RESONANCE SINGULARITIES OF SURFACE IMPEDANCE INDUCED BY FERMI-LIQUID INTERACTION IN A METAL IN A MAGNETIC FIELD

A. V. KOBELEV

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted March 24, 1971

Zh. Eksp. Teor. Fiz. 61, 1203–1210 (September, 1971)

Corrections to the asymptotic expansion of the surface impedance of a metal in a magnetic field perpendicular to its surface are calculated in the frequency range of the anomalous skin effect. The corrections appear when allowance is made for the spin-independent Fermi-liquid interaction of the electrons. The resonance singularities of the surface impedance in the vicinity of the cyclotron-resonance point and the threshold frequency of the Fermi-liquid mode are determined. The results can be used to explain the experimentally-observed position of the surface-impedance peak of potassium and may help explain its magnitude.

THE constants of the theory of a degenerate electron liquid[4], which characterize the electron correlation, have been determined phenomenologically in a number of recent investigations by measuring the surface impedance of alkali metals in a magnetic field[1-3]. In[1,2] they plotted the dispersion curve of cyclotron waves propagating across a magnetic field in potassium by determining the maximum points of the derivative of the real part of the impedance, corresponding to a wavelength that is a multiple of the sample thickness. The experimentally observed difference between the limiting frequency of these waves at k = 0 and the gyroscopic electron frequency has made it possible the second constant of the Fermi-liquid interaction, $\alpha_2 = -(0.025 \pm 0.005)$. In[3] they measured the real part of the surface impedance of potassium in a magnetic field perpendicular to the surface, in a frequency range including the region of existence of the cyclotron waves, and found a positive peak at the point of the limiting frequency of the Fermi-liquid cyclotron mode. It should be noted that in the absence of correlation a cyclotron wave can propagate along the magnetic wave, so that the presence of a peak in the impedance is a direct consequence of the Fermi-liquid interaction of the electrons, and a study of the structure of the impedance near the cyclotron frequency makes it possible to determine the parameters of this interaction. In the theoretical calculations of[5], based on a variational method, it was impossible to reconcile the position and relative magnitude of the peak with experiment without changing the ideas concerning the interaction between the electrons and the surface.

We have calculated here the surface impedance of a semi-infinite isotropic electron liquid of a metal in a magnetic field orthogonal to the plane boundary, under the assumption that the electrons are specularly scattered. In the case when a plane transverse circularly polarized electromagnetic wave is incident along the direction of the magnetic field, the surface impedance is expressed in terms of the transverse component of the conductivity tensor of the plasma[6]. By solving the kinetic equation for a degenerate Fermi liquid in an approximation wherein the function characterizing the electron interaction is approximated by two nonzero moments, we can obtain the transverse conductivity in the form (cf.[7]):

$$\sigma^{tr}(\eta, x) = \frac{3}{8\pi} i \frac{\omega_{Le}^2}{\omega + i\nu} \eta \frac{M(\eta, x)}{D(\eta, x)}, \tag{1}$$

where

$$D(\eta,\ x) = 1 - {}^3/_2\eta A_1 q_{11}(x) - {}^5/_2\eta A_2(1 - \eta A_1)x^{-2}[3q_{11}(x) - 2], \tag{2}$$

$$M(\eta,\ x) = q_{11}(x) - {}^5/_3\eta A_2 x^{-2}[3q_{11}(x) - 2], \tag{3}$$

$x = kv/\psi$ is the variable ($\psi = \omega \pm \Omega + i\nu$), $\eta = (\omega + i\nu)/\psi$ is a parameter, k and ω are the wave number and frequency of the electromagnetic wave, Ω is the gyroscopic frequency, ω_{Le} the Langmuir frequency, ν the electron collision frequency, v the electron Fermi velocity, $A_n \equiv \alpha_n/(1 + \alpha_n)$, the Fermi-liquid interaction parameters α_n are defined as in[7], and

$$q_{11}(x) = \frac{1}{2} x^{-3}\left[2x - (1 - x^2)\ln\frac{1+x}{1-x}\right]. \tag{4}$$

The logarithm in (4) is taken to be an analytic function of the variable x, such that when $x > 1$

$$\ln\frac{1+x}{1-x} = \ln\left|\frac{1+x}{1-x}\right| + i\pi.$$

The function $q_{11}(x)$ is proportional, apart from a constant, to the transverse conductivity in the absence of correlation.

Putting

$$\varkappa(x) = 2M(\eta,\ x)\ /\ D(\eta,\ x), \tag{5}$$

we can write the surface impedance in the form

$$Z(\omega) = \frac{4iv}{c^2}\frac{\omega}{\omega + i\nu}\eta \int_{-\infty/\psi}^{\infty/\psi} \frac{dx}{x^2 + \xi\varkappa(x)}, \tag{6}$$

where the parameter is

$$\xi = \frac{3}{4}\frac{\omega_{Le}^2\omega}{\psi^3}\frac{v^2}{c^2},$$

and c is the speed of light.

At $\Omega = 0$ the parameter $\xi(\Omega = 0) = -\xi_R$, as follows from the theory of the anomalous skin effect[8], has a large absolute value in the frequency region where an important role is played by the spatial dispersion of the conductivity tensor in the calculation of the surface impedance and where the skin effect differs from the classical one. At the parameter values characteristic of the experimental conditions[3], namely $2\pi\omega_{Le} = 6.07 \times 10^{15}$ sec^{-1}, $2\pi\omega = 1.5 \times 10^{11}$ sec^{-1}, $v = 7.1 \times 10^7$ cm/sec, and $\nu = (1.1\text{—}0.4) \times 10^8$ sec^{-1}, the parameter is $|\xi| \sim 10^{10}$ near the cyclotron-resonance point $\omega = \Omega$. Therefore in the region of the limiting anomalous skin effect, in which we are interested, when $|\xi| \gg 1$, it is possible to obtain the surface impedance in the form of an asymptotic series in decreasing powers of ξ with coefficients that depend on the parameters A_1 and A_2, using the method proposed in[9] for the calculation of the impedance in the free-electron model.

We assume without proof that in the sector bounded by the lines $(0, \infty/\psi)$ and $(0, i\infty)$ the integrand of (6) has no poles. Then, using the fact that the function $\kappa(x)$ is even, as is obvious from (2) and (3), and applying the Cauchy theorem, we find that the surface impedance (6) is expressed in terms of integrals of the type

$$I_1 = \frac{2}{\pi}\int_0^\infty \frac{dt}{t^2 \pm \xi\varkappa(t)}. \tag{7}$$

We note that the Mellin transform of (7) with respect to ξ, where ξ is a real positive parameter (using the known tabulated value of the integral[10]), can be written in the form

$$M(z) = \int_0^\infty \xi^{z-1} I_1\, d\xi = \frac{2}{\pi}\int_0^\infty \frac{\xi^{z-1}}{t^2 \pm \xi\varkappa}\, dt\, d\xi = \begin{cases} \dfrac{2}{\sin \pi z}\displaystyle\int_0^\infty t^{2z-2}\varkappa^{-z}\, dt \\ \\ 2 \operatorname{ctg} \pi z \displaystyle\int_0^\infty t^{2z-2}\varkappa^{-z}\, dt. \end{cases} \tag{8}$$

From the formula for the inverse transform[10] we have

$$I_1 = \frac{1}{2\pi i}\int_{\beta - i\infty}^{\beta + i\infty} M(z)\xi^{-z}\, dz = \frac{1}{\pi i}\int_{\beta - i\infty}^{\beta + i\infty} \frac{\xi^{-z}}{\sin \pi z} \times (\cos \pi z)^{1/2(1 - \operatorname{sign} \xi)} \int_0^\infty t^{2z-2}\varkappa^{-z}\, dt\, dz, \tag{9}$$

where β is a real positive number.

The gist of the subsequent procedure for calculating the impedance is that the region of integration with respect to t in (9) is subdivided into two, from 0 to 1 and from 1 to ∞ (the point $t = \pm 1$, as seen from (4), is a logarithmic branch point). The function $[\kappa(t)]^{-z}$ is then expanded in increasing powers of t and the resultant series are integrated term by term. Expanding the logarithm in powers of x, we obtain the following expression for $\kappa(x)$ at $|x| < 1$:

$$\varkappa(x) = \frac{4}{3}\left\{3\sum_{n=0}^\infty \frac{x^{2n}}{2n+3}\left(\frac{1}{2n+1} - \frac{5\eta A_2}{2n+5}\right)\right\} \times \left\{1 - 3\sum_{n=0}^\infty \frac{x^{2n}}{2n+3}\left[\frac{\eta A_1}{2n+1} + \frac{5\eta A_2(1-\eta A_1)}{2n+5}\right]\right\}^{-1}. \tag{10}$$

We write out the first three terms of the quotient obtained by dividing the power series in (10):

$$\varkappa(x) = \frac{4}{3}\left[\frac{1}{1-\eta A_1} + \frac{1}{5}\frac{1}{(1-\eta A_1)^2(1-\eta A_2)}x^2 + \frac{3}{35}\frac{1}{(1-\eta A_1)^3(1-\eta A_2)^2}\left(1 - \frac{8}{15}\eta A_1\right)x^4 + \ldots\right]. \tag{11}$$

We note that the series (11) has singularities at $\eta A_1 = 1$ and $\eta A_2 = 1$.

Expanding the functions $D(\eta, x)$ and $M(\eta, x)$ in decreasing powers of x and dividing, we obtain for $|x| > 1$

$$\varkappa(x) = x^{-1}\{i\pi + (4 - {}^3/_4\pi^2\eta A_1 + {}^{20}/_3\eta A_2)x^{-1} - i\pi[1 + 10\eta A_2(1 - \eta A_1) - 3\eta A_1(2 - {}^3/_{16}\pi^2\eta A_1)]x^{-2} - \ldots\}. \tag{12}$$

Using the multinomial theorem[11] concerning the raising of a power series to a negative fractional power, we calculate the coefficients defined by the identities

$$\sum_{\mu=0}^\infty Q_\mu(z)x^\mu \equiv \left\{\frac{3}{4}\varkappa(x<1)\right\}^{-z}, \tag{13}$$

$$\sum_{\nu=0}^\infty R_\nu(z)x^{-\nu} \equiv \left\{\frac{x}{\pi}\varkappa(x>1)\right\}^{-z}. \tag{14}$$

They turn out to equal

$$Q_0 = (1-\eta A_1)^z, \quad Q_1 = 0,$$
$$Q_2 = \frac{z}{5}\frac{(1-\eta A_1)^{z-1}}{1-\eta A_2}, \quad Q_3 = 0,$$
$$Q_4 = \frac{z}{50}\frac{(1-\eta A_1)^{z-2}}{(1-\eta A_2)^2}\left(1 - \frac{23}{7}z + \frac{16}{7}z\eta A_1\right), \tag{15}$$
$$Q_5 = 0, \ldots;$$

$$R_0 = 1, \quad R_1 = \frac{4}{\pi}z\left(1 - \frac{3}{16}\pi^2\eta A_1 + \frac{5}{3}\eta A_2\right)$$
$$R_2 = \frac{8z(z+1)}{\pi^2}\left(1 - \frac{3}{16}\pi^2\eta A_1 + \frac{5}{3}\eta A_2\right)^2 - z\left[1 + 10\eta A_2(1-\eta A_1) - 3\eta A_1\left(2 - \frac{3}{16}\pi^2\eta A_1\right)\right], \ldots \tag{16}$$

From (9), using (13) and (14), we obtain

$$2\pi i I_1 = \sum_\mu \int_{\beta'-i\infty}^{\beta'+i\infty} 2\left(\frac{4\xi}{3}\right)^{-z}\frac{Q_\mu(z)}{2z+\mu-1} \times \frac{dz}{\sin \pi z}(\cos \pi z)^{1/2(1-\operatorname{sign}\xi)} + \sum_\nu \int_{\beta''-i\infty}^{\beta''+i\infty} 2(\pi\xi)^{-z} \times \frac{R_\nu(z)}{-3z+\nu+1}\frac{dz}{\sin \pi z}(\cos \pi z)^{1/2(1-\operatorname{sign}\xi)}.$$

The expansion of I_1 in decreasing powers of ξ is the sum of the residues at the poles lying to the right of the lines $\operatorname{Re} z = \beta'$ and $\operatorname{Re} z = \beta''$, multiplied by $-2\pi i$. The contribution made to the expansion in decreasing powers of ξ by the poles $(\sin \pi z)^{-1}$ at $z = m$ ($m = 1, 2, 3, \ldots$) in the first integral is given by the expression

$$-\frac{2}{\pi}\left(-\frac{3}{4\xi}\right)^m \sum_\mu \frac{Q_\mu(m)}{2m+\mu-1}$$

The second integral has poles at $z = (1+\nu)/3$ and from $\sin \pi z$ at integer z. If z is not an integer, then the poles do not coincide, and in the opposite case a second order pole is produced. The pole $(-3z+\nu+1)^{-1}$ gives a residue $-1/3$, so that when z is not an integer the contribution is

$$\frac{2}{3}(\pi\xi)^{-(1+\nu)/3}R_\nu({}^1/_3 + {}^1/_3\nu)/(\sin({}^1/_3 + {}^1/_3\nu)\pi).$$

In the case when m = 1, 2, 3, ..., the contribution from the second-order pole is

$$\frac{2}{3\pi}(-1)^m\left[\frac{\partial}{\partial z}(\pi\xi)^{-z}R_{3m-1}(z)\right]$$
$$=\frac{2}{3\pi}(-\pi\xi)^{-m}\{R'_{3m-1}(m)-R_{3m-1}\ln(\pi\xi)\}.$$

At z = m, the other values of ν make a contribution

$$-\frac{2}{\pi}\sum_{\nu}(-\pi\xi)^{-m}\frac{R_\nu(m)}{-3m+\nu+1}.$$

Thus, the series expansion of the impedance, accurate to terms of order ξ^{-1}, is

$$\begin{aligned}2\pi i I_1 &= \frac{2}{3}(\pi\xi)^{-1/3}\frac{R_0(1/3)}{\sin(\pi/3)}+\frac{2}{3}(\pi\xi)^{-2/3}\frac{R_1(2/3)}{\sin(2\pi/3)}\\ &+ 3/2(\pi\xi)^{-1}[Q_0(1)+1/3Q_2(1)+1/5Q_4(1)+\ldots]\\ &-\frac{2}{3\pi}(\pi\xi)^{-1}[R_2'(1)-R_2(1)\ln(\pi\xi)]\\ &+\frac{2}{\pi}(\pi\xi)^{-1}[1/2R_0(1)+R_1(1)+R_3(1)]+\ldots\end{aligned}\tag{17}$$

Using the values of the coefficients (15) and (16), we obtain ultimately

$$\begin{aligned}Z(\omega)&=\frac{4\pi v}{c^2}\eta\frac{\omega}{\omega+i\nu}\left\{\frac{4}{3\sqrt{3}}(\pi\eta^3\xi_R)^{-1/3}\right.\\ &+\frac{32}{9\pi\sqrt{3}}(\pi\eta^3\xi_R)^{-2/3}\left(1-\frac{3}{16}\pi^2\eta A_1+\frac{5}{3}\eta A_2\right)\\ &+\frac{2}{3\pi}(\pi\eta^3\xi_R)^{-1}\ln(\pi\eta^3\xi_R)\\ &\times\left[\frac{16}{\pi^2}-1-3\eta A_1+\frac{16}{3\pi^2}\eta A_2\left(2-\frac{3}{8}\pi^2+\frac{5}{3}\eta A_2\right)\right]\\ &-\frac{2}{3\pi}(\pi\eta^3\xi_R)^{-1}\left[\frac{12}{\pi}\left(\frac{2}{\pi}-1\right)-\frac{5}{2}\right.\\ &-3\eta A_1\left(2-\frac{3}{4}\pi-\frac{3}{32}\pi^2\eta A_1\right)\\ &\left.-5\eta A_2\left(2+\frac{4}{\pi}\left(1-\frac{4}{\pi}\right)-5\eta A_1-\frac{40}{3\pi^2}\eta A_2\right)\right]\\ &\left.+\frac{3}{2}(\pi\eta^3\xi_R)^{-1}\left[1-\eta A_1+\frac{1}{15}(1-\eta A_2)^{-1}-\frac{8}{875}(1-\eta A_2)^{-2}\right]\right\}.\end{aligned}\tag{18}$$

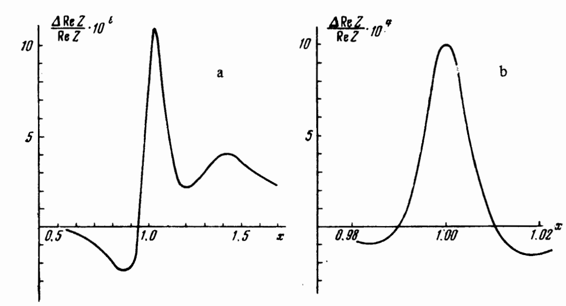
The term $\sim(\xi_R)^{-1/3}$ in the impedance, as seen from (18), does not depend on the magnetic field and coincides fully with the result obtained in the electron-gas model for the extremely anomalous skin effect[8]. In the second term, $\sim(\xi_R)^{-2/3}$, there is a contribution due to the Fermi-liquid interaction and proportional to A_1 and A_2 raised to the first power. This contribution is independent of the magnetic field. In the term $\sim\xi_R^{-1}\ln\xi_R$ there appear increments, of which the one proportional to $(\eta A_2)^2$ has a logarithmic resonant singularity at the cyclotron frequency. Finally, in the term $\sim(\xi_R)^{-1}$ there is an increment with a resonant singularity at a frequency determined by the relation $\eta A_2 = 1$. In the absence of a Fermi-liquid interaction, when $A_1 = A_2 = 0$, the surface impedance is a monotonic function of the frequency, and the influence of the magnetic field is evidenced only by the vanishing of the terms of order $(\xi_R)^{-2/3}$ and above. As can be readily seen, this property of the impedance is retained also in the model in which $A_1 \neq 0$, $A_n = 0$, and $n > 1$.

The peak observed in the real part of the surface impedance[3] occurs when the ratio of the frequencies of the gyroscopic rotation of the electron and of the electromagnetic wave is $\Omega/\omega = -1.025 \pm 0.005$. This value corresponds to the limiting (at k = 0) frequency of a cyclotron wave propagating along the magnetic field in a Fermi liquid: $\eta A_2 = 1$ or $\omega = \Omega(1+\alpha_2)$, if we assume the parameter value $\alpha_2 = -0.03$ obtained from spectral measurements. The relative magnitude of the experimental peak turned out to be $\Delta\,\mathrm{Re}\,Z/\mathrm{Re}\,Z \sim 10^{-2}$. A variational method was used in[5] to calculate the increment to the impedance term $\sim(\xi_R)^{-2/3}$; this increment is connected with the Fermi-liquid interaction. It turned out to be independent of the magnetic field in first order in ηA_2; this agrees with (18). The second order of perturbation theory in ηA_2 results in a correction that has a resonant singularity in the vicinity of the cyclotron frequency,

$$\sim\exp\left(-\frac{2\pi}{3}i\right)(|\xi_R|)^{-2/3}A_2^2\left[\frac{\omega}{\omega\mp\Omega+i\nu}+\omega\frac{(\omega\mp\Omega(1+\alpha_2)+i\nu)^{1/2}}{(\omega\mp\Omega+i\nu)^{3/2}}\right].$$

As seen from (16) and (17), there is no such contribution in the term $\sim(\xi_R)^{-2/3}$, and the resonant singularity appears only in the terms $\sim(\xi_R)^{-1}$. The maximum of the real part of the resonant singularity, given in[5], coincide with the experimental position only at a parameter value $-\Omega\alpha_2/\nu \equiv -\Omega\tau\alpha_2 \sim 1$, and the relative peak is in this case smaller by one order of magnitude than the experimental one. Attempts to increase the relative height of the peak in calculation without allowance for the electron correlation in the uneven-surface model[12], when it is assumed that the electron collision frequency increases near the limit as a result of the collisions with the surface roughnesses, lead to a significant shift of the peak relative to the experimental position.

The fact that a relatively large and sharp peak is observed at the limiting frequency of the Fermi-liquid cyclotron mode indicates that the electron correlation effects are large in the experiment in comparison with effects of collision damping, which cause broadening of the peak. The parameter $-\Omega\tau\alpha_2$, which characterizes the relative significance of these effects, must therefore be regarded as large. In this case the surface impedance, according to the experimental data[3], has a resonant singularity at the frequency $\Omega(1+\alpha_2)$, and not at the cyclotron-resonance frequency. Let us write out



The variable $x = (1-\Omega/\omega)A_2$ is chosen such that the cyclotron-resonance point corresponds to x = 0, and the limiting frequency of the Fermi-liquid wave corresponds to x = 1. a) $\alpha_2 = -0.03$, $-\Omega\tau\alpha_2 = 10$; b) $\alpha_2 = -0.03$, $-\Omega\tau\alpha_2 = 10^2$.

the resonant increments to the value $Z(0) \sim (\pi \xi_R)^{-1/3}$ in the expression for the impedance (18):

$$\Delta Z(\omega) \sim \alpha_2^2 |\xi_R|^{-1} \ln\left[- |\xi_R| \left(\frac{\omega + i\nu}{\omega \mp \Omega + i\nu} \right)^3 \right] + |\xi_R|^{-1} \left\{ 7 \frac{(\omega \mp \Omega + i\nu)^3}{(\omega + i\nu)^2 [\omega \mp \Omega(1+\alpha_2) + i\nu]} - \frac{(\omega \mp \Omega + i\nu)^4}{(\omega + i\nu)^2 [\omega \mp \Omega(1+\alpha_2) + i\nu]^2} \right\}. \quad (19)$$

At parameter values $-\Omega\tau\alpha_2 \gtrsim 10$ the term in (19), which has a resonant logarithmic singularity, is small compared with the terms that follow. A plot of the ratio of the real part of (19) to Re Z ($\Omega = 0$) is shown in the figure for values of $-\Omega\tau\alpha_2$ equal to 10 and 100. The peak in the real part of the surface impedance lies at the point of limiting frequency $\omega = \Omega(1 + \alpha_2)$, and at the parameter value $\alpha_2 = -0.025 \pm 0.005$, which agrees with the spectral measurements, its position coincides with the experimental value. The maximum of the real part of (19) equals $\sim \alpha_2^2 |\xi_R|^{-1} (\Omega\tau\alpha_2)^2$. Estimates of the relative magnitude of the peak at the experimental electron-collision frequency ($-\Omega\tau\alpha_2 \sim 6$–$20$, corresponding to a ratio 5000–13000 of the residual conductivity to the room-temperature conductivity), yield Δ Re Z/Re Z $\sim 10^{-5}$, and at the $-\Omega\tau\alpha_2 \sim 10^2$ the relative magnitude of the peak is close to the experimental value.

In conclusion, I take the opportunity to thank Professor V. P. Silin for directing this work.

[1] W. M. Walsh and P. M. Platzman, Phys. Rev. Lett. 15, 784, 1965.
[2] P. M. Platzman, W. M. Walsh and E. N. Foo, Phys. Rev. 172, 689, 1968.
[3] G. A. Baraff, C. C. Crimer and P. M. Platzman, Phys. Rev. Lett. 22, 590, 1969.
[4] V. P. Silin, Fiz. Met. Metallov. 29, 681 (1970).
[5] G. A. Baraff, Phys. Rev. B1, 4307, 1970.
[6] P. M. Platzman and K. C. Jacobs, Phys. Rev. 134, A, 974, 1964.
[7] V. P. Silin, Zh. Eksp. Teor. Fiz. 54, 1016 (1968) [Sov. Phys.-JETP 27, 541 (1968)].
[8] G. E. Reuter and E. H. Sondheimer, Proc. Roy. Soc., A195, 336, 1948.
[9] R. B. Dingle, Appl. Sci. Res., B3, 69, 1953.
[10] P. S. Gradshteĭn and I. M. Ryzhik, Tablitsy integralov (Tables of Integrals), Fizmatgiz, 1962.
[11] E. P. Adams, Smithsonian mathematical formular, Washington, 1922.
[12] G. A. Baraff, Phys. Rev. 187, 1861, 1969.

Translated by J. G. Adashko
123