SURFACE MAGNETOSTATIC WAVES IN UNIAXIAL ANTIFERROMAGNETS

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The spectrum of surface magnetostatic waves is obtained for a semi-infinite uniaxial antiferromagnet. Ranges of existence are found for these waves in magnetic fields perpendicular and parallel to the surface of the crystal, for antiferromagnets with magnetic anisotropy characterized by an axis or a plane of easy magnetization.

1. INTRODUCTION

It was Damon and Eshbach[1] who first calculated the spectrum of surface magnetostatic oscillations in a semi-infinite ferromagnet. Those authors showed that surface waves, in contrast to volume waves, possess the so-called independence property; that is, they are so propagated that the projections of their wave vectors on the axis $\mathbf{n}_0 \times \mathbf{H}$ are always positive ($\mathbf{n}_0$ = vector normal to the surface, $\mathbf{H}$ = vector magnetic field). It was also shown[1] that the spectrum of surface magnetostatic waves lies above the spectrum of volume oscillations. At present, investigations are to be made into the use of these properties of the surface-wave spectrum for the production of nonreciprocal elements in the microwave range.[2]

The present paper considers the problem of finding the spectrum of surface magnetostatic oscillations in semi-infinite uniaxial antiferromagnets, for magnetic fields parallel and perpendicular to the crystal surface. It is found that the frequencies of surface waves may lie either inside or outside the range of existence of volume oscillations. In antiferromagnets, in contrast to ferromagnets, surface waves can exist also in the case in which the magnetic field is perpendicular to the crystal surface.

2. DISPERSION EQUATIONS

To seek the spectrum of surface oscillations, we shall use Maxwell's equations in the magnetostatic approximation:

$$\operatorname{rot}\mathbf{h} = 0, \quad \operatorname{div}\hat{\mu}\mathbf{h} = 0, \tag{1}$$

where $\hat{\mu}(\omega)$ is the magnetic permeability tensor, and where $\mathbf{h}$ is the alternating magnetic field. We shall suppose that outside the antiferromagnet, $\hat{\mu} = 1$. On introducing, as usual, the scalar potential φ instead of the magnetic field

$$\mathbf{h} = \nabla\varphi,$$

we write equation (1) in the form

$$\mu_{ik}(\omega)\nabla_i\nabla_k\varphi^{(i)} = 0, \quad \nabla^2\varphi^{(e)} = 0, \tag{2}$$

where $\varphi^{(i)}$ and $\varphi^{(e)}$ denote the potentials inside and outside the ferromagnet, respectively.

By choosing a system of coordinates XYZ with the Z axis along the external normal to the surface, one can write the boundary conditions on the magnetic potentials $\varphi^{(i)}$ and $\varphi^{(e)}$ in the following form:

$$\varphi^{(i)}|_{z=0} = \varphi^{(e)}|_{z=0}, \quad \varphi^{(e)}|_{z\to+\infty} = 0, \quad \varphi^{(i)}|_{z\to-\infty} = 0,$$
$$\mu_{zx}\frac{\partial\varphi^{(i)}}{\partial X}\Big|_{z=0} + \mu_{zY}\frac{\partial\varphi^{(i)}}{\partial Y}\Big|_{z=0} + \mu_{zz}\frac{\partial\varphi^{(i)}}{\partial Z}\Big|_{z=0} = \frac{\partial\varphi^{(e)}}{\partial Z}\Big|_{z=0}. \tag{3}$$

We shall seek a solution of equations (2) in the form:

$$\varphi^{(i)} = A\exp\{qZ + ik_XX + ik_YY\},$$
$$\varphi^{(e)} = A\exp\{-fZ + ik_XX + ik_YY\}. \tag{4}$$

On substituting (4) into equations (2), we get

$$\mu_{XX}k_X^2 + \mu_{YY}k_Y^2 - \mu_{ZZ}q^2 = 0, \quad f = k_\perp \equiv \sqrt{k_X^2 + k_Y^2}. \tag{5}$$

From the boundary conditions (3) there follows a second equation, which relates the frequency of the surface waves with wave vector $\mathbf{k}$ to the value of q:

$$ik_X\mu_{ZX} + ik_Y\mu_{ZY} + q\mu_{ZZ} + k_\perp = 0. \tag{6}$$

On solving for q in equation (6) and substituting in (5), we get the dispersion equation for a surface magnetostatic wave,

$$\mu_{XX}\mu_{ZZ} - \mu_{ZZ}(\mu_{XX} - \mu_{YY})y^2 - (1 + i\mu_{ZX}x + i\mu_{ZY}y)^2 = 0 \tag{7}$$

and the equation for the imaginary component of the wave vector, q:

$$q/k_\perp = -\mu_{ZZ}^{-1}(1 + i\mu_{ZX}x + i\mu_{ZY}y), \tag{8}$$

This determines the range of existence of surface waves (from the condition $q > 0$). Here $x = k_X/k_\perp$, $y = k_Y/k_\perp$, $x^2 + y^2 = 1$.

3. FREQUENCIES OF SURFACE OSCILLATIONS IN ANTIFERROMAGNETS WITH MAGNETIC ANISOTROPY OF THE "EASY AXIS" TYPE

Because the spectrum of surface oscillations depends significantly on the orientation of the external magnetic field with respect to the surface of the crystal, we shall consider two situations: when the magnetic field is directed perpendicular to the surface, and when it is directed parallel to it.

A. If the magnetic field is parallel to the anisotropy axis n and perpendicular to the crystal surface ($\mathbf{H} \parallel \mathbf{Z}$), the magnetic permeability tensor has the form

$$\hat{\mu} = \begin{bmatrix} \mu & i\mu' & 0 \\ -i\mu' & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Turning to Eq. (8), we see that in the case under consideration, surface waves are nonexistent, because the inequality $q > 0$ cannot be satisfied when $\mu_{ZZ} = 1$ and $\mu_{ZX} = \mu_{ZY} = 0$.

B. In a magnetic field parallel to the crystal surface ($\mathbf{H} \parallel \mathbf{X}$) and to the anisotropy axis, the tensor $\hat{\mu}$ will be [3]

$$\hat{\mu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & i\mu' \\ 0 & -i\mu' & \mu \end{bmatrix}, \tag{9}$$

where the form of the components μ and μ' depends on the value of the applied magnetic field.

If the field H is less than the field H_{AE} for "flop" of the magnetic moments, the values of μ and μ' will be

$$\mu(\omega) = 1 + \chi_0\omega_0\left(\frac{\varepsilon_1}{\varepsilon_1^2 - \omega^2} + \frac{\varepsilon_2}{\varepsilon_2^2 - \omega^2}\right),$$

$$\mu'(\omega) = \chi_0\omega_0\omega\left(\frac{1}{\varepsilon_2^2 - \omega^2} - \frac{1}{\varepsilon_1^2 - \omega^2}\right),$$

$$\varepsilon_{1,2} = \omega_0 \pm \omega_H, \quad \omega_0 = \gamma H_{AE}, \quad \omega_H = \gamma H, \quad H_{AE} = \sqrt{H_A H_E},$$
$$H_A = (\beta - \beta')M_0, \quad H_E = 2\delta M_0, \quad \chi_0 = 2\pi/\delta. \tag{10}$$

If, however, $H > H_{AE}$, then

$$\mu = 1 + 2\chi_0\frac{\omega_H^2}{\omega_H^2 - \omega^2}; \qquad \mu' = 2\chi_0\frac{\omega\omega_H}{\omega_H^2 - \omega^2}. \tag{10'}$$

Here γ is the gyromagnetic ratio, δ is a dimensionless constant of uniform exchange between the two magnetic sublattices ($\delta \sim 10^2$ to 10^3), β and β' are anisotropy constants, and M_0 is the magnetic-moment density of a single magnetic sublattice.

On substituting the values of the components of the tensor $\hat{\mu}$ from (9) into equations (7) and (8), we get

$$\mu + \mu(\mu - 1)y^2 - (1 + y\mu')^2 = 0, \tag{11}$$

$$q/k_\perp = -(1 + y\mu')/\mu. \tag{11'}$$

A qualitative analysis of equations (11) and (11′) in the case in which $\mu(\omega)$ and $\mu'(\omega)$ are determined by formulas (10) was carried out in paper [4].

If $H < H_{AE}$, the frequency of a surface wave is determined by the following equation:

$$\omega = \{[1 + 2\chi_0 y^2/(1 + y^2)]\omega_0^2 - [(1 - y^2)/(1 + y^2)]^2\omega_H^2\}^{1/2} - 2y\omega_H/(1 + y^2), \tag{12}$$

whence it is seen that the value of ω depends on the sign of the projection of the wave vector along the Y axis. The condition (11′) determines the range of existence of surface oscillations. Analysis of the expression (11′) shows that the permissible values of the frequencies for $y < 0$ lie inside the region of volume oscillations

$$\omega_c(H) < \omega < \omega_H + \omega_0\sqrt{1 + \chi_0}, \tag{13}$$

whereas for $y > 0$ surface oscillations exist in the magnetic-field intervals

$$0 < H < H_1 \equiv {}^1\!/_2 H_{AE}(\sqrt{1 + \chi_0} - \sqrt{1 - \chi_0}), \tag{14}$$
$${}^1\!/_2 H_{AE}(\sqrt{1 + \chi_0} + \sqrt{1 - \chi_0}) \equiv H_2 < H < H_{AE}.$$

The limiting frequency $\omega_c(H)$ is determined from the equation

$$1 - \mu - (\mu')^2 = 0 \tag{15}$$

and corresponds to a wave with $q = 0$. If $H \ll \chi_0 H_{AE}$, the dependence of ω_c on magnetic field has the form

$$\omega_c^2 \approx \omega_0^2[1 + 2(\chi_0 H^2/H_{AE}^2)^{1/3}]. \tag{16}$$

In the case of quite strong fields, $\chi_0 H_{AE} \ll H < H_{AE}$, we have

$$\omega_c^2 \approx \varepsilon_1^2 + \chi_0\omega_0\varepsilon_1 - \chi_0^2\omega_0^2\frac{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_1\varepsilon_2}{\varepsilon_1^2 - \varepsilon_2^2}. \tag{17}$$

In magnetic fields larger than the "flop" field ($H > H_{AE}$), the values of μ and μ' are determined by formulas (10′), substitution of which into (11) and (11′) gives

$$\omega = \omega_H(\chi_0 y + {}^1\!/_2(y + y^{-1})), \tag{18}$$

$$\omega_H\sqrt{1 + 2\chi_0} < \omega < \omega_H(1 + \chi_0), \quad (1 + 2\chi_0)^{-1} < y < 1. \tag{18'}$$

Hence it is seen that when $H > H_{AE}$, the values of the frequencies of surface magnetostatic waves lie above the range of existence of volume oscillations.

In Fig. 1, the regions of existence of surface and volume waves are pictured schematically for antiferromagnets with magnetic anisotropy of the "easy axis" type, in the case in which the magnetic field **H** is parallel to the magnetic-anisotropy axis **n**.

C. If $\mathbf{H} \parallel \mathbf{Z}$ and $\mathbf{H} \perp \mathbf{n} \parallel \mathbf{X}$, the magnetic permeability tensor will have the form

$$\hat{\mu} = \begin{bmatrix} \mu_1 & i\mu' & 0 \\ -i\mu' & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix},$$

$$\mu_1 = 1 + 2\chi_0\frac{\omega_H^2}{\omega_1^2 - \omega^2}, \quad \mu_2 = 1 + 2\chi_0\frac{\omega_1^2}{\omega_1^2 - \omega^2}, \quad \mu_3 = 1 + 2\chi_0\frac{\omega_2^2}{\omega_2^2 - \omega^2},$$

$$\mu' = 2\chi_0\frac{\omega\omega_H}{\omega_1^2 - \omega^2}, \qquad \omega_1^2 = \omega_0^2 + \omega_H^2, \qquad \omega_2^2 = \omega_0^2\left(1 - \frac{H^2}{H_E^2}\right). \tag{19}$$

In this case we have, for the frequencies of volume magnetostatic oscillations, the expression

$$\Omega_{1,2}^2 = {}^1\!/_2\{\omega_1^2 + \omega_2^2 + 2\chi_0(x^2\omega_H^2 + y^2\omega_1^2 + z^2\omega_2^2)$$
$$\pm [(\omega_1^2 - \omega_2^2)^2 + 4\chi_0(\omega_1^2 - \omega_2^2)(x^2\omega_H^2 + y^2\omega_1^2 - z^2\omega_2^2)$$
$$+ 4\chi_0^2(x^2\omega_H^2 + y^2\omega_1^2 + z^2\omega_2^2)^2]^{1/2}\}, \tag{20}$$
$$x = k_X/k, \quad y = k_Y/k, \quad z = k_Z/k, \quad k = \sqrt{k_X^2 + k_Y^2 + k_Z^2}.$$

Analysis of formula (20) shows that the frequencies $\Omega_{1,2}$ lie in the following intervals:

$$\omega_{1,2}^2 \leqslant \Omega_{1,2}^2 \leqslant \omega_{1,2}^2(1 + 2\chi_0). \tag{21}$$

On substituting the values of the components of the tensor $\hat{\mu}$ from (19) into (7) and (8), we obtain the equations that determine the frequencies of the surface oscillations:

$$\mu_1\mu_3 - \mu_3(\mu_1 - \mu_2)y^2 - 1 = 0, \tag{22}$$

$$q/k_\perp = -\mu_3^{-1}. \tag{22'}$$

The solution of (22) has the form

$$\omega^2 = \omega_2^2\left[1 + 2\chi_0 - \frac{\omega_2^2(1 + \chi_0) - \omega_1^2}{\omega_1^2 + \omega_2^2 - \omega_0^2(1 - y^2)}\right], \tag{23}$$

and (22′) determines the range of existence of surface waves:

$$0 < H^2 < H_0^2 = \frac{2\chi_0 H_{AE}^2}{1 + (1 + 2\chi_0)H_A/H_E} \tag{24}$$

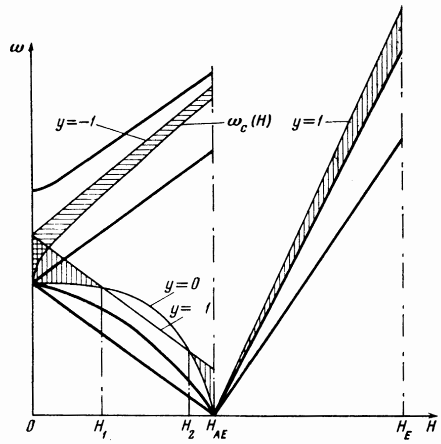

FIG. 1. Regions of existence of surface and volume oscillations in antiferromagnets of the "easy axis" type for the case H || n || X. The solid bold line shows the boundaries of the spectrum of volume waves; the horizontal hatching corresponds to surface waves with $y < 0$, the vertical with $y > 0$.

It is easily shown that the frequencies of surface waves lie inside the region of existence of volume waves. In Fig. 2 the regions of existence of surface and volume oscillations are pictured schematically.

D. If the anisotropy axis is perpendicular to the crystal surface ($\mathbf{n} \parallel \mathbf{Z}$), while the magnetic field is parallel to the surface ($\mathbf{H} \parallel \mathbf{X}$), the magnetic permeability tensor has the form

$$\hat{\mu} = \begin{bmatrix} \mu_3 & 0 & 0 \\ 0 & \mu_2 & i\mu' \\ 0 & -i\mu' & \mu_1 \end{bmatrix},$$

where the expressions for the values of μ_1, μ_2, μ_3, and μ' are determined by formulas (19). In this case, we have the following equation for the frequencies of surface waves:

$$(1-y^2)\omega_2^2(\omega_1^2-\omega^2) + (\omega_2^2-\omega^2)[\omega_H^2 + (\omega_1^2 + 2\chi_0\omega_H^2)y^2 - 2y\omega_H\omega] + 2\chi_0\omega_H^2\omega_2^2(1-y^2) = 0, \quad (25)$$

the frequency of surface oscillations must satisfy one of the following inequalities: either

$$\chi_0\omega_H y + \sqrt{\omega_1^2 + \chi_0^2\omega_H^2 y^2} < \omega < \sqrt{\omega_1^2 + 2\chi_0\omega_H^2},$$

or

$$\sqrt{\omega_1^2 + 2\chi_0\omega_H^2} < \omega < \chi_0\omega_H y + \sqrt{\omega_1^2 + \chi_0^2\omega_H^2 y^2}. \quad (26)$$

It is found that in the case $H_A < 8\pi M_0$, the frequencies of surface waves with negative values of y are enclosed in the region I, bounded by the curves (in the plane (ω^2, H^2)):

$$\omega^2 = \omega_2^2, \quad \omega^2 = \omega_2^2 \frac{H_{AE}^2 + 2(1+\chi_0)H^2}{H_{AE}^2 + (1 - H_A/H_E)H^2};$$
$$\omega^2 = \omega_1^2 + 2\chi_0\omega_H^2 \frac{\omega_2^2(1+2\chi_0) - \omega_1^2}{\omega_2^2 + 2\chi_0\omega_H^2 - \omega_1^2}; \quad (27)$$

while those with positive y are enclosed in three regions: when $H^2 < H_{AE}^2(1 + H_A/H_E)^{-1}$ (region II),

$$\omega^2 = \omega_1^2 + 2\chi_0\omega_H^2, \quad \omega^2 = \omega_2^2 \frac{H_{AE}^2 + 2(1+\chi_0)H^2}{H_{AE}^2 + (1 - H_A/H_E)H^2},$$
$$\omega^2 = \omega_1^2 + 2\chi_0\omega_H^2 \frac{\omega_2^2(1+2\chi_0) - \omega_1^2}{\omega_2^2 + 2\chi_0\omega_H^2 - \omega_1^2}; \quad (28)$$

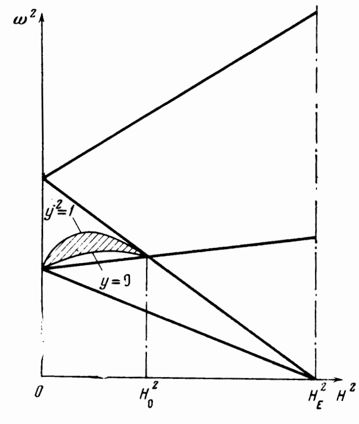

FIG. 2. Regions of existence of surface and volume waves in antiferromagnets of the "easy axis" type for the case H || Z, H ⊥ n || X. The region of surface waves is hatched.

when $(1 + H_A/H_E)^{-1}H_{AE}^2 < H^2 < (2\chi_0)^{-1}H_{AE}^2$ (region III),

$$\omega_1^2 + 2\chi_0\omega_H^2 < \omega^2 < \omega_1^2 + 2\chi_0\omega_H^2 \frac{\omega_2^2(1+2\chi_0) - \omega_1^2}{\omega_2^2 + 2\chi_0\omega_H^2 - \omega_1^2}; \quad (29)$$

when $(2\chi_0)^{-1}H_{AE}^2 < H^2 < H_E^2$ (region IV),

$$\omega_1^2 + 2\chi_0\omega_H^2 < \omega^2 < (1+\chi_0)^2\omega_H^2 + (1+\chi_0)\omega_0^2 + \omega_0^4/4\omega_H^2. \quad (30)$$

In regions I–III, the frequencies of surface oscillations lie inside the region of existence of volume waves, whereas in region IV they go partly outside it. If $H_A > 8\pi M_0$, the region IV is absent.

Figure 3 shows the regions of existence of surface and volume waves for the case considered.

4. FREQUENCIES OF SURFACE OSCILLATIONS IN ANTIFERROMAGNETS WITH MAGNETIC ANISOTROPY OF THE "EASY PLANE" TYPE

The characteristic frequencies of volume magnetostatic oscillations in an antiferromagnetic plate with magnetic anisotropy of the "easy plane" type were calculated in [5,6] To calculate the spectrum of surface waves, we consider first the case in which the magnetic field is perpendicular to the crystal surface ($\mathbf{H} \parallel \mathbf{Z}$), while the magnetic-anisotropy axis is parallel to it. Then the magnetic permeability tensor has the form[7]

$$\hat{\mu} = \begin{bmatrix} \mu_1 & i\mu' & 0 \\ -i\mu' & \mu_1 & 0 \\ 0 & 0 & \mu_2 \end{bmatrix},$$

$$\mu_1 = 1 + 2\chi_0\frac{\omega_H^2}{\omega_H^2 - \omega^2}, \quad \mu_2 = 1 + 2\chi_0\frac{\varepsilon^2}{\varepsilon^2 - \omega^2}, \quad \mu' = 2\chi_0\frac{\omega\omega_H}{\omega_H^2 - \omega^2},$$
$$\varepsilon^2 = \gamma^2 H_{AE}^2\left(1 - \frac{H^2}{H_E^2}\right). \quad (31)$$

On substituting the expressions for the components of the tensor $\hat{\mu}$ into formulas (7) and (8), we get equations for determination of the frequencies of surface waves:

$$1 - \mu_1\mu_2 = 0, \quad (32)$$

$$q/k_\perp = -\mu_2^{-1} > 0. \quad (32')$$

Solution of (32) and (32′) gives

$$\omega^2 = 2\gamma^2 H_{AE}^2(1+\chi_0)\frac{H^2(1 - H^2/H_E^2)}{H_{AE}^2 + H^2(1 - H_A/H_E)}$$

$$q/k_\perp = (1+2\chi_0)\frac{H_{k2}^2}{H_{k1}^2}\frac{H^2 - H_{k1}^2}{H_{k2}^2 - H^2}. \quad (33)$$

$$H_{k1}^2 = H_{AE}^2(1+2\chi_0 + H_A/H_E)^{-1},$$
$$H_{k2}^2 = H_{AE}^2[(1+2\chi_0)^{-1} + H_A/H_E]^{-1}. \tag{34}$$

As is seen from (34), surface magnetostatic waves exist in the case under consideration only in the magnetic-field interval

$$H_{k1} < H < H_{k2}, \tag{35}$$

where, as was shown in [6], volume oscillations do not exist. When $H \to H_{k1}$, the spectrum of surface waves joins on to the spectrum of volume oscillations ($q \to 0$); and when $H \to H_{k2}$, a surface wave is concentrated mainly at the surface of the crystal ($q \to \infty$). The region of existence of surface oscillations when $\mathbf{H} \parallel \mathbf{Z}$ is shown schematically in Fig. 4 (dotted line)

If the magnetic field is parallel to the crystal surface ($\mathbf{H} \parallel \mathbf{X}$), while the anisotropy axis is perpendicular to it, the magnetic permeability tensor $\hat{\mu}$ will be

$$\hat{\mu} = \begin{bmatrix} \mu_2 & 0 & 0 \\ 0 & \mu_1 & i\mu' \\ 0 & -i\mu' & \mu_1 \end{bmatrix},$$

and Eqs. (7) and (8) will take the form

$$\mu_1\mu_2 + \mu_1(\mu_1 - \mu_2)y^2 - (1 + y\mu')^2 = 0, \tag{36}$$

$$q/k_\perp = -\mu_1^{-1}(1 + y\mu') \tag{36'}$$

Equations (36) and (36′) give several regions of existence of surface waves:

1) a region with $(1 + 2\chi_0)^{-1} < y < 1$ enclosed in the interval

$$\omega_H\sqrt{1+2\chi_0} < \omega < \omega_H(1+\chi_0); \tag{37}$$

2) a region with $-1 < y < 0$ bounded by the following curves:

$$\omega^2 = \gamma^2 H_{AE}^2(1 - H^2/H_E^2),$$

$$\omega^2 \leqslant 2\gamma^2 H_{AE}^2(1+\chi_0)\frac{H^2(1 - H^2/H_E^2)}{H_{AE}^2 + H^2(1 - H_A/H_E)},$$

$$\omega^2 = \omega_H^2\left\{1 + \frac{2\chi_0[(1+2\chi_0)H_{AE}^2(1 - H^2/H_E^2) - H^2]}{H_{AE}^2(1 - H^2/H_E^2) - (1 - 2\chi_0)H^2}\right\}; \tag{38}$$

3) a region with $0 < y < 1$ bounded by the same curves as the second region, except that the inequality sign in the second formula (38) must be reversed.

Figure 4 gives a schematic representation of the regions of existence of surface and volume oscillations in the case under consideration. On comparing the results obtained in this section with the results of [6], we see that the interval $H_{k1} < H < H_{k2}$, which has remained "empty" for volume oscillations, is filling up with surface waves; the frequency of the surface waves satisfies the inequality $\omega(y, H) \neq \omega(-y, H)$. In the region of overlap of the first and third regions, for given frequency and value of the magnetic field, there can be excited two surface waves, with different (positive) projections y of the wave vectors, and volume oscillations.

5. FREQUENCIES OF SURFACE OSCILLATIONS IN ANTIFERROMAGNETS WITH WEAK FERROMAGNETISM

In a whole series of antiferromagnets ($MnCO_3$, $CoCO_3$, $FeBO_3$, α-Fe_2O_3, NiF_2), the magnetic moments of the

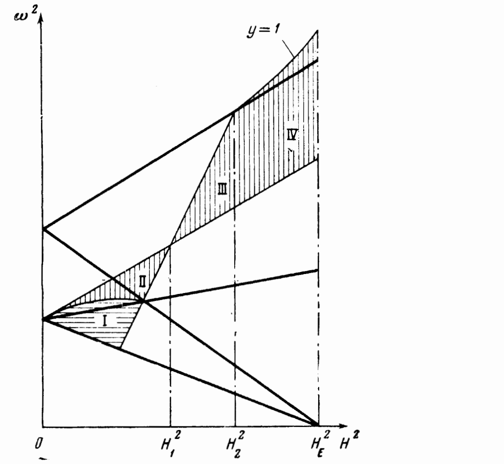

FIG. 3. Regions of existence of surface and volume waves in antiferromagnets of the "easy axis" type for the case $\mathbf{H} \parallel \mathbf{X}$, $\mathbf{n} \parallel \mathbf{Z}$, $H_1^2 = H_{AE}^2(1 + H_A/H_E)^{-1}$, $H_2^2 = H_{AE}^2(2\chi_0)^{-1}$.

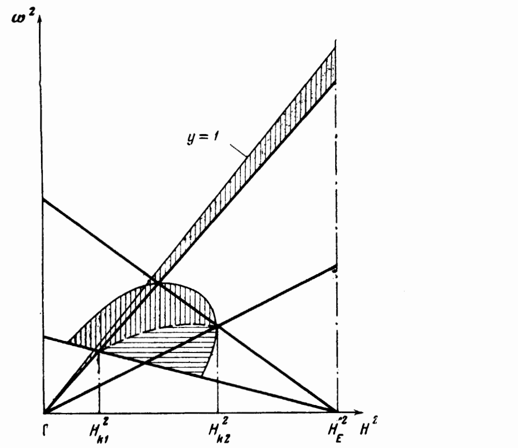

FIG. 4. Regions of existence of surface and volume waves in antiferromagnets of the "easy plane" type. The hatched regions correspond to the case $\mathbf{H} \parallel \mathbf{X}$, the dotted line to the case $\mathbf{H} \parallel \mathbf{Z}$.

sublattices are noncollinear in the absence of a magnetic field (antiferromagnets with weak ferromagnetism). The magnetic permeability tensor for such antiferromagnets, in the case in which the magnetic field is parallel to the weak moment and is directed along the crystal surface ($\mathbf{H} \parallel \mathbf{X}$), has the form

$$\hat{\mu} = \begin{bmatrix} \mu_X & 0 & 0 \\ 0 & \mu_Y & i\mu' \\ 0 & -i\mu' & \mu_Z \end{bmatrix},$$

$$\mu_X = 1 + 2\chi_0\frac{\varepsilon_2^2}{\varepsilon_2^2 - \omega^2}, \quad \mu_Y = 1 + 2\chi_0\frac{(\omega_H + \omega_d)^2}{\varepsilon_1^2 - \omega^2},$$

$$\mu_Z = 1 + 2\chi_0\frac{\varepsilon_1^2}{\varepsilon_1^2 - \omega^2}, \quad \mu' = 2\chi_0\frac{\omega(\omega_H + \omega_d)}{\varepsilon_1^2 - \omega^2}, \tag{39}$$

where

$$\varepsilon_1^2 = \omega_H(\omega_H + \omega_d), \ \varepsilon_2^2 = \gamma^2 H_{AE}^2 + \omega_d(\omega_H + \omega_d), \ \omega_d = \gamma H_d,$$

H_d is the Dzyaloshinskiĭ field.

On substituting the components of the tensor $\hat{\mu}$ from formula (39) into equations (7) and (8), we get

$$\mu_X\mu_Z - \mu_Z(\mu_X - \mu_Y)y^2 - (1 + y\mu')^2 = 0, \tag{40}$$

$$q / k_\perp = -\mu_z^{-1}(1 + y\mu'). \tag{40'}$$

We shall seek a solution of equations (40) and (40′) in the region of small magnetic fields ($H \ll H_d$). Then the frequency of a surface wave will satisfy the inequality $\omega \ll \epsilon_2$, and equations (40) and (40′) will take the form

$$(1-y^2)\omega^2 + 2y(\omega_H+\omega_d)\omega - \{\varepsilon_1^2[(1-y^2)(1+2\chi_0)+1] + (1+2\chi_0)(\omega_H+\omega_d)^2y^2\} = 0, \tag{41}$$

$$\frac{q}{k_\perp} = -\frac{\omega^2 - 2\chi_0\omega y(\omega_H+\omega_d) - \varepsilon_1^2}{\omega^2 - (1+2\chi_0)\varepsilon_1^2}. \tag{41'}$$

Equation (41) gives the following dependence of the frequency on H and y:

$$\omega^2 = \frac{1}{1-y^2}\{y^2(\omega_H+\omega_d)^2 + (1-y^2)(1+2\chi_0)[(\omega_H+\omega_d)^2y^2 + \varepsilon_1^2(1-y^2)] + \varepsilon_1^2(1-y^2)\}^{1/2} - \frac{\omega_H+\omega_d}{1-y^2}y, \tag{42}$$

while equation (41′) gives the region of existence of surface waves ($q > 0$):

$$(1-\chi_0)\sqrt{\frac{H}{H+H_d}} < y < (1+3\chi_0)\sqrt{\frac{H}{H+H_d}}. \tag{43}$$

We note that the inequality (43) was obtained in a linear approximation with respect to χ_0. Thus Eq. (42) and inequality (43) determine the region of existence of surface magnetostatic waves in antiferromagnets with weak ferromagnetism in weak magnetic fields.

[1] R. W. Damon and J. R. Eshbach, J. Appl. Phys. **31**, 104S (1960).
[2] P. Young, Electr. Lett. **4**, 566 (1968).
[3] M. I. Kaganov and V. M. Tsukernik, Zh. Eksp. Teor. Fiz. **41**, 267 (1961) [Sov. Phys.-JETP **14**, 192 (1962)].
[4] K. A. Ryazantsev and I. A. Gilinskiĭ, Fiz. Tverd. Tela **12**, 2217 (1970) [Sov. Phys.-Solid State **12**, 1770 (1970)].
[5] N. I. Gordon, A. M. Kadigrobov, and M. A. Savchenko, Zh. Eksp. Teor. Fiz. **48**, 864 (1965) [Sov. Phys.-JETP **21**, 576 (1965)].
[6] V. G. Bar'yakhtar, M. A. Savchenko, and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. **49**, 1631 (1965) [Sov. Phys.-JETP **22**, 1115 (1966)].
[7] E. A. Turov, Fizicheskie svoĭstva magnitouporyaochennykh kristallov (Physical Properties of Magnetically Ordered Crystals), Izd. AN SSSR, 1963 (Translation: Academic Press, New York, 1965).

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