

NATURE OF TURBULENT HEATING IN THE TOKAMAK TM-3 DEVICE

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An explanation of the experimental data obtained in ^[11] is proposed on the basis of the theory of anomalous resistance arising as a result of ion-acoustic current instability in a plasma.^[8, 12] In a strong magnetic field, the anomalous resistance may be due to instability of short-wave oscillations for which $k v_{Te}/\omega_{He} > 1$. The laws of turbulent ion and electron heating are explained. It is shown on the basis of the experimental data that the characteristic time for hot ion escape is defined by "classical" diffusion due to scattering by electric-field fluctuations. It is predicted that because of overheating instability the current should flow along thin filaments in which the heat is evolved. In this case the experimental data can be reconciled with the "classical" losses.

1. INTRODUCTION

THE phenomenon of anomalous resistance, which results in turbulent heating in Tokamak devices, was reported in ^[1]. These measurements, made at relatively low intensities of the heating electric field, supplemented with very important data the picture of the phenomenon first observed in experiments on turbulent heating of plasma by magnetosonic waves,^[2] and then investigated in studies of turbulent heating by a current flowing along the magnetic field.^[3, 4] An analysis of these investigations during the period from 1961 through 1967 can be found in the review of Zavoiskii and the author.^[5] During recent years, the number of experimental and theoretical investigations of anomalous resistance and turbulent heating increased still more. Under conditions when the heating is carried out by an electric field of high intensity ($E \approx 0.1-1$ cgs esu), it was possible to measure the level of the ion-acoustic turbulence of the plasma by several methods.^[6, 7] It turned out to be sufficient to explain the value of the anomalous resistance by means of the formulas of the theory of Korablev and the author^[8], based on the ion-acoustic instability of the current.

The results of the first measurements of the anomalous resistance on the TM-3 Tokamak were represented in the form of plots of the degree of anomaly σ_C/σ_{eff} (σ_C —Coulomb conductivity, calculated from the measured temperature of the electrons; σ_{eff} —measured conductivity) as functions of the parameter ω_{He}/ω_{pe} (ratio of the electronic plasma and cyclotron frequencies). The anomaly of the conductivity appeared at $\omega_{He}/\omega_{pe} > 1$ and increased with this parameter.^[1]

Bearing these results in mind, Kadomtsev and Pogutse^[9] analyzed the role of the ion-acoustic instability in the phenomenon of flow of electric current through a plasma along a strong magnetic field ($\omega_{He}/\omega_{pe} \gg 1$) and reached the conclusion that it cannot prevent "run-away" of the electrons and greatly lower the conductivity of the plasma. They therefore proposed a different explanation for this phenomenon, based on the assumption that the current transports a weak beam of high-energy electrons. However, a specially performed experiment did not confirm the existence of such a beam.

In ^[10] they investigated the influence of the magnetic field on the anomalous resistance in experiments on

turbulent heating with a strong electric field. This experiment has shown that the resistance of the plasma is practically independent of the magnetic field in a wide range of values of the parameter ω_{He}/ω_{pe} .

In a paper delivered at the Dubna Conference in the fall of 1969 and in ^[11], Razumova and Bobrovskii reported that the plasma resistance at fixed n and T_e depends little on the magnetic field, in accordance with the result of ^[10]. They also measured the distribution function of the ions.

As will be shown below, most experimental data given in ^[11] concerning the anomalous resistance and turbulent heating of electrons and ions can be explained on the basis of the ion-acoustic theory of this phenomenon, developed in ^[8, 12].

The ideas advanced below were reported by the author in a review paper at the Dubna Conference in September 1969.

2. FUNDAMENTAL EQUATIONS OF TURBULENT HEATING

An ion acoustic instability sets in the plasma when the current velocity $u = j/ne$ exceeds a critical value. In a plasma with developed ion-acoustic turbulence in a weak magnetic field, the fast electrons with $v \gg \omega/k$ are acted upon, owing to the induced Cerenkov effect, by a reaction force that leads to quasielastic scattering of the particles with respect to the angles, with frequency

$$\nu_e(v) = \frac{\pi e^2}{m^2 v^2} \int k^2 |\varphi_k|^2 \delta(\omega - kv) dk \approx \omega_{pe} \frac{W}{nT} \left(\frac{v_{Te}}{v} \right)^3. \quad (1)$$

Here φ_k is the Fourier component of the fluctuations of the potential, W the energy density of the oscillations, and v_{Te} the mean thermal velocity. The estimate of $\nu_e(v)$ is given here for the characteristic $k r_D \sim 1$, where $r_D = v_{Te}/\omega_{pe}$.

The influence of the magnetic field H on the motion of the electrons in the oscillations can be neglected if the oscillation frequency acting on a particle moving along H exceeds the cyclotron frequency. The electrons can interact with oscillations for which $k_z v_z / \omega_{He} \ll 1$ if $\omega = k_z v_z$. In other words, the resonance condition and the instability condition $\partial f / \partial v_z > 0$ can be satisfied only for a fraction of the electrons on the order of u/v_{Te} . The free acceleration of particles

with $v_z > u$ and $v_z < 0$ at $E > E_D$ (E_D is the field at which runaway takes place) cannot be prevented by such oscillations, a conclusion reached by Kadomtsev and Pogutse.^[9]

The interaction of the electrons with the shorter-wave oscillations $k_z v_z / \omega_H \sim 1$ is more effective. In this case, cyclotron resonance $\omega - l\omega_{He} = k_z v_z$ ($l = \pm 1, \pm 2, \dots$), is possible, and leads to scattering with respect to the angle of both the particles with $v_z > 0$ and the particles with $v_z < 0$. Simultaneous allowance for several resonances in the formulas of the quasilinear theory leads to equations describing the quasielastic scattering of the electrons with frequency given by formula (1).

In a strong magnetic field, $\omega_{He}/\omega_{pe} > 1$, the phase velocity of the short-wave oscillations with $k_z v_{Te}/\omega_{He} > 1$ is smaller than the velocity of the ion sound and decreases with increasing k_z . At a fixed ion temperature, such oscillations experience a stronger Landau damping by resonant ions than the long-wave oscillations. However, under conditions when, during the course of time of plasma heating with the electric current, the temperature of the ions and electrons increases by tens and hundreds of times, there may be established a heating regime such that the condition of instability is satisfied at all times for oscillations with $k_z v_{Te}/\omega_{He} > 1$. A detailed theoretical analysis of such a regime can be found in [8, 5, 12]. We present here only the main relations.

In the steady state, the action of the electric field in a turbulent plasma is balanced by the recoil force upon excitation of ion-acoustic waves by the electrons

$$neE - 2 \sum_{\mathbf{k}} \gamma_{\mathbf{k}}^e p_{\mathbf{k}} = 0, \quad (2)$$

$p_{\mathbf{k}} = kW_{\mathbf{k}}/\omega_{\mathbf{k}}$ is the momentum of the wave and $\gamma_{\mathbf{k}}^e$ is the contribution of the electrons to the instability increment. In the estimate of $\gamma_{\mathbf{k}}^e$ it must be borne in mind that the oscillations are generated by the main mass of the electrons (see [8, 12]), and the electric current is transported by the particles with energy higher than the mean-thermal one. The latter is a consequence of the fact that $\nu_{\text{eff}} \sim v^{-3}$. Therefore in the estimate

$$\gamma_{\mathbf{k}}^e \approx \frac{\omega_{pe}}{(kr_D)^2} \frac{u^* - \omega/k}{v_{Te}} \quad (3)$$

the value of u^* is smaller than the current velocity. In the case when the average frequency of the Coulomb collisions ν_C^e is much larger than $\tau_{\text{heat}}^{-1} = eEu/T_e$, we have

$$u/u^* = 2 \quad (4)$$

(in analogy with the known ratio of the Coulomb conductivities perpendicular and parallel to the magnetic field). This ratio can be even larger if $\nu_C^e \leq \tau_{\text{heat}}^{-1}$. The extreme expression of this effect is the runaway phenomenon.

Thus, if we assume that $\omega/k = c_S$, then in the TM-3 Tokamak the current velocity exceeds the characteristic threshold velocity (which, allowing for the statements made above, should be considered to be larger than $2\omega/k$) by less than five times.

If the heating time is much larger than the characteristic time of development of the instability γ_e^{-1} , then

it is necessary to satisfy in the steady state the condition

$$\delta\gamma_{\mathbf{k}} \ll \gamma_{\mathbf{k}}^e, \quad \delta\gamma_{\mathbf{k}} = \gamma_{\mathbf{k}}^e + \gamma_{\mathbf{k}}^i \approx \frac{\omega_{pi}}{(kr_D)^2} \left[\frac{u^* - \omega/k}{v_{Te}} - \frac{\omega}{k} \frac{T_e}{M} f_i \left(\frac{\omega}{k} \right) \right], \quad (5)$$

$\gamma_{\mathbf{k}}^i$ is the decrement of the damping of the oscillations by the ions. Relation (5) means that the ions absorb oscillations produced by electrons. Thus, the ions transfer momentum and energy. It follows from (2) and (5) that the balance of the forces acting on the ions is given by

$$n_e E - \sum_{\mathbf{k}} 2\gamma_{\mathbf{k}}^i W_{\mathbf{k}} / \left(\frac{\omega}{k_i} \right) = 0. \quad (6)$$

In the linear approximation, the oscillations can interact only with resonant ions, i.e., ions for which $v > (\omega/k)_{\text{min}}$. There are few such ions: $n'/n \ll 1$ (the prime will henceforth denote resonant ions), and they cannot take on the momentum of the oscillations. Formally this is expressed in the fact that the decrement $\delta\gamma_{\mathbf{k}} = \gamma_{\mathbf{k}}^i + \gamma_{\mathbf{k}}^e$ is always positive in a narrow cone of angles near the direction of the current-velocity vector.^[8] Therefore the momentum of the oscillations will increase in accordance with the law

$$\frac{d}{dt} \mathbf{p} = -neE,$$

until the weak effects of linear collision damping and nonlinear damping of the oscillations by the main mass of the nonresonant ions stops their growth. Oscillations of high intensity

$$W \sim \frac{neE}{\delta\gamma} \frac{\omega}{k},$$

but concentrated in a small solid angle will not noticeably influence the value of the resistance. In the steady state, the heating of the resonant and nonresonant ions and electrons will be determined by the relations

$$\frac{3}{2} \frac{d}{dt} n'T' \approx \frac{1}{2} neE \frac{\omega}{k} - \frac{n'T'}{\tau'}, \quad (7)$$

$$\frac{3}{2} \frac{d}{dt} nT_i \approx \frac{1}{2} neE \frac{\omega}{k} - \frac{nT_i}{\tau_i}, \quad (8)$$

$$\frac{3}{2} \frac{d}{dt} nT_e = neEu - \frac{nT_e}{\tau_e}. \quad (9)$$

In these equations we have introduced phenomenologically the energy lifetimes. The complete heating equation for the ionic component of the plasma was first obtained by R. Z. Sagdeev, who started from the momentum balance equations (6). The coefficient $1/2$ in Eqs. (7) and (8) reflects only the qualitative aspect of the described phenomenon. Its exact value can be obtained from a complete solution. Such a solution was obtained in the quasilinear approximation by Kovrizhnykh.^[12] It should be borne in mind only that the heating of the main mass of the ions as a result of oscillation damping by the collision viscosity, which was taken into account there, is in practice always smaller than the heating due to nonlinear induced scattering of the oscillations by the ions. This was shown by Kingsep.^[13] It should be noted here that the turbulent-heating regime indicated by Sagdeev and Vekshtein,^[14] which does not affect the bulk of the ions, contradicts the solution of Kovrizhnykh.

3. INTERPRETATION OF THE EXPERIMENTAL DATA OBTAINED WITH THE TM-3 TOKAMAK

We note first that the proposed theory explains the non-Coulomb anomalously strong heating of the electrons and ions in the experiments described in [11]. The ion distribution function obtained from an analysis of the energy spectrum of the charge-exchange neutrals escaping from the plasma has, under certain conditions, a clearly pronounced kink. Such a kink is also predicted by the theory in question.

The Maxwellian form of the distribution function of the nonresonant ions to the left of the kink point can be attributed to the influence of the Coulomb collisions. For the case shown in Fig. 10 of [11] ($n = 4 \times 10^{-12}$) we have $\tau_{\text{C}}^i \approx 10^{-3}$ sec, which is smaller than the heating time. For a Maxwellian distribution, Eq. (5) can be written in the form

$$\frac{u^* - \omega/k}{v_{Te}} \approx \frac{\omega}{kv_{Ti}} \frac{T_e}{T_i} \exp\left(-\frac{\omega^2}{k^2 v_{Ti}^2}\right). \quad (10)$$

If we substitute in this equation the experimental values [11]

$$u^* = 5c, \quad \frac{T_e}{T_i} = 5, \quad \frac{\omega}{kv_{Ti}} \approx 2.2, \quad \exp\left(-\frac{\omega^2}{k^2 v_{Ti}^2}\right) \approx 10^{-2},$$

then this equation is satisfied. From (10) we can determine with logarithmic accuracy the phase velocity of the unstable oscillations

$$\frac{\omega}{k} = v_{Ti} \left(\ln \left[\sqrt{\frac{M}{m}} \frac{\omega}{ku} \left(\frac{T_e}{T_i} \right)^{1/2} \right] \right)^{1/2}. \quad (11)$$

The resonant ions are to the right of the kink (see Fig. 10 of [11]). Their relative concentration is small. However, according to Eqs. (7) and (8) they obtain energy to the same extent as the nonresonant ions. Were it possible to neglect the ion energy loss during the heating time ($\tau_i' > \tau_{\text{heat}}$), and were the lifetimes τ_i and τ_i' determined by some macroscopic instability that leads to approximately the same velocity of outflow of ions of different energy, then the temperature of the resonant ions would have to be high, $T' > T_i n/n'$; this, however, is not the case.

The difference between the temperatures of the resonant and nonresonant ions will be small if one makes the natural assumption that the ions lose energy at a rate determined by the frequency of their effective collisions. This may be the collisional diffusion calculated by Galeev and Sagdeev. [15] Under this assumption, the kinetic equation describing the heating of the resonant ions takes the form

$$\frac{\partial f_i'}{\partial t} = \frac{1}{\omega^2} \frac{\partial}{\partial v} v^2 v_{eff}^i \frac{\omega^2}{k^2} \frac{\partial f_i'}{\partial v} + \frac{1}{r} \frac{\partial}{\partial r} r (\Delta r)^2 \eta v_{eff}^i \frac{\partial f_i'}{\partial r}. \quad (12)$$

The resonant particles fall in the region of "rare" collisions. Therefore $\Delta r \approx r_{H_i} H_z / H_\phi$, $\eta = \sqrt{r/R}$ is the fraction of the captured particles, and α is a numerical coefficient that depends on the form of the noise spectrum. The solution of Eq. (12) takes the form

$$f_i' \sim \exp\left(-\frac{\mathcal{E}}{T_i'}\right), \quad T_i' = \frac{M(\omega/k)^2 r}{2} \frac{H_\phi}{r_{H_i} H_z} \frac{1}{\sqrt{\eta \alpha}}. \quad (13)$$

For the parameters corresponding to Fig. 10, [11] $r_{H_z}/r_{H_i} H_\phi = 2$. This solution explains the Maxwellian

form of the "tail," results in numerical agreement, and gives the dependence of the temperature of the "tail" T_i' on the value of the current I .

The rate of energy loss in the discussed experiments is larger by one order of magnitude than the rate of "classical" losses in toroidal systems owing to effective collisions. Generally speaking, under the conditions of the anomalous resistance caused by the ion-acoustic instability of the current, the macroscopic instabilities inherent in a current-carrying plasma may become appreciably amplified. An example is the current-convective instability investigated theoretically and experimentally in [16], where it increases the electronic thermal conductivity along the magnetic field by one order of magnitude and limits the turbulent heating of the plasma by the current in the probkotron machine.

However, the arguments presented by us in the discussion of the mechanism for heating resonant ions gives grounds for preferring another instability, one not connected with the macroscopic motion of the plasma, namely overheating instability that leads only to a redistribution of the current over the cross section of the plasma column.

Assuming that the electron energy losses are determined by the diffusion of the blocked particles as a result of scattering by the electric fields of the ion-acoustic oscillations, we can represent the energy balance equation in the form

$$\frac{3}{2} \frac{d}{dt} n T_e = e n u E + \frac{1}{r} \frac{\partial}{\partial r} r D_e \frac{\partial}{\partial r} n T_e. \quad (14)$$

In practice, the value of ν_e corresponds to "rare" collisions, and therefore

$$D_e \approx \nu_e \eta r_{He}^2 H_z^2 / H_\phi^2.$$

In the case of a smooth distribution of the temperature and current density over the cross section, the energy release, as already noted, is larger by one order of magnitude than the losses. Under these conditions, and when $\partial u / \partial T > 0$, the heat will be released predominantly in the current tube in which the temperature is higher, i.e., the temperature difference will increase. The current density will then likewise become redistributed. It will be larger in the region with the higher temperature. The redistribution of the current and the overheating in individual current tubes may occur within a time that is longer than the skin time for the given transverse dimension of the tube. The minimum tube dimension r_0 is determined from the condition that the heating and heat loss be equal:

$$r_0 / r \approx \beta_i \eta^{1/2}, \quad \beta_i = 8\pi n T_e / H_\phi^2. \quad (15)$$

The larger thermal conductivity along the magnetic field also hinders this process. Therefore the vicinity of the magnetic surfaces, where the force lines are closed, is most favorable for the increased heating.

The phenomenon of additional local heating and redistribution of the current can occur both in the ordinary Coulomb heating regime and in the turbulent regime. In the latter case it should be more clearly pronounced. Indeed, since the diffusion coefficient assumed by us in (12) for the resonant ions is larger than D_e by a factor $(MT_e/mT_i)^{1/2}$, the distribution function

of the resonant ions should be smooth. We then get from (5) Ohm's law for a thin current filament

$$j \approx env_{Te} \frac{\omega T_e}{k M} f' \sim T_e^2. \quad (16)$$

It should be remembered, however, that j can vary only in the range

$$v_{Te} > j/en > c_s.$$

Thus, there are grounds for expecting the greater part of the current to flow in the experiments in question along thin tubes located predominantly in the vicinity of the degenerate magnetic surfaces. It is in these tubes that the heat is released, and from them that it is rapidly carried away by collision diffusion into the relatively cold surrounding plasma, where the resistances and the thermal conductivity are high. Nonetheless, this region determines the average temperature of the plasma column, since the summary cross section of the tubes is small. Compiling the energy balance on the basis of these considerations, we obtain for β_I the estimate

$$\beta_I \approx \eta^{-1/2} (u/\bar{u})^{1/2}, \quad (17)$$

where \bar{u} is the current velocity averaged over the column cross section, and u is the current velocity in the "cold" part of the plasma. From the condition for the existence of ion-acoustic instability we have $u \geq \omega/k$ and $\omega/k \approx c_s$. In the discussed experiments, $\bar{u}/c_s \approx 10$ and $\eta^{1/2} \approx 2/3$, so that $\beta_I \approx 0.5$.

Equations (9) and (14), like Eqs. (8) and (12) for resonant ions, do not give us any information on the effective collision frequencies. One can attempt to extract an estimate for the conductivity from Eq. (8) for the heating of the bulk of the ions. At a given current, the electric field should be such that the ion temperature satisfies condition (10) during the process of turbulent heating. From Eq. (8) there follows for ν_{eff} the estimate

$$\nu_{\text{eff}} \approx 3T_e/m \frac{\omega}{k} \bar{u} \tau_i, \quad (18)$$

which contains the quantity τ_i , which is not determined experimentally. In light of the remark made, we can substitute the "classical" lifetime of the ion in a toroidal system^[15] for τ_i in the discussion of heating of resonant ions. Under the conditions corresponding to Fig. 10 of [11], it falls on the "plateau" of the $\tau_i(\nu_i)$ dependence and its order of magnitude is

$$\tau_i = \frac{R^2 r}{r_{ii}^2} \frac{H_0}{H_e \nu_{Te}}.$$

The estimate of ν_{eff} obtained with this value of τ_i is smaller than the experimental value, but it must be borne in mind that T_e was determined experimentally with large inaccuracy, and $\nu_{\text{eff}} \sim T_e^{5/2}$.

In the foregoing comparison of the proposed theory with the experimental data we used an experimental value of the phase velocity, determined from the position of the kink on the function $f'_i(v)$. If all the conclusions concerning turbulent ion heating are valid for any value of ω/k , then we can explain the nature of the anomalous resistance in a strong magnetic field, $\omega_{pe} < \omega_{He}$, only by assuming that $\omega/k < c_s \omega_{pe}/\omega_{He}$. This means that on Fig. 10 of [11] the limiting permissible value of the energy at the kink should decrease by

a factor of 4–6 when the magnetic field is changed from 10 to 26 kOe, since nT_e is practically independent of H_z in the experiments under discussion. Although the accuracy with which the kink position is determined is low, it is nevertheless difficult to reconcile this requirement with the experimental data. This does not mean that we should forgo the ion-acoustic theory of anomalous resistance. It is possible that an increase in the accuracy of the experimental data will eliminate this partial contradiction. There also remains the possibility of improving the theory and of introducing into it a collective dynamic friction that prevents the runaway of the leading front of the electron distribution function. We shall then be able to forgo the "non-magnetization" condition.

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