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MAGNETIC IMPURITIES IN AN ALMOST MAGNETIC METAL

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We study the properties of magnetic impurities in metals which are nearly ferromagnetic so that their magnetic susceptibility is anomalously high. The impurity spin magnetizes the conduction electrons in regions which are much larger than the interatomic distances. The effective electron-impurity exchange interaction becomes long-range and the scattering amplitude has a steep maximum for small angles. The Kondo effect in almost magnetic metals can thus differ qualitatively from the Kondo effect in normal metals. In the most characteristic case the impurity resistivity and the effective magnetic moment decrease when the temperature is lowered, regardless of the sign of the exchange interaction constant. This kind of behavior has been observed experimentally.[1]

THE exchange interaction of conduction electrons with each other is large in Pd and Pt and only slightly smaller than is necessary for the occurrence of a ferromagnetic transition. As a result these metals have an anomalously high magnetic susceptibility. Doniach and Engelsberg[2] have shown that the spatial and frequency dispersion of the susceptibility in almost magnetic metals starts at smaller values of the frequencies and wave vectors than in normal metals. This effect can be interpreted as the existence of low-frequency Bose-type excitations—paramagnons—in almost magnetic metals.

We study below the properties of magnetic impurities in almost magnetic metals. The impurity spin magnetizes the conduction electrons in the vicinity of the impurity. The size of this region is determined by the spatial dispersion of the susceptibility and appreciably exceeds the lattice constant in almost magnetic metals. As a result the impurity acquires a "gigantic" magnetic moment.

The interaction of the impurities with the paramagnons starts to affect the temperature dependence of the magnetic moment and the resistivity at higher temperatures than the Kondo corrections[3-5] to the scattering amplitude. In this temperature range the effective magnetic moment and the resistivity decrease logarithmically when the temperature is lowered independent of the sign of the electron-impurity exchange interaction. When the temperature is further lowered the Kondo corrections become important and in the very low temperature region the behavior of the resistivity and of the magnetic moment must become the same as in normal metals.

We consider in Secs. 3 to 6 the impurity-paramagnon interaction and study in Secs. 7 and 8 the Kondo corrections to the scattering amplitude. We assume that the impurity concentration is fairly small and we neglect the impurity-impurity interactions.

1. CORRELATION FUNCTION OF AN ALMOST MAGNETIC METAL

The magnetic properties of a metal are determined by the spin correlation function which in the temperature-dependent technique has the form

$$D(\tau, \mathbf{r})\delta^{ij} = \langle \sigma^i(00)\sigma^j(\tau, \mathbf{r})\rangle. \quad (1)$$

The angle brackets indicate an ensemble average. The spin density operators $\sigma^i(\tau, \mathbf{r})$ can be expressed in terms of Pauli matrices $\sigma^i_{\alpha\beta}$ and of operators, $\psi^+_\alpha(\mathbf{r})$ and $\psi_\alpha(\mathbf{r})$, creating or annihilating an electron with spin α at the point $\mathbf{r}$:

$$\sigma^i(0, \mathbf{r}) = \psi_\alpha{}^+(\mathbf{r})\sigma_{\alpha\beta}{}^i\psi_\beta(\mathbf{r}), \quad \sigma^i(\tau, \mathbf{r}) = e^{H\tau}\sigma^i(0, \mathbf{r})e^{-H\tau}. \quad (2)$$

Here H is the electron-electron interaction Hamiltonian. We also introduce a symbol for the Fourier component of the correlation function

$$D(\omega_n) = \int_0^\beta d\tau\, e^{\omega_n\tau}D(\tau, \mathbf{r}=0), \quad \omega_n = 2\pi i n T, \quad \beta = 1/T \quad (3)$$

and

$$D(\omega_n, \mathbf{k}) = \int_0^\beta d\tau \int d^3\mathbf{r}\, e^{\omega_n\tau - i\mathbf{k}\mathbf{r}}D(\tau, \mathbf{r}). \quad (4)$$

The static magnetic susceptibility is connected with D through the formula

$$\chi = \mu_B{}^2 D(\omega=0, \mathbf{k}\to 0), \quad \mu_B = e\hbar/mc. \quad (5)$$

In the Fermi-liquid theory the static spin correlation function can be expressed in terms of the exchange scattering amplitude of the electrons, F:[6]

$$D(\omega=0, \mathbf{k}\to 0) = \frac{2\nu}{1+F}, \quad \nu = \frac{mp_F}{2\pi^2}, \quad (6)$$

where ν, m, and p_F are the density of states, the mass, and the momentum of the electrons at the Fermi surface. A metal is stable as regards the ferromagnetic transition, if $F > -1$.[7] In almost magnetic metals F is close to -1 which causes a large value of the susceptibility. Owing to the same circumstance, D starts to depend on ω and k even when $\omega \ll kv$ and $k \ll p_F$. Neglecting the quantities ω/kv and k/p_F in comparison with 1, but not in comparison with $1 + F$, we find

$$D(\omega_n, \mathbf{k}) = 2\nu[1 + F + (ak)^2 + \pi|\omega_n|/2kv]^{-1}, \quad v = p_F/m. \quad (7)$$

The coefficient of $|\omega_n|/kv$ in the denominator of (7) is equal to $\pi/2$, as follows from Fermi-liquid theory. The coefficient a can not be expressed in terms of Fermi-liquid parameters and is of the order of magnitude of the interatomic distance. An equation similar to (7) was derived in ref. 2 by an approximate summation of one class of diagrams. More complex diagrams are not small and the connection between F and the interaction constant and between a and p_F which was obtained in that paper in actual fact does not occur. We can consider the function $D(\omega_n, \mathbf{k})$ as a Green function for Bose-type paramagnon excitations.

We shall need an expansion of $D(\omega_n)$ for small frequencies:

$$D(\omega_n)-D(0)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}[D(\omega_n,\mathbf{k})-D(0,\mathbf{k})]\approx-\frac{\pi\nu^2|\omega_n|}{2(ap_F)^2(1+F)}. \tag{8}$$

This expression is valid when $|\omega_n| \ll \epsilon_0$, where

$$\varepsilon_0=\varepsilon_F(1+F)^{3/2}. \tag{9}$$

When ω_n increases further we can use the formula

$$D(\omega_n)-D(0)\approx-\frac{2^{7/3}\pi^{4/3}\nu^2\varepsilon_F}{3^{3/2}(ap_F)^{8/3}}\left(\frac{|\omega_n|}{\varepsilon_F}\right)^{1/3}, \tag{10}$$

$$\varepsilon_0\ll|\omega_n|\ll\varepsilon_F.$$

2. EVALUATION OF THE IMPURITY SPIN CORRELATORS

We assume that the interaction between the impurity spin and the electrons in the metal is described by the Hamiltonian

$$J\mathbf{S}\boldsymbol{\sigma}(0, 0), \tag{11}$$

where J is the exchange interaction constant, **S** the impurity spin operator, and $\boldsymbol{\sigma}(0, 0)$ the electron spin density at the point $\mathbf{r} = 0$ (see (2)). We shall evaluate below different quantities as series in the constant J. There will then appear impurity spin correlators for which Abrikosov[4] has proposed a pseudo-fermion diagram technique. An impurity was described by a fermion field with 2S + 1 components. The Green function had the form

$$G_{0\alpha\beta}(\omega_n+\mu)=\frac{\delta_{\alpha\beta}}{\omega_n+\mu},\quad \omega_n=(2n+1)\pi iT, \tag{12}$$

where μ is the chemical potential. The spin correlator corresponding to a well-defined order of impurity spins was represented by one pseudo-fermion loop, the vertices of which were associated with the impurity spin operators and the lines connecting the vertices by the functions $G_{0\alpha\beta}$. To obtain a physical result it was necessary to take the limit as $\mu \to -\infty$ and calculate the coefficient of $e^{\beta\mu}$.

It is convenient to take that limit at the start. To do this we sum over the internal frequency of the loop for fixed incoming frequencies. We replace the sum over the frequency ω_n by a contour integral encircling the poles of the fermion function $n(z) = (e^{\beta z} + 1)^{-1}$:

$$T\sum_n f(\omega_n+\mu)=-\frac{1}{2\pi i}\oint dz\, n(z) f(z+\mu). \tag{13}$$

The function $f(z + \mu)$ consisting of a product of functions of the kind (12) has only poles as singularities. The integral (13) reduces thus to the residues in those poles. Bearing in mind that as Re $z \to +\infty$, the function $n(z) \to e^{-\beta z}$, we have

$$T\sum_n f(\omega_n+\mu)\approx e^{\beta\mu}\sum \mathrm{Res}\, e^{-\beta z}f(z),\quad \mu\to-\infty. \tag{14}$$

We have thus explicitly split off the factor $e^{\beta\mu}$. The evaluation of the correlator reduces now to associating to each line in the loop a factor $(z + \omega_n)^{-1}$, after that, multiplying the expression obtained by $e^{-\beta z}$, and taking the sum of all residues over z.

We express the free energy per spin in terms of the impurity Green function G(z). The number of impurities can be expressed in terms of μ by the formula

$$N=N_0e^{\beta\mu}\sum \mathrm{Res}\, e^{-\beta z}G_{\alpha\alpha}(z), \tag{15}$$

where N_0 is the number of sites available for impurities. Evaluating μ from (15) and using the relation $\mathcal{F} = \mu$, where $\mathcal{F}$ is the free energy per impurity, we find

$$\mathcal{F}=-T\ln\sum \mathrm{Res}\, e^{-\beta z}G_{\alpha\alpha}(z). \tag{16}$$

We have dropped terms which do not contribute to the magnetic moment or the specific heat. The sum over the residues in (15) and (16) means that first we must calculate the impurity correlator in each perturbation theory diagram.

3. PERTURBATION THEORY

The impurity-paramagnon interaction can turn out to be the most important one for almost magnetic metals. In that case we must, when averaging the density matrix over the electron states, replace the electron correlator in each order of perturbation theory by a sum of products of the spin correlation functions (1). The perturbation theory series we obtain can be written in symbolic form:

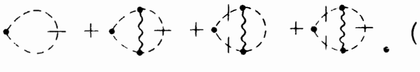
$$\mathrm{Sp}_{\mathrm{el}}\exp\{-\beta[J\mathbf{S}\boldsymbol{\sigma}(0,0)+H-\mathcal{F}_0]\}$$
$$\approx T_\tau\exp\left[\frac{J^2}{2}\int_0^\beta\int_0^\beta d\tau\, d\tau'\, \mathbf{S}(\tau)D(\tau-\tau',0)\mathbf{S}(\tau')\right]. \tag{17}$$

Here H and $\mathcal{F}_0$ are the Hamiltonian and the free energy of the pure metal. The criterion for the applicability of this approximation will be given below. When evaluating any physical quantity one expands the right-hand side of (17) in a series in J^2 and in each order the spin correlator occurs. For instance, we have for the pseudo-fermion Green function in first-order perturbation theory

$$G(\omega)=G_0(\omega)+J^2S(S+1)T\sum_{\omega'}G_0^2(\omega)D(\omega')G_0(\omega+\omega'). \tag{18}$$

The sum of residues occurring in Eq. (16) can be depicted graphically after substituting (18):

$$\sum \mathrm{Res}\, e^{-\beta z}G(z)=$$



(19)

The dashed lines correspond to G_0, the wavy ones to D, and residues are taken at the poles of the lines that are crossed out. If several lines are crossed out, the poles are multiple ones. Analytically one can write (19) in the form

$$\sum \mathrm{Res}\, e^{-\beta z}G(z)=1+J^2S(S+1)T\sum_{\omega\neq0}D(\omega)/\omega^2$$
$$+J^2S(S+1)T\sum_{\omega\neq0}D(\omega)\frac{d}{dz}\frac{e^{-\beta z}}{(z+\omega)}\Big|_{z=0}+\frac{1}{2}J^2S(S+1)TD(0)\frac{d^2}{dz^2}e^{-\beta z}. \tag{20}$$

The main contribution in this equation comes from the low frequency region where, according to (8),

$$J^2D(\omega) \approx J^2D(0) - \pi g|\omega|/2. \qquad (21)$$

Here g is the effective interaction constant,

$$g = (J\nu)^2/(ap_F)^2(1+F). \qquad (22)$$

The contribution from the first term in Eq. (21) can be taken into account in general form in all orders of perturbation theory, since this term is proportional to $\delta(\tau)$ in the τ-representation, and it therefore corresponds in the integrand in (17) to a term of the form

$$\tfrac{1}{2}J^2D(0)\mathbf{S}(\tau)\delta(\tau-\tau')\mathbf{S}(\tau') \equiv \tfrac{1}{2}J^2D(0)S(S+1)\delta(\tau-\tau').$$

This last expression is not an operator and can therefore trivially be taken out from under the T_τ-product sign to give a constant contribution equal to $\frac{1}{2}J^2D(0)$, to the free energy. We must thus replace $D(\omega)$ in Eq. (20) by $D(\omega) - D(0)$. The last term in (20) then vanishes and, with logarithmic accuracy, the second one is equal to

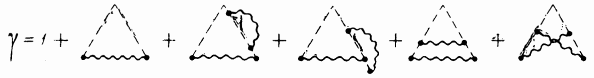
$$-\frac{\pi}{2}gS(S+1)\sum_{\omega\neq 0}\frac{|\omega|}{\omega^2} \approx \frac{1}{2}gS(S+1)\ln\frac{\varepsilon_0}{T}, \qquad (23)$$

where ϵ_0 is defined by Eq. (9). The first two terms of (17), equal to $1 + \frac{1}{2}gS(S+1)\ln(\epsilon_0/T)$, are an expansion of the vortex γ. The expansion parameter is $g\ln(\epsilon_0/T)$. We note that, in contrast to the Kondo effect in normal metals, g is always positive. The third term in Eq. (20) gives the contribution from the self-energy part.

In subsequent orders the perturbation theory series contains powers of g as well as powers of $g\ln(\epsilon_0/T)$. We shall assume that $g \ll 1$ and collect a series in powers of $g\ln(\epsilon_0/T)$.

4. EQUATION FOR THE VERTICES

Up to terms of order g^2 the vertex γ is equal to



(24)

After summation over the frequencies, large quantities of order $g^2\epsilon_0/T$ occur in the last two diagrams; they originate from the regions where the frequencies of the two pseudo-fermion lines entering the vertex are of order T while the frequencies of the paramagnon lines are of order ϵ_0. These quantities have the same form as the second term in Eq. (24) if we replace in them the wavy line, depicting $D(\omega) - D(0)$, by $D(0)$. The large quantities in the last graphs are thus removed just as $D(0)$ was removed above. Apart from the term $J^2D(0)$ there occur then in the free energy unimportant constants of the form $J^4\sum_\omega[D(\omega) - D(0)]^2/\omega^2$, and in the last two graphs of Eq. (24) there remains only a summation over the region where the frequencies of the two pseudo-fermion lines entering the vertex are larger than or of the order of the other two pseudo-fermion lines. After this renormalization it turns out that the last graph of Eq. (24) contains a lower power of the logarithm than of the constant g. This is also true of the more complex graphs containing intersecting wavy lines. In the main approximation we can thus neglect these graphs. The penultimate graph is proportional to $[g\ln(\epsilon_0^0/T)]^2$ and it can be considered to be the second term in Eq. (24) with a complicated vertex γ. Similarly the third and fourth terms can be considered to be complications of the second one when one takes corrections to the Green function and the vertex γ_S into account.

Summing the main logarithms we are thus led to the solution of the following set of equations for γ_S and γ:

(25)

The dashed line here depicts the exact Green function which by means of the Ward identity $\partial G^{-1}/\partial\omega = \gamma(\omega)$ can with logarithmic accuracy be expressed in terms of $\gamma(\omega)$:

$$G(\omega) = 1/\omega\gamma(\omega). \qquad (26)$$

Using (26), it is convenient to write Eq. (25) in analytical form after introducing a logarithmic variable:

$$\gamma_s(x) = 1 + \frac{1}{2}g[S(S+1) - 1]\int_0^x \frac{\gamma_s^3(y)}{\gamma^2(y)}dy; \qquad (27)$$

$$\gamma(x) = 1 + \frac{1}{2}gS(S+1)\int_0^x \frac{\gamma_s^2(y)}{\gamma(y)}dy;$$

$$x = \ln(\varepsilon_0/|\omega|).$$

Solving these equations we get

$$\gamma_s(x) = (1+gx)^{[S(S+1)-1]/2}, \quad \gamma(x) = (1+gx)^{S(S+1)/2}. \qquad (28)$$

The effective interaction

$$g(\gamma_s/\gamma)^2 = g[1 + g\ln(\varepsilon_0/T)]^{-1} \qquad (29)$$

tends to zero when the temperature is lowered independent of the sign of the interaction J.

5. MAGNETIC MOMENT AND SUSCEPTIBILITY

The electrons of an almost magnetic metal change the interaction between an impurity and the magnetic field in two ways. First, they change the vertex γ_S. Second, they themselves are magnetized by a static magnetic field, and as a result the effective gyromagnetic ratio of an impurity becomes equal to

$$\tilde{\mu} = 2\mu_B[1 + J\nu/(1+F)], \qquad (30)$$

if we assume that the gyromagnetic ratios of the impurity and of the electrons are equal to $2\mu_B$. In almost magnetic metals, even when $J\nu \ll 1$, it can happen that $\tilde{\mu} \gg 2\mu_B$. In these cases one says that the impurity has a "gigantic" magnetic moment.

With logarithmic accuracy we can write the Green function in a magnetic field in the form

$$G(\omega) = [\omega\gamma - \gamma_s\tilde{\mu}\mathbf{S}\mathbf{H}]^{-1}.$$

Substituting this expression into (16) we get the free energy in a magnetic field

$$\mathcal{F} = -T\{\ln \operatorname{sh}[(S+\tfrac{1}{2})\mu\beta H] - \ln \operatorname{sh}[\tfrac{1}{2}\mu\beta H]\}, \qquad (31)$$

where we introduced the notation

$$\mu = 2\mu_B(\gamma_s/\gamma)[1 + J\nu/(1+F)]. \qquad (32)$$

It is clear that the free energy is expressed by the same formula as for a free spin but now the gyromagnetic ratio μ depends logarithmically on the temperature. In a strong field when $\mu H \gg T$ we must replace T by μH under the logarithm sign in the function γ_S/γ.

When T = 0, the magnetic moment is

$$M = (\gamma_s/\gamma)\tilde{\mu}S = \tilde{\mu}S[1 + g\ln(\varepsilon_0/\mu H)]^{-1/2}. \qquad (33)$$

The susceptibility in a weak field is

$$\chi = \left(\frac{\gamma_s}{\gamma}\right)^2 \frac{\tilde{\mu}^2 S(S+1)}{3T} = \frac{\tilde{\mu}^2 S(S+1)}{3T[1 + g\ln(\varepsilon_0/T)]}. \qquad (34)$$

It is clear from (34) that the effective magnetic moment decreases when the temperature is lowered, independent of the sign of the interaction.

6. RESISTIVITY

In the Born approximation, the dimensionless amplitude for the exchange scattering of an electron by an impurity at small angles θ in almost magnetic metals is equal to

$$v(\theta) = J\nu[1 + F + (ak)^2]^{-1}, \quad k^2 = 2p_F^2(1 - \cos\theta) \approx p_F^2\theta^2. \qquad (35)$$

This equation is obtained in the same way as Eq. (7). We can neglect the frequency dependence of v(θ) as in the conductivity small energy transfers of the order of the temperature are important. The cross section for scattering through an angle θ with energy transfer ϵ can be expressed in terms of v(θ) and the impurity spin correlator

$$\sigma(\theta, \varepsilon) = \frac{\pi v^2(\theta)}{2p_F^2} \int_{-\infty}^{\infty} dt\, e^{i\varepsilon t}\langle SS(t)\rangle. \qquad (36)$$

We write the exchange part of the resistivity in the form

$$\rho = \frac{2\pi p_F c}{e^2 n} \int_{-1}^{1} d\cos\theta(1 - \cos\theta) \int_{-\infty}^{\infty} d\varepsilon\, \sigma(\theta, \varepsilon)\beta\varepsilon[1 - e^{-\beta\varepsilon}]^{-1} \qquad (37)$$

where n is the electron concentration and c the impurity concentration. The determination of the resistivity has thus been reduced to the evaluation of the spin correlator which can be found by means of the technique expounded above. In the energy range $\epsilon \lesssim T$, which is important in Eq. (37), the spin correlator as well as the correlator entering into the expression (34) for the susceptibility differ from the corresponding value for the free spin only through the factor $(\gamma_S/\gamma)^2$. As a result we get for the resistivity

$$\rho = \left(\frac{\gamma_s}{\gamma}\right)^2 \rho_0, \qquad \rho_0 = \frac{\pi^3 (J\nu)^2 c S(S+1)}{2e^2 n p_F (a p_F)^4} |\ln(1 + F)|, \qquad (38)$$

or

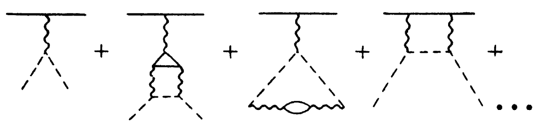
$$\rho = \rho_0[1 + g\ln(\varepsilon_0/T)]^{-1}. \qquad (39)$$

Independent of the sign of J the exchange part of the resistivity decreases when the temperature is lowered.

7. CORRECTIONS TO THE SCATTERING AMPLITUDE

We used above the Born approximation for the calculation of the scattering amplitude for the electron by an impurity. In the same approximation the electron correlators were replaced by a product of paramagnon correlation functions. To elucidate the criterion for the applicability of the Born approximation and the generalization of the results to the case when it is inapplicable, we consider the earlier neglected corrections to the scattering amplitude. The first order graphs for the amplitude have the form

$\Gamma =$



(40)

The first graph describes the Born approximation; for vanishing frequency transfer it corresponds to Eq. (35) and depicts the scattering of an electron by a Yukawa potential. If the frequency transfer is not zero we must add $\pi|\omega|/2kv$ to the denominator in Eq. (35). The frequency dependence of the amplitude becomes important for $\omega \sim kv(1 + F) \ll kv$. This fact turns out to be important for estimating the second graph in (40). One can show that for zero frequency transfer the second graph gives a relative correction to the Born approximation of order Jν. (Maleev and Shender[8] have considered similar corrections to the Born approximation.) These corrections do not contain a logarithmic dependence on the temperature and are always small, so that we shall neglect them below.

We may thus assume that the problem of a magnetic impurity in an almost magnetic metal is equivalent to the problem of a magnetic impurity interacting with free electrons through the Yukawa type potential (35). The correction to the vertex analogous to the second term in (24) is depicted by the third graph of (40). The earlier paramagnon line now corresponds to an electron loop connected with an impurity line by two potentials (35). We must retain in the loop (as earlier in the paramagnon line) only the part linear in $|\omega|$, which is equal to $-\pi|\omega|/2kv$. This representation of the paramagnon correlation function corresponds to an expansion of D(ω), linear in $|\omega|$, which leads to a large logarithm. Further consideration of the dispersion of the potential determines the upper limit of the logarithm cut-off, equal to ϵ_0.

The last graph in (40) depicts the usual Kondo correction to the amplitude. The evaluation of such graphs is made easier by a change to partial amplitudes Γ_l with angular momentum l:

$$\Gamma(\theta) = \sum_{l=0} \frac{2l+1}{2} \Gamma_l P_l(\cos\theta), \qquad (41)$$

where P_l are Legendre polynomials. For the Born approximation partial amplitudes we introduce the notation

$$v_l \equiv \int_{-1} v(\theta) P_l(\cos\theta)\, d\cos\theta = \frac{J\nu}{(p_F a)^2} Q_l\left(1 + \frac{1+F}{2(p_F a)^2}\right). \qquad (42)$$

Here the Q_l are Legendre functions of the second kind. The relative correction from the last graph in (40) to the partial amplitude v_l is of order of magnitude $v_l \ln(\epsilon_0/T)$. The Kondo corrections are thus small, if

$$J\nu \ln(\varepsilon_0/T) < 1. \qquad (43)$$

We have omitted here the factor arising from Q_l in (42). If $1 + F \ll 1$ it depends logarithmically on $1 + F$ and for estimates we shall assume this logarithm to be of the order of unity.

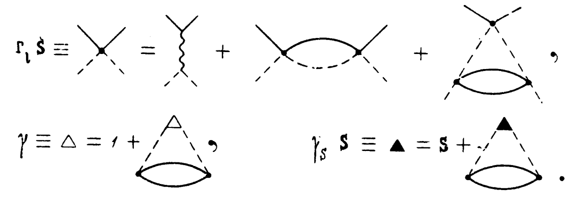
Earlier we collected the series in powers of $g\ln(\epsilon_0/T)$, neglecting Kondo corrections. A necessary condition for this is $g > J\nu$, or, using Eq. (22) for g,

$$1 + F < J\nu. \qquad (44)$$

Comparing (44) with (30) we see that condition (44) means the existence of a "gigantic" magnetic moment. If this condition is not satisfied the paramagnon corrections will be smaller than the Kondo ones and for impurities with such a small J an almost magnetic metal does not differ from a normal one. Even if condition (44) is satisfied, consideration of the Kondo corrections becomes important at low temperatures.

8. SOLUTION OF THE PARQUET EQUATIONS

When we take the paramagnon and the Kondo terms simultaneously into account the equations for the vertices have the form



(45)

The solid lines depict electrons, and the dashed ones exact impurity Green functions which are connected with γ through Eq. (26). Analytically it is convenient to write Eqs. (45) down after changing to a logarithmic variable:

$$\Gamma_l(x)=v_l-\int_0^x\frac{\Gamma_l^2(y)}{\gamma(y)}dy+[S(S+1)-1]\int_0^x\Gamma_l(y)\Sigma(y)dy,$$

$$\gamma=1+S(S+1)\int_0^x\gamma(y)\Sigma(y)dy,$$

$$\gamma_S=1+[S(S+1)-1]\int\gamma_s(y)\Sigma(y)dy, \tag{46}$$

where we have written

$$\Sigma(x)\equiv\sum_{l=0}\frac{2l+1}{2}\tilde{\Gamma}_l^2(x),\quad \tilde{\Gamma}_l\equiv\frac{\Gamma_l}{\gamma},\quad x\equiv\ln\frac{\varepsilon_0}{|\omega|}. \tag{47}$$

Differentiating Eqs. (46) and changing to the functions $\tilde{\Gamma}_l$, using (47), we get

$$\tilde{\Gamma}_l'(x)=-\tilde{\Gamma}_l^2(x)-\tilde{\Gamma}_l(x)\Sigma(x),\quad \tilde{\Gamma}_l(0)=v_l. \tag{48}$$

Using this equation we can express $\tilde{\Gamma}_l$ in terms of $\Sigma(x)$:

$$\tilde{\Gamma}_l=v_l\exp\Big[-\int_0\Sigma(y)dy\Big]\Big\{1+v_l\int_0^x\exp\Big[-\int_0^{x'}\Sigma(y)dy\Big]dx'\Big\}. \tag{49}$$

Expressing $\tilde{\Gamma}_l$ and Σ in terms of the auxiliary function

$$\alpha(x)\equiv\int_0^x\exp\Big[-\int_0^{x'}\Sigma(y)dy\Big]dx' \tag{50}$$

and its derivatives and substituting these formulae into (47) we get an equation for α:

$$\alpha''=-(\alpha')^3\sum_{l=0}^{\infty}\frac{2l+1}{2}\frac{v_l^2}{[1+v_l\alpha]^2} \tag{51}$$

This equation has the solution

$$x=\alpha+\sum_{l=0}^{\infty}\frac{2l+1}{2}[v_l\alpha-\ln(1+v_l\alpha)]. \tag{52}$$

We express $\tilde{\Gamma}_l$ and γ_S/γ in terms of α:

$$\tilde{\Gamma}_l=\frac{\gamma_S(\alpha)}{\gamma(\alpha)}\frac{v_l}{1+v_l\alpha},$$

$$\frac{\gamma_S(\alpha)}{\gamma(\alpha)}=\Big[1+\sum_{l=0}^{\infty}\frac{2l+1}{2}\frac{v_l^2\alpha}{1+v_l\alpha}\Big]^{-1}, \tag{53}$$

which together with (52) gives us a parametric solution of Eqs. (46). One can show that Eqs. (32) to (34) for the free energy, the magnetic moment, and the susceptibility, obtained neglecting Kondo corrections, remain valid also when these corrections are taken into account, if we assume that in them γ_S/γ is determined by Eqs. (52) and (53).

The exchange part of the resistivity can be expressed in terms of the dimensionless amplitude, $\tilde{\Gamma}_l$, for the scattering of an electron by an impurity by the formula

$$\rho=\frac{2\pi^3c}{e^2np_F}\sum_{l=0}^{\infty}\tilde{\Gamma}_l\Big[\frac{2l+1}{2}\tilde{\Gamma}_l-(l+1)\tilde{\Gamma}_{l+1}\Big]. \tag{54}$$

The transition to the previously-considered case when the Kondo corrections could be neglected can be made in (52) and (53) as follows. It is necessary to expand the logarithm in (52) up to terms quadratic in α, and to discard in the denominators of (53) $v_l\alpha$ compared to unity. If we bear in mind that

$$\sum_{l=0}^{\infty}\frac{2l+1}{2}v_l^2\equiv\frac{g}{2}, \tag{55}$$

the function $\gamma_S(\alpha)/\gamma(\alpha)$ expressed in terms of x is after these transformations the same as the one obtained from (28).

When condition (44) is satisfied the solution (28) and (29) is applicable in a wide range of temperatures and when $g\ln(\epsilon_0/T) \gg 1$ the effective magnetic moment, which is proportional to the ratio γ_S/γ, can be decreased appreciably. When the temperature is further lowered, when $(1 + F)\ln(\epsilon_0/T) \sim 1$, the Kondo corrections become important. In that range of temperatures the behavior of the impurity depends essentially on the sign of the constant J. If $J > 0$, Eqs. (53) are valid for all temperatures. When $T \to 0$, the effective amplitude $\tilde{\Gamma}$ tends to zero, and

$$\gamma_S/\gamma=[1+J\nu/(1+F)]^{-1}. \tag{56}$$

The effective gyromagnetic ratio determined by Eq. (32) decreases thus with decreasing temperature from the gigantic value (30) to the free-spin gyromagnetic ratio $2\mu_B$.

If $J < 0$, the gyromagnetic ratio μ decreases down to a value of order of μ_B when $(1 + F)\ln(\epsilon_0/T) \sim 1$. When the temperature is further lowered the zeroth harmonic $\tilde{\Gamma}_0$ increases faster than the others and reaches a value of order unity where the parquet approximation and, hence, Eqs. (53) become incorrect. At such low temperatures the behavior of the magnetic moment of an impurity must be the same as in non-magnetic metals for $T < T_K$ which was studied, for instance, by Abrikosov and Migdal.[5]

It follows from Eqs. (52) and (53) that when $gx \gg 1$ the gyromagnetic ratio μ is independent of the magnitude of the bare exchange interaction J, but depends on its sign.

When $J\nu \sim (1 + F)$ the amplitude $\tilde{\Gamma}$ behaves in the same way as in non-magnetic metals, but the magnetic moment changes to an appreciable magnitude of order $J\nu/(1 + F)$ even in the region where the parquet approximation can be applied.

The resistivity ρ decreases with decreasing temperature proportional to the square of the magnetic moment as long as $(1 + F)\ln(\epsilon_0/T) < 1$. When the temperature is further lowered the resistivity decreases, if $J > 0$, and increases, if $J < 0$.

CONCLUSION

The behavior of magnetic impurities in almost magnetic metals has a number of peculiarities. These peculiarities show up most characteristically in the case when the interaction between the impurities and the conduction electrons is not too small and condition (44) is satisfied. In that case the impurity has at high temperatures a "gigantic" magnetic moment which is large compared to the moment of a free impurity. When the temperature is lowered the gyromagnetic ratio decreases as follows:

$$\mu = 2\mu_B / (1+F)^{1/2} \ln^{1/2}(T^*/T), \quad T^* \sim \varepsilon_0 \exp(1/g).$$

This behavior is independent of the sign of the exchange interaction J and must be valid at temperatures $T > T_K \sim \epsilon_0 \exp[-1/(1 + F)]$. In this range the resistivity is proportional to μ^2 and also decreases with decreasing temperature. Such results agree well with experiments[1] in which a logarithmic dependence of the resistivity on temperature was observed for Co impurities in Pt and it was noted that ρ was proportional to μ^2. Such behavior is not explained in the usual Kondo-effect theory, according to which depending on the sign of the constant J, either the resistivity decreases when the temperature is lowered while the magnetic moment is unchanged, or the magnetic moment decreases but the resistivity increases. To explain this paradox it was proposed to take into account the existence of two conduction bands in Pt[9] or the presence of strong potential scattering of electrons by Co atoms.[10,11] The fact that this effect was observed in platinum, which is an almost magnetic metal, makes the explanation considered in the present paper plausible. To choose between these three explanations it is necessary to study in detail the temperature dependence of the resistivity and of the magnetic moment in a wider range.

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