ESTIMATED RADIATION INTENSITY EMITTED BY A MODULATED ELECTRON BEAM

B. Ya. ZEL'DOVICH

P. N. Lebedev Physics Institute, USSR Academy of Sciences

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The intensity of radiation emitted by an electron beam modulated at light frequency is estimated. It is shown that theoretical models of radiation by a beam of this kind which have recently been put forward to explain the results of the Schwarz-Hora experiments are, in fact, inconsistent with these results.

A number of theoretical papers have recently been published (see[1-8]), dealing with the radiation emitted when an electron beam, whose density is modulated at light frequencies, is incident on a nonluminescing screen. The purpose of these papers was to explain the results obtained by Schwarz and Hora[9], who reported the experimental observation of this type of radiation. We shall use a few simple estimates to show that the proposed theories cannot explain the results reported in[9] at least in the following two respects:

1) The total intensity of the modulation radiation estimated from the proposed theories turns out to be lower by factors of 10^4–10^5 than the experimentally measured value of 10^{-10} W.

2) When an electron beam is incident on a nonluminescing screen one should observe white radiation with a continuous spectrum in the visible range (made up of the transition radiation of the individual electrons). Under the conditions of the experiment described in[9] the intensity of this white radiation should be greater by factors of 10^5–10^6 than the intensity of the monochromatic (modulation) radiation predicted in the theoretical papers.[1-3]

Let us now prove the above statements.

1. The mechanism assumed in[1-3] was emission by the modulated current and its mirror image in the surface of the screen (transition radiation).[1)] To be specific, consider normal incidence of an electron beam on a surface and suppose that the transverse size D of the beam is less than or of the order of $\lambdabar = \lambda_0/2\pi$, where $\lambda_0 = 2\pi c/\omega_0$ is the wavelength of the modulating light. We shall write the electron current in the form

$$j(z,t) = e[i_= + i_\sim \cos(\omega_0 t - \omega_0 z/v)], \qquad (1)$$

where v is the electron velocity, i is the total (dc) current (electrons per sec), and the ratio $(i_=/i_\sim)$ is determined by the modulation depth. If we calculate the intensity of the radiation emitted by this current and by its mirror image, we obtain, assuming that $v \ll c$,

$$dW_{\text{mod}} = \frac{e^2}{2\pi}\left(\frac{v}{c}\right)^2 \frac{i_\sim^2}{c}\sin^2\theta\, do, \qquad (2)$$

where do is the solid angle element and θ is the angle between the direction of emission and the normal to the metal surface. Integration over a hemisphere then yields

$$W_{\text{mod}} = \frac{2}{3}\frac{e^2}{c}\left(\frac{v}{c}\right)^2\left(\frac{i_\sim}{i_=}\right)^2 i_=^2. \qquad (3)$$

According to the estimates based on the modulation theory given in[2,3], the modulation depth $(i_\sim/i_=)$ under the conditions of the experiment described in[9] was of the order of 0.1. If we substitute the experimental velocity $v = 10^{10}$ cm/sec and the experimental current 0.5 μA ($i_= = 3\times10^{12}$ electrons/sec) we find that the theoretical value of the total intensity of the monochromatic modulation radiation is $W_{\text{mod}} = 5\times10^{-15}$ W, which is lower by a factor of 2×10^4 than the figure of 10^{-10} W reported in the original work.[2)] Even if we assume that $(i_\sim/i_=) = 1$ the discrepancy by a factor of about 200 still remains. The fact that the transverse beam size was a few microns in[9] should reduce the theoretical estimate of the total intensity of the modulation radiation by a further factor of $(D/\lambdabar_0)^2 \approx 100$.

The mechanism based on the coherent (multielectron) transition radiation proposed in the above theoretical papers is thus incapable of explaining the high observed intensity.

2. Each individual electron incident on a metal screen should produce a continuous spectrum of transition radiation even in the absence of any systematic modulation. The energy radiated by the dipole formed by an electron and its mirror image into a solid angle do into the frequency range $d\omega$ is (see[10]; Section 69)

$$d\varepsilon = \frac{e^2}{\pi^2 c}\left(\frac{v}{c}\right)^2 \sin^2\theta\, do\, d\omega. \qquad (4)$$

Integration over a hemisphere and the frequency interval $\Delta\omega$ gives the total intensity of noncoherent radiation:

$$W_{\text{incoh}} = \frac{4}{3\pi}\frac{e^2}{c}\left(\frac{v}{c}\right)^2 i_=\Delta\omega. \qquad (5)$$

The ratio of the monochromatic component (for given depth of modulation $i_\sim/i_=$) to the noncoherent white radiation in a frequency band $\Delta\omega$ which arises independently of the systematic modulation is

$$\frac{W_{\text{mod}}}{W_{\text{incoh}}} = \frac{\pi}{2}\frac{i_=}{\Delta\omega}\left(\frac{i_\sim}{i_=}\right)^2. \qquad (6)$$

This formula gives the ratio not only for the transition radiation but also for the Cerenkov radiation (considered

1)In actual fact, the experimental screen was the dielectric Al_2O_3 and not a metal. If the theory is developed for the transition radiation at a dielectric this will only increase the disagreement with regard to 1, whereas as far as 2 is concerned the situation will be the same as for a metal screen.

2)This was reported by H. Schwarz at the Conference on Laser Plasma, Moscow, 1970.

in[8]) and for any other radiation mechanism provided the mechanism is not frequency selective. This ratio can be evaluated from the Schottky formula for shot noise in a frequency band $\Delta\omega$ (see[11]; Sec. 7.4):

$$\overline{(i_{\sim}\cos\omega_0 t)^2}=\frac{1}{2}i_{\sim}^2\to 2i_{=}\frac{\Delta\omega}{2\pi}. \qquad (7)$$

If we suppose that $\Delta\omega$ corresponds to the width of the visible part of the spectrum, $\Delta\omega \approx \omega_0 = 2\pi/T_0$, where ω_0 corresponds to the wavelength of the modulating light (blue line of argon laser, $\lambda_0 = 4880$ Å) we find that

$$\frac{W_{\text{mod}}}{W_{\text{incoh}}}=\frac{i_{=}T_0}{4}\left(\frac{i_{\sim}}{i_{=}}\right)^2, \qquad (8)$$

where $T_0 = 2\pi/\omega_0$ is the modulation period. In[9] the quantity $i_{=}T_0/4$ was approximately 0.001 (for the central undiffracted spot) since $T_0 = 1.6\times10^{-15}$ sec. Thus, even for 100% modulation depth, the above theoretical descriptions of the Schwarz-Hora experiment predict that one should observe largely white radiation unconnected with modulation, and the intensity of the monochromatic modulation component should be only 10^{-3} of the total radiation intensity in the visible region. Any difference from the 100% modulation depth, the effect of finite transverse-beam size (for $D \gtrsim \lambda\!\!\!^{-}_0$), and the reduced $i_{=}$ for the Laue diffraction spots observed in the experiment will only reduce the theoretical estimate for $W_{\text{mod}}/W_{\text{incoh}}$.

Within the framework of the theory given in[1-8] one could develop the following hypothesis which would avoid, at least partially, the conflict with experiment. However, we must emphasize that we do not regard this as the likely explanation.

Let us suppose that the main electron current $i_{=} = 3\times10^{12}$ sec^{-1} was not constant in the experiment but had sharp bursts with a duty ratio $Q \gtrsim 10^4-10^5$ and time interval between bursts $\Delta t \gtrsim 10^{-10}$ sec. Due to the increase in the instantaneous $i_{=}(t)$ up to about $10^{17}-10^{18}$ sec^{-1} one could then ensure that the order of magnitude of the theoretical estimates for W_{mod} and W_{incoh} would be of the same order of magnitude, and so will W_{mod} and the experimental figure of 10^{-10} W. This behavior of the current $i_{=}(t)$ can readily be detected from its noise properties in the microwave range, so that this hypothesis can readily be checked experimentally.

We have so far used the classical theory of radiation to calculate the intensity of monochromatic and noise components. A rigorous quantum electrodynamic analysis of the process of emission of quanta with energies of about 3 eV (visible range) by a modulated beam of electrons of energy ~ 50 keV is found to give the same results.

The present author considers that the quantum theory of modulation of an electron beam in the theory of coherent (multielectron) transition radiation is not internally inconsistent and will be useful for the analysis of the above effects.

At the same time, we may conclude that the theoretical models put forward so far[1-8] are incapable of explaining the Schwarz-Hora results.

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