ELECTROACOUSTOMAGNETIC EFFECT AND HALL EFFECT IN SEMICONDUCTORS IN A STRONG ELECTRIC FIELD

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Large-scale properties of a semiconducting crystal in external electric and magnetic fields are considered for the case when the carrier drift velocity exceeds the phase velocity of the sound wave and phonon generation occurs. It is shown that the generated phonon flux results in an additional force (besides the Lorentz force) which acts on the electrons and which is, generally speaking, not a potential force. It is precisely the nonpotentiality of the acoustoelectric force connected with the spatial anisotropy of the phonon emission diagram which leads to the appearance in the sample of an annular current component, and, as a consequence, to formation of a magnetic moment (electroacoustomagnetic effect). The Hall effect in a strong electric field under phonon-generation conditions is considered for two limiting cases: for Hall-shorted and Hall-open samples. It is shown that in the case of a Hall-open sample the Hall constant decreases and reverses sign with increasing electric field (the absolute value of the Hall constant in this case may exceed the value in a weak field). The current-voltage characteristic is also found, and it is shown that under phonon-generation conditions the current is saturated in a Hall-open sample. The case of a Hall-shorted sample with a Corbino disc geometry is considered and it is shown that in a strong magnetic field the current in the source circuit rises sharply. In weak magnetic fields the current is saturated. The theoretical results are compared with available experimental data and good qualitative agreement is found. Kinematic effects connected with phonon generation are also mentioned.

EXPERIMENTAL investigations of the Hall effect in semiconductors and semimetals in a strong electric field have shown that it is accompanied by a number of new singularities and phenomena which are quite difficult to interpret theoretically (see, for example, the monograph [1]). Most theoretical work on the Hall effect in a strong electric field initially begins with the premise that in a strong electric field the electron (hole) distribution function is significantly altered, so that the electric field can no longer be regarded as a small correction, and the electron temperature comes to depend on the electric field. By considering further some particular type of carrier scattering, with different dependences of the scattering time on the carrier energy, it is naturally possible to obtain different corrections to the Hall constant, necessitated by the action of the strong electric field. The galvanomagnetic properties and current-voltage characteristics of semiconductors in strong magnetic and electric fields were investigated in detail by Bass[2] (see also the references therein), who has shown that in a strong electric field, when the heating of the electrons is appreciable, the current-voltage characteristic of a semiconductor varies and it becomes possible to obtain negative differential resistance. Analogous phenomena were investigated earlier in plasma physics in connection with the problem of runaway electrons.[3]

At the same time, there is one more mechanism whereby the Hall constant can be altered in a strong electric field; this mechanism is connected with generation of acoustic phonons by supersonic drift of the electrons.[4] Indeed, if a sufficiently strong electric field is applied to the sample, such that the average directional velocity of the electrons or holes exceeds the phase velocity of the acoustic wave, then, as is well known,[5] acoustic phonons, sometimes also called acoustic noise,[1)] is produced in a crystal with relatively strong electron-phonon interaction. Owing to the acoustoelectric effect, the growing flux of acoustic phonons in the crystal reacts on the electrons, and an additional force produced by the generated phonons will act on the electrons in addition to the Lorentz force.[2)] Under these conditions the Hall emf and the Hall constant determined from it are significantly altered. Moreover, as will be shown below, a case is possible when the sign of the Hall constant is reversed (without a change in the type of carriers). Since the acoustoelectric force exerted on the electrons by the phonons generated in the crystal depends on the coordinates, it is natural that the experimentally measured Hall potential difference and Hall constant will also depend on the coordinates.

It should be noted that an investigation of the Hall effect in piezosemiconducting p-Te recently carried out by Tanaka and Hojo[8] has shown that in a strong electric field the Hall constant actually decreases and reverses sign, and that the region in which the indicated phenomena are observed corresponds quite closely to the region of phonon generation. Apparently, in a number of other experiments, where a decrease of the Hall constant

[1)]The influence of uneven distribution of the phonons in strong electric and magnetic fields was also investigated by Chuenkov [6], who has shown that the mutual dragging of the electrons and phonons has a strong influence on the galvanomagnetic properties of a semiconductor. However, he considered only the case of subsonic motion of the electrons, and there was no phonon generation.

[2)]The influence of an external monochromatic acoustic wave on different kinetic coefficients (particularly the Hall effect in an acoustoelectric current) was considered by Gulyaev [7].

was likewise observed in a strong electric field, phonon generation should also play a noticeable role (see [9,10]).

Also closely connected with phonon generation is another effect, namely the appearance of a magnetic moment in a sample when the spatial distribution of the generated phonons is anisotropic inside the Cerenkov cone. (We call this the electroacoustomagnetic effect, in analogy with the acoustoelectric effect.[3)]) The physical reason for the occurrence of a magnetic moment is as follows. We consider a piezoelectric anisotropic crystal in which the direction corresponding to the largest value of the phonon generation intensity does not coincide with the direction of the drift-velocity vector. It is then obvious that phonon generation will occur mainly at a certain angle to the drift velocity, and consequently the acoustoelectric force exerted on the electrons by the generated phonons will be directed not against the drift velocity but at a certain angle. Since the phonon generation intensity depends on the coordinates, it is clear that the curl of the acoustoelectric force will differ from zero in such an anisotropic crystal. This in turn produces in the crystal a solenoidal component of the current due entirely to the solenoidal component of the acoustoelectric force. It is the solenoidal component of the current which produces the magnetic moment in the sample. We emphasize that the magnetic moment also appears in the absence of an external magnetic field.

In this paper we consider the simplest case, when we can confine ourselves to the hydrodynamic equations in the description of the electronic subsystem and, in addition, we disregard all mechanisms governing the dependence of the relaxation time on the electron energy, i.e., in other words, the frequency of electron collisions with all the scattering centers is assumed to be constant. In calculating the large-scale properties of the electron subsystem, we did not use the explicit form of the acoustoelectric force; it turns out that all the main physical results can be obtained in general form without specifying for the acoustoelectric force a concrete form that is valid, of course, in some approximation. In this sense, the results are sufficiently general and apparently remain in force also in a more rigorous kinetic analysis of this problem.

We consider two limiting cases, those with the Hall effect open-circuited and short-circuited in the sample. For both cases we obtain the Hall constants under conditions of phonon generation and determine the current-voltage characteristics. We demonstrate the close connection between the current saturation effect and the Esaki "kink effect" in a strong magnetic field, on the one hand, and with the effect of variation of the Hall constant on the other.

1. LARGE-SCALE PROPERTIES OF THE MEDIUM UNDER PHONON GENERATION CONDITIONS. ELECTROACOUSTOMAGNETIC EFFECT

Under phonon generation conditions, the electrons (or holes) are acted upon by an additional force, which can be determined from the exact hydrodynamic equations after the latter are averaged over the fluctuations. We represent all the quantities characterizing the electron subsystem, i.e., the electric field, the carrier density, and the hydrodynamic velocity, in the form of two components

$$A(\mathbf{r},t)=\langle A(\mathbf{r},t)\rangle+A_{\sim}(\mathbf{r},t), \tag{1}$$

where $A_{\sim}$ is the rapidly-oscillating part and $A \equiv \langle A(\mathbf{r},t)\rangle$ is an averaged value, the averaging being carried out over time and space whose scales, on the one hand, are much larger than the period and wavelength of the generated acoustic noise, and on the other hand much smaller than the reciprocal values of the characteristic growth (or damping) increments of these quantities in time and in space. For growing acoustic fluctuations, such an averaging scale always exists by virtue of the conditions $|\mathrm{Re}\,\omega| \gg |\mathrm{Im}\,\omega|$, $|\mathrm{Re}\,q| \gg |\mathrm{Im}\,q|$, where ω and q are the frequency and wave vector of the generated waves. It is easily seen that the hydrodynamic equations for quantities averaged in this manner are as follows:*

$$\mathbf{j}+[\mathbf{hj}]=e\mu n\mathbf{E}+e\mu n_0\mathbf{F}-\mu T\nabla n, \tag{2}$$

$$\mathrm{rot}\,\mathbf{E}=0,\ \mathrm{div}\,\mathbf{j}=0, \tag{3}$$

$$\mathrm{div}\,\mathbf{E}=4\pi e\varepsilon_0^{-1}(n-n_0), \tag{4}$$

where $\mathbf{j} = e\langle n\mathbf{v}\rangle$ is the current density, $\mathbf{h} = \mu\mathbf{B}/c$, $\mu = e/m\nu$ is the mobility, e the charge of the electron, m its mass, ν the frequency of electron collisions with all the scattering centers, here assumed constant and independent of the electron energy, **E** the electric field, **B** the magnetic field, ϵ_0 the dielectric constant of the lattice, c the velocity of light in vacuum, $n_0 = \int dVn(\mathbf{r})$ the electron density averaged over the volume, and

$$e\mathbf{F}(\mathbf{r})=en_0^{-1}\langle n_{\sim}(\mathbf{r},t)\mathbf{E}_{\sim}(\mathbf{r},t)\rangle \tag{5}$$

the acoustoelectric force exerted on the electrons by the phonons. In the quasilinear approximation, i.e., when we can confine ourselves in the expansion of the carrier density to terms linear in the alternating field of the wave, the acoustoelectric force $e\mathbf{F}(\mathbf{r})$ is expressed in terms of the energy density of the generated acoustic noise in the form[5]

$$e\mathbf{F}=\frac{1}{n_0}\sum_{\alpha}\int d\mathbf{q}\frac{\mathbf{q}}{\omega}\gamma_{\mathrm{el}}^{\alpha}(\mathbf{q})\mathscr{E}^{\alpha}(\mathbf{q},\mathbf{r}), \tag{6}$$

where $\gamma_{\mathrm{el}}^{\alpha}(\mathbf{q})$ is the electronic increment (decrement) of generation of acoustic waves with polarization α (for details see [5]), $\mathscr{E}^{\alpha}(\mathbf{q},\mathbf{r})$ is the energy density of the generated acoustic noise, which can be obtained by solving the stationary kinetic equation for phonons

$$\mathbf{v}_g^{\alpha}\frac{\partial\mathscr{E}^{\alpha}}{\partial\mathbf{r}}+2(\gamma_{\mathrm{el}}^{\alpha}+\gamma_{\mathrm{Lat}}^{\alpha})\mathscr{E}^{\alpha}=Q^{\alpha}(\mathbf{q}). \tag{7}$$

Here $\mathbf{v}_g^{\alpha} = \partial\omega^{\alpha}/\partial\mathbf{q}$ is the group velocity of the acoustic waves, $\gamma_{\mathrm{lat}}^{\alpha}(\mathbf{q})$ is the lattice absorption decrement due, for example, to viscosity, and $Q^{\alpha}(\mathbf{q})$ the phonon source (see [5]). Under phonon generation conditions $\gamma_{\mathrm{el}}^{\alpha} + \gamma_{\mathrm{lat}}^{\alpha} < 0$, and therefore the energy density of the acoustic waves $\mathscr{E}^{\alpha}(\mathbf{q},\mathbf{r})$ increases exponentially in space. Under these conditions, as seen from (6), the electroacoustic force $e\mathbf{F}(\mathbf{r})$ also increases in space.

Eliminating the carrier density $n(\mathbf{r})$ from (2) with

[3)]The occurrence of a magnetic moment under the influence of an external surface acoustic wave was considered by Gulyaev et al.[11].

*$[\mathbf{hj}] = \mathbf{h}\times\mathbf{j}$.

the aid of the Poisson equation (4), we obtain an equation for the electric field

$$\left(1+\frac{\varepsilon_0}{4\pi e n_0}\operatorname{div}\mathbf{E}\right)\mathbf{E}-r_D^2\Delta\mathbf{E}+\frac{1}{\sigma_0}\{[\mathbf{jh}]-\mathbf{j}\}+\mathbf{F}(\mathbf{r})=0, \qquad (8)$$

where r_D is the Debye radius of the electrons. It is impossible to obtain a solution of the nonlinear equation (8). We therefore consider below a limiting case of practical importance, when the inhomogeneity scale of the electric field l satisfies the conditions

$$l\gg r_D, \qquad l\gg\frac{v^\alpha v}{\omega_0^2}\frac{\mu E^{max}}{v^\alpha}, \qquad (9)$$

where E^{max} is the maximum value of the electric field in the inhomogeneity, $\omega_0 = (4\pi e^2 n_0/m\epsilon_0)^{1/2}$ is the plasma frequency of the electron gas, and v_α is the speed of sound. When the conditions (9) are satisfied, Eq. (8) is greatly simplified. From (8) and (3), with conditions (9) satisfied, we get the relations

$$\mathbf{j}+[\mathbf{hj}]=\sigma_0(\mathbf{E}+\mathbf{F}), \qquad (8a)$$

$$\operatorname{div}(\mathbf{E}+\mathbf{F})+\mathbf{h}\operatorname{rot}\mathbf{F}=0, \qquad (10)$$

$$\operatorname{rot}\mathbf{j}=\frac{\sigma_0}{1+h^2}\{\operatorname{rot}\mathbf{F}-\mathbf{h}\operatorname{div}(\mathbf{E}+\mathbf{F})\}, \qquad (11)$$

from which it follows that

$$\operatorname{rot}\mathbf{j}=\sigma_0\operatorname{rot}\mathbf{F}. \qquad (12)$$

Thus, the spatially-inhomogeneous electric force $e\mathbf{F}$ leads to the appearance of solenoidal current components. It is important that the magnetic field does not enter explicitly in (12), so that the appearance of annular currents in the sample can also occur without any external magnetic field, provided, of course, that the piezoelectric properties of the crystal have the required anisotropy. Let us explain this in greater detail. We consider a case when the direction of the maximum phonon generation intensity, say the direction of the vector **n**, does not coincide with the local value of the electron drift velocity. It is then obvious that the distribution of the phonon-radiation intensity inside the Cerenkov cone will have a maximum somewhere near the direction of the vector **n**, shown schematically by the heavy line in Fig. 1. Since the intensity of acoustic wave generation is maximal near the direction of the vector **n**, it follows that the direction along which the acoustoelectric force $e\mathbf{F}$ acts will also be near the vector **n**. The latter means that a force component normal to the drift velocity is produced in such a piezoelectric anisotropic crystal. This component depends on the coordinate along the drift-vector direction and, consequently, curl **F** turns out to differ from zero.[4)]

The appearance of an annular current under phonon generation conditions produces in the sample a magnetic moment whose magnitude will obviously be[12]

$$\mathbf{M}=\frac{1}{2c}\int dV[\mathbf{rj}^r]=\frac{\sigma_0}{2c}\int dV[\mathbf{rF}^r],$$

where $\mathbf{j}^r$ is the density of the solenoidal component of

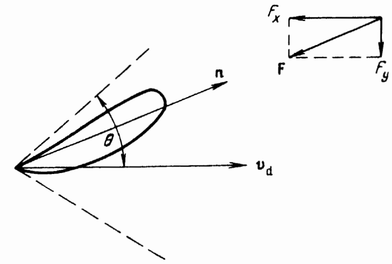

FIG. 1. Directivity pattern of phonon radiation and direction of electroacoustic force in a crystal where the phonon generation intensity is maximal in the direction of the vector **n**. The Cerenkov angle θ is determined by the condition $\cos\theta = v^\alpha/v_d$.

the current and $\mathbf{F}^r$ is the solenoidal component of acoustoelectric field. The magnetic field corresponding to this magnetic moment is

$$\mathbf{H}=\frac{\sigma_0}{c}\int\frac{[\mathbf{F}^r\mathbf{R}]}{R^3}dV,$$

where **R** is the radius vector drawn from the volume element dV to the point of observation of the field. Thus, even in the presence of only an electric field, a magnetic moment is produced in a piezosemiconducting sample as a result of phonon generation, provided this generation is suitably anisotropic. In analogy with the acoustoelectric effect, where phonon generation gives rise to a redistribution of the electric field in the sample, the appearance of a magnetic moment can be called the electroacoustomagnetic effect. To estimate the order of magnitude of the magnetic field that can occur at the surface of the sample, we assume that the solenoidal component of the current is of the order of en_0v^α (this is the maximum possible value of the acoustoelectric current in general). Then, at a carrier density $n_0 \approx 10^{16}-10^{17}$ cm^{-3} we obtain 100–1000 Oe respectively for the magnetic field at the surface of a sample with linear dimensions on the order of 1 cm.

The appearance of a magnetic moment in the sample should cause it to be rotated in an external magnetic field to a spatial position at which the energy of the magnetic moment in the external field is minimal.

It is convenient to break up the density of the total current into potential and solenoidal components: $\mathbf{j} = \mathbf{j}^p + \mathbf{j}^r$, with the dc component included in the potential part. It is then obvious that

$$\mathbf{j}^r=\sigma_0\mathbf{F}^r, \qquad (13)$$

$$\mathbf{j}^p=\mathbf{j}-\mathbf{j}^r=\frac{\sigma_0}{1+h^2}\{\mathbf{E}+\mathbf{F}+[\mathbf{E}+\mathbf{F},\mathbf{h}]\}-\sigma_0\mathbf{F}^r. \qquad (14)$$

Here $\mathbf{F} = \mathbf{F}^p + \mathbf{F}^r$, where $\mathbf{F}^r$ and $\mathbf{F}^p$ are respectively the solenoidal and potential parts of the acoustoelectric force; they satisfy the relations

$$\operatorname{rot}\mathbf{F}^p=0,\ \operatorname{div}\mathbf{F}^r=0. \qquad (15)$$

In the two-dimensional problem under consideration $F_z \equiv 0$, and it therefore follows immediately from (8), (13), and (15) that curl **E** = 0. Thus, such a breakup of the quantities **j** and **F** makes it possible immediately to satisfy the equation for the curl of the electric field without having to solve the complicated Poisson equation for the potential φ of the electric field:

$$\Delta\varphi=\operatorname{div}\mathbf{F}+\mathbf{h}\operatorname{rot}\mathbf{F}.$$

[4)]It is obvious that such a radiation pattern of the phonons in the crystals should be accompanied also by transverse acoustoelectric effects, i.e., by the occurrence of the transverse electric field in the crystal when a sufficiently strong longitudinal electric field is applied to it.

The microscopic properties of the medium, or more accurately the dielectric tensor, which determines the electronic increment of phonon generation, as well as the source of phonons in the kinetic equations, depend strongly on the local value of the electron drift velocity $v_d = \langle v \rangle$, which can readily be shown to be determined by the expression

$$v_d = \frac{\mu}{1+h^2}(E + [Eh]). \quad (16)$$

It is important that the drift velocity is determined by the local value of the electric field and differs in direction from the current vector; if we introduce the difference $j - en_0 v_d \equiv s$, then s satisfies the relation

$$s + [hs] = \sigma_0 F. \quad (16a)$$

The behavior of the crystal in the external field depends strongly on the boundary conditions. We therefore consider two limiting cases, in which the sample is open-circuited and closed-circuited to the Hall effect.

2. OPEN-CIRCUITED HALL EFFECT

We consider a sample in the form of a parallelepiped with dimensions L_x, L_y, and L_z along the edges x, y, and z, respectively. The boundary conditions for a sample with open-circuited Hall effect are

$$\int_0^{L_x} dx\, E_x = V, \quad j_y{}^p = j_z{}^p = 0, \quad h \| z. \quad (17)$$

where V is the potential difference across the sample. Using the boundary conditions (17), we obtain from (14) the potential part of the current density in the x direction:

$$j_x{}^p = \sigma_0 \left\{ \frac{V}{L_x} + \frac{1}{L_x} \int_0^{L_x} (F_x + hF_y{}^r - F_x{}^r)\, dx \right\}. \quad (18)$$

The total current in the source circuit is then

$$I_x = L_y L_z \left\{ \frac{\sigma_0}{L_x} V + \frac{\sigma_0}{L_x L_y} \int_0^{L_x} dx \int_0^{L_y} dy\, F_x{}^p \right\}, \quad (19)$$

since the integral of the solenoidal component of the current over the volume is equal to zero.

The Hall emf, by definition, is

$$\mathscr{E}_H = \int_0^{L_y} E_y\, dy,$$

from which we get for the Hall constant

$$R^H(x) = \frac{\mathscr{E}_H^{(x)}/L_y}{BI_x/L_y L_z} = R_0{}^H - \frac{L_z}{I_x B} \int_0^{L_y} (F_y - F_y{}^r - hF_x{}^r)\, dy, \quad (20)$$

where $R_0^H = 1/en_0c$ is the Hall constant in the absence of phonon generation. It is clear, however, that the solenoidal component of the current density makes no contribution to the total current, and therefore

$$\frac{1}{\sigma_0} \int_0^{L_y} j_x{}^r\, dy = \int_0^{L_y} F_x{}^r(x, y)\, dy \quad (21)$$

and consequently the final formula for the Hall coefficient will be

$$R^H(x) = R_0{}^H - \frac{L_z}{BI_x} \int_0^{L_y} F_y{}^p(x, y)\, dy. \quad (22)$$

It is seen from (22) that under phonon generation conditions the Hall constant depends on the coordinate x, and, in addition, it may even reverse sign under certain conditions.

Let us examine the physical nature of this phenomenon in greater detail. From expression (6) for the electron drift velocity it follows that the drift velocity in the case of a sample with open-circuited Hall effect has the following nonzero components:

$$v_{dx} = \mu \left[\frac{V}{L_x} + \frac{1}{L_x} \int_0^{L_x} F_x{}^p\, dx \right] - \frac{\mu}{1+h^2}(F_x + hF_y) + \mu F_x{}^r, \quad (23)$$

$$v_{dy} = \frac{\mu}{1+h^2}(hF_x - F_y) + \mu F_y{}^r. \quad (24)$$

Since the acoustoelectric force $e\mathbf{F}$ depends on the drift-velocity vector, expressions (23) and (24) are in fact the functional equations with respect to the drift-velocity components. These equations cannot be solved even in the quasilinear approximation for $\mathbf{F}$, and we therefore confine ourselves below only to qualitative conclusions that can be deduced from (23) and (24) by perturbation theory. We put in the zeroth approximation $\mathbf{F} = 0$. It is then obvious that in this approximation $v_{dx}^{(0)} = \mu V/L_x$ and $v_{dy}^{(0)} = 0$. Then, substituting these values of the drift velocity in the expressions for $\mathbf{F}$, we can obtain the first-approximation formulas, etc. It is obvious that in the first approximation we obtain for the drift velocity

$$v_{dx}^{(1)} = \mu \frac{V}{L_x} + \frac{\mu}{L_x} \int_0^{L_x} F_x{}^{p(0)}\, dx - \frac{\mu}{1+h^2} F_x^{(0)} + \mu F_x{}^{r(0)}, \quad (25)$$

$$v_{dy}^{(1)} = \frac{\mu}{1+h^2} hF_x^{(0)}, \quad (26)$$

where $\mathbf{F}$ with the zero superscript denotes that it is necessary to substitute here the zeroth approximation for the drift velocity. In the derivation of (25) and (26) we also took into account the fact that the direction in which the intensity of the generated acoustic phonons is maximal coincides with the direction of x. For this reason, the acoustoelectric field component $F_y^{(0)} = 0$, since in the zeroth approximation the electrons drift only in the x direction; naturally, the phonons are also generated in the same direction. It follows even from the first-approximation formulas that a nonzero drift velocity component v_{dy}^1 is produced. If we now substitute this value of the drift velocity into the expression for the acoustoelectric field $\mathbf{F}$, then we see that in the first approximation $F_y^{(1)}$ also differs from zero, and thus a correction to the Hall constant also appears in the same approximation (see (22)). Let us now establish the sign of $F_y^{(1)}$. It is physically clear that in the zeroth approximation $F_x^{(0)} < 0$, since the generated acoustic noise slows down the supersonic motion of the electrons (for details see the review by one of the authors[5]), and therefore $v_{dy}^{(1)} < 0$. Under phonon-generation conditions in a crystal where the piezoelectrically-active direction coincides with the x axis, the vector of the acoustoelectric force is directed opposite to the drift-velocity vector, and consequently $F_y^{(1)}$ should be larger than zero. It follows directly from this that the Hall constant decreases with increasing electric field when F_y increases.

We shall now show that in a strong electric field, in which, in accordance with the experimental data (see the review [5]), current saturation sets in, so that we can put approximately

$$I_x / L_yL_z \sim en_0v_s \tag{27}$$

(v_s is the velocity of a certain acoustic wave), the Hall coefficient can become negative. Relation (27) means physically that all the electrons are "decelerated" by the generated phonon flux to the velocity of sound. Using now relation (27), we obtain from (19) an estimate for the x component of the acoustoelectric field $\overline{F_x^p}$ averaged over the volume:

$$\overline{F_x^p} = \frac{1}{L_xL_y}\int_0^{L_x} dx \int_0^{L_y} dy\, F_x^p \approx \frac{en_0v_s}{\sigma_0} - \frac{V}{L_x}. \tag{28}$$

We assume now that the x and y components of the acoustoelectric field are of the same order. Since we are interested below only in order-of-magnitude estimates, such an assumption is obviously admissible. Substituting (28) in the expression (22) for the Hall constant and integrating (22) with respect to x from zero to L_x, we obtain for the Hall constant averaged over the x coordinate

$$\overline{R_H(x)} \approx R_H^0\left[1+\frac{1}{h}-\frac{\mu V}{L_xv_sh}\right]. \tag{29}$$

In the derivation of (29) we took into account the fact that F_y^p and F_x^p have opposite signs. It follows thus from (29) that if the potential difference V across the crystal satisfies the condition

$$\mu V/L_xv_s \gg h+1, \tag{30}$$

then the Hall constant averaged over the coordinate x becomes negative; it is obvious that this condition is sufficient for reversal of the sign of the Hall constant at any point of the crystal.

The estimate obtained above agrees well with the data obtained experimentally in p-Te,[8] where reversal of the sign of the Hall constant occurred at a potential difference such that $\mu V/L_Xv_S \approx 4$ and the magnetic field corresponded to the parameter value h = 3 [5]). Moreover, the experimental values $R_H(x)/R_X^0 \approx 6$ observed in an electric field with $\mu V/L_Xv_S \approx 25$ and a magnetic field with h = 3 also agree well with the estimating formula (29), which yields a value 7 for the ratio $R_H(x)/R_H^0$.

Thus, the experimentally measured Hall constant not only reverses sign under phonon-generation conditions, but also greatly exceeds the absolute value of the constant in a weak electric field.

3. SAMPLE WITH SHORT-CIRCUITED HALL EFFECT

Such a sample is easiest to produce in the Corbino-ring geometry (see Fig. 2). The electric field is applied

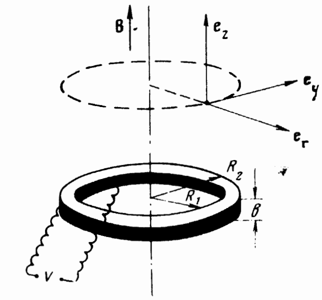

FIG. 2. Sample with short-circuited Hall effect (Corbino ring).

along the radius of the ring, in the magnetic field perpendicular to its plane. We note immediately that in such a geometry, in a sufficiently strong magnetic field, the electron drift is tangential to the periphery; the phonons are generated in the same direction.

The boundary conditions in such a sample are

$$E_\varphi(r) = 0, \qquad \int_{R_1}^{R_2} E_r(r)\,dr = V, \qquad j_z = 0, \tag{31}$$

where E_r and E_φ are respectively the radial and axial projections of the electric-field vector in a cylindrical coordinate frame. It is seen from the symmetry of the problem that $\partial E_r/\partial\varphi = 0$ and therefore the equation curl **E** = 0 is identically satisfied ($E_z = 0$, $\partial/\partial z \to 0$, $E_\varphi = 0$).[6)] As above, we shall consider a case when the criteria (9) are satisfied. We then get from (2) and (3)

$$j_r = \frac{\sigma_0}{1+h^2}\{E_r+F_r+hF_\varphi\}, \qquad j_\varphi = \frac{\sigma_0}{1+h^2}\{F_\varphi - h(E_r+F_r)\}. \tag{32}$$

Substituting this value of the current in the equation div **j** = 0, we obtain a linear equation for the radial component of the electric field E_r. Integrating this equation with allowance for the boundary condition (31), we get

$$E_r = \frac{1}{r}\left(\ln\frac{R_2}{R_1}\right)^{-1}\left[V+\int_{R_1}^{R_2}(F_r+hF_\varphi)\,dr\right] - F_r - hF_\varphi. \tag{33}$$

Substituting (33) into formulas (32) for the current, we obtain

$$j_r = \frac{\sigma_0}{1+h^2}\frac{1}{r}\left(\ln\frac{R_2}{R_1}\right)^{-1}\left[V+\int_{R_1}^{R_2}(F_r+hF_\varphi)\,dr\right],$$
$$j_\varphi = \sigma_0F_\varphi - \frac{h\sigma_0}{(1+h^2)r}\left(\ln\frac{R_2}{R_1}\right)^{-1}\left[V+\int_{R_1}^{R_2}(F_r+hF_\varphi)\,dr\right]. \tag{34}$$

The current depends only on the radius like 1/r, as it should. The total current I_r in the source circuit will obviously be (see Fig. 2)

$$I_r = 2\pi rbj_r. \tag{35}$$

The total current in the ring is determined analogously:

$$I_\varphi = b\int_{R_1}^{R_2} j_\varphi\,dr = \frac{b\sigma_0}{1+h^2}\int_{R_1}^{R_2}F_\varphi\,dr - \frac{bh\sigma_0}{1+h^2}\left[V+\int_{R_1}^{R_2}F_r\,dr\right]. \tag{36}$$

From the general relation (16) for the drift velocity and from the obtained expression for the electric field (33) it follows that

$$v_{dr} = \frac{\mu}{1+h^2}E_r, \qquad v_{d\varphi} = -\frac{\mu h}{1+h^2}E_r. \tag{37}$$

5)The physical explanation of the observed phenomena given in [8] is based on the paper of Gulyaev and Epshteĭn [13], who considered the propagation of an external acoustic wave in a transverse magnetic field (in the absence of an electric field). As shown in [13], an acoustoelectric emf transverse to the wave vector and the magnetic-field vector is produced only if the electron-scattering relaxation time depends on the energy. In our opinion, this physical explanation of the experimentally observed phenomena given in [8] has no bearing on the experiment itself. The authors are grateful to Professor S. Tanaka (Tokyo University) for the opportunity to read his paper prior to publication.

6)The piezoelectric properties of the crystal in the plane of the ring are assumed to be isotropic. The latter can be realized, for example, in a crystal having a symmetry C_{6v}, if the C_6 axis is directed along the magnetic field.

From these drift-velocity components we determine the direction of the acoustoelectric-force vector; it is obvious that if $E_r > 0$, then $F_r < 0$ and $F_\varphi > 0$. If the magnetic field is strong enough and $h > 1$, then $|v_{d\varphi}| > |v_{dr}|$, and consequently phonons are generated mainly in directions tangent to the peripheries. Under these conditions, obviously, $|F_\varphi| > |F_r|$ and therefore, according to (34), a sharp increase of the current in the source circuit takes place. This indeed is the Esaki effect.[14] The circular component of the current I_φ, to the contrary, decreases under the generation conditions. It is also seen from (34) that in a weak magnetic field, when $h < 1$, we have $|F_\varphi| < |F_r|$ and the current saturates (Smith saturation[15]). This is precisely the behavior observed by Moore[16] for a current flowing in a CdS ring in a magnetic field: the current saturated in a weak magnetic field whereas in a strong one, when $h > 1$, a kink appeared on the current-voltage characteristic with increasing current, just as in Esaki's first experiment. Unfortunately, it is difficult to carry out a quantitative comparison of the theory with Moore's experiment,[16] since the maximum value of the magnetic field corresponded to $h = 1.6$.

We note that if account is taken of the nonlinear interaction of the phonons with phonons, the circular phonon flux generated in the ring should lead to a kinematic effect whereby the sample as a whole acquires a momentum opposite to the momentum of the phonon flux.

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[1] E. M. Conwell, Kinetic Properties of Semiconductors in Strong Electric Fields (Russ. transl.), Mir, (1970).

[2] F. G. Bass, Zh. Eksp. Teor. Fiz. 48, 275 (1965) [Sov. Phys.-JETP 21, 181 (1965)].

[3] A. V. Gurevich, ibid. 36, 624 (1959) [9, 434 (1959)].

[4] V. I. Pustovoĭt and Yu. P. Mukhortov, ZhETF Pis. Red. 13, 211 (1971) [JETP Lett. 13, 149 (1971)].

[5] V. I. Pustovoĭt, Usp. Fiz. Nauk 97, 257 (1969) [Sov. Phys.-Usp. 12, 105 (1969)].

[6] V. A. Chuenkov, Fiz. Tverd. Tela 7, 2469 (1965) [Sov. Phys.-Solid State 7, 1990 (1966)].

[7] Yu. V. Gulyaev, ibid. 8, 3366 (1966) [8, 2687 (1967)].

[8] A. Hojo and S. Tanaka, Proc. Int. Conf. Phys. Semicond., Boston, USA, 1970.

[9] I. Akasaki and T. Hara, Trudy IX Mezhdunarodnoĭ konferentsii po fizike poluprovodnikov (Trans. Ninth International Conference on Semiconductor Physics), Moscow, 1968, p. 833.

[10] R. Yamamoto, M. Ikeda, and H. Sato, J. Phys. Soc. Japan 20, 229 (1965).

[11] Yu. V. Gulyaev, A. Yu. Karabanov, A. M. Kmita, A. V. Medved', and Sh. S. Tursunov, Fiz. Tverd. Tela 12, 2595 (1970) [Sov. Phys.-Solid State 12, 2085 (1971)].

[12] L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), Fizmatgiz, 1967.

[13] Yu. V. Gulyaev and É. P. Épshteĭn, Fiz. Tverd. Tela 9, 376 (1967) [Sov. Phys.-Solid State 9, 288 (1967)].

[14] L. Esaki, Phys. Rev. Lett. 8, 4 (1962).

[15] R. W. Smith, Phys. Rev. Lett. 9, 87 (1962).

[16] A. R. Moore, Phys. Rev. Lett. 12, 47 (1964).

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