6}$ cm. For λ = 1296 Å we have $\tau = 2.8 \times 10^{-9}$ sec, f = 0.27, and $l_0 = 2.5 \times 10^{-6}$ cm. At a tube radius R = 2.5 cm, we get $D = 3.2 \times 10^5$ cm²/sec, $T = 6.5 \times 10^{-6}$ sec and $u^* = 4.5$ km/sec (these quantities are almost independent of the pressure): $I^* = 9.0$ eV.

There are no experimental data on the cross sections for ionization of the excited atoms by electron impact, and we shall estimate α by using the universal Gryzinski formula for $\sigma_i(\epsilon)$, derived on the basis of classical mechanics. The formula describes well the measured cross sections for ionization from the ground state of many atoms, including cesium,[11] to which the excited xenon atom should be similar. The Gryzinski curve is given in [11], as is also the experimental curve for cesium. Calculation yields an ionization rate constant $\alpha \approx 4 \times 10^{-8}$ cm³/sec.

The frequency of the elastic collisions of the electrons is $\nu_m \approx 2.4 \times 10^{10}$ sec⁻¹, and the field frequency in the experiments of [1,2] is $\omega = 5.3 \times 10^{10}$ sec⁻¹. The microwave absorption coefficients calculated with these data in the main dissipation zone can be approximated by the formula $\mu = 3.5 \times 10^{-6} N^{1/2}$ cm⁻¹, and this determines b and β. The experiments of [1,2] offer evidence that the initial electron density is $N_0 < 10^{10}$ cm⁻³, but at the same time it is smaller by not many orders of magnitude than the final density 10^{12}–10^{13} cm⁻³. We assume for the calculations $N_0 = 5 \times 10^8$ cm⁻³, corresponding to

2) A relation having a similar meaning was obtained also in the case of a high-frequency discharge [8], which constitutes another limit with respect to field description. A similar "step" approximation was used in [8].

S_1, W/cm²	$N_f \cdot 10^{-12}$, cm⁻³	$N^*_m \cdot 10^{-12}$, cm⁻³	$1-\rho_0$	$1-\rho$	S_0, W/cm²	$u \cdot 10^{-4}$, cm/sec	$u_{exp} \cdot 10^{-4}$ cm/sec
0,5	1.8	0.8	0.56	0.79	0.63	0.68	2.5
1	2.6	1.6	0.48	0.71	1.4	1.4	6.0
3	4.7	4.7	0.35	0.65	4.4	4.1	20
5	6,0	9.3	0.31	0.46	11	6.8	50
10	8.1	15	0.27	0.42	24	14	80
15	9.0	23	0.25	0.38	40	21	110

an extrapolated value[3)] $\mu_0 = 0.08\ \text{cm}^{-1}$. The first equation of (20) then yields $\gamma_c = 7$. At the same time, $\mu_1 = 2.4\ \text{cm}^{-1}$ and $N_1 = 0.45 \times 10^{12}\ \text{cm}^{-3}$. The ionization of excited xenon ceases at $S_f \approx 0.1\ \text{W/cm}^2$. We emphasize that the results depend weakly (logarithmically) on either S_f or μ_0.

The results of the calculation of the wave velocity u and of the final electron density N_f are listed in the table. The fluxes S_0 in the incident microwave were calculated from S_1 in terms of the reflection coefficients ρ. The latter were made more precise, just as in [4], in comparison with the values of ρ_0 for a sharp plasma boundary; the smearing of the boundary was taken into account. The last column of the table gives the experimental velocities u_{exp} from [2], corresponding to the same values of S_0.

The calculation gives true values of the electron densities in the plasma, which are in fair agreement with [2]. The calculated velocities u increase with increasing microwave power, just as in the experiment, but turn out to be lower by a factor 4–7. It must be assumed that this is due to underestimation of the assumed constant α (N_f does not depend on α, and therefore the calculation gave the correct figures). Stepwise ionization proceeds rapidly via successive increases of the degree of excitation by electron impact. The strongly excited atom, combining with the usual gas atom, then produces with high probability a molecular ion and an electron.[2] This, according to the estimates, is fully capable of increasing the ionization rate by a total of several times. The equation for electron production can be written as before in the form (1), but now this equation will describe the summary kinetics, and α will represent a certain effective constant of the resultant ionization rate, and can be several times larger than the value assumed in the calculation. Partial reflection of the resonant quanta from the tube walls can increase somewhat the time T of the drift of the excitation to the walls as well as the velocity u, since $u \sim T$. We note that in Xe at 3 mm Hg the breakdown field corresponds to a flux of 43 W/cm² in the traveling wave, but in this case there is also the field of the reflected wave, which may become superimposed on the field of the incident wave, i.e., the flux threshold in the incident wave can be lower. An additional ionization mechanism, not taken into account at all, comes into play at fields close to breakdown.

As to the threshold for the existence of the ionization-wave regime, it is determined mainly by the elastic electron energy losses, and according to the estimate made in Sec. 2 it amounts to several tenths of a W/cm² in agreement with the experimental results.[2]

We assume that the theory developed above describes correctly the most essential features of the phenomenon in question; its results are in reasonable agreement with experiment, but, of course, not all the problems have been solved here. In particular, the velocity jumps observed under certain conditions[3] remain unexplained.

3) As follows from the derivation of the fundamental formulas, μ_0 should be taken to mean not the true coefficient of N_0, but a value extrapolated to N_0 by means of the formula used in the main dissipation zone.

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