NONLINEAR PROPAGATION OF HIGH-FREQUENCY MONOCHROMATIC ULTRASOUND IN SOLIDS WITH HIGH MOBILITY CURRENT CARRIERS

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Nonlinear ultrasonic propagation is investigated in solids under the condition $kl \gg 1$, where k is the wave number of the sound, and l the free path length of the current carriers (electrons) As has been shown earlier,[1,2] nonlinear amplification restriction begins at comparatively low sound intensities, which are lower for narrow sound spectral lines. Monochromatic sound of sufficiently high frequency is studied in detail in the present paper. A modified kinetic equation is derived which can describe the interaction between the electrons and such sound. The distribution function of the electrons, the current, and the electronic absorption coefficient are determined from this equation. The absorption decreases with increase in the sound intensity in proportion to the inverse of the square root of the intensity. In contrast, the acousto-electric current increases proportionally to the square root of the intensity, but is small in comparison with the total current. Electron heating by monochromatic sound is extremely small. For n-InSb at 77°K, the sound intensity required for nonlinearity is found to be 3×10^{-2} W/cm²; for Bi at 4.2°K, the respective quantity is 5×10^{-4} W/cm².

THE propagation in solids of ultrasound of sufficiently high frequency, for which

$$kl \gg 1, \quad (1)$$

is accompanied by specific nonlinear effects. These effects are connected with the violent disruption, under the action of the sound, of the equilibrium of the electrons with respect to momentum, and the subsequent effect of the nonequilibrium electrons on the conditions of propagation of the sound itself. Such a nonlinearity has been studied in [1,2] for sound with rather broad spectral lines, Δk. As was shown, the nonlinear effects begin at lower sound intensities the smaller the value of Δk. Therefore, they should be very significant for monochromatic sound. The theoretical description of such sound, however, encounters certain difficulties. It is known that resonance energy denominators appear in the terms of interaction with the scatterer in the derivation of the kinetic equation for the momentum distribution of the electrons $f(\mathbf{p})$. Usually, the detailed behavior of these denominators at the point of resonance is not important, inasmuch as they are integrated and it suffices to know only the rule for bypassing the singularities corresponding to resonance. For monochromatic sound, however, the integral over its wave vector is removed and the denominators are not integrated. Then the vicinity of the resonance can be important. Such is also the case in nonlinear theory, in which it is necessary to take into account the perturbation of the distribution $f(\mathbf{p})$ by the sound. It turns out that $f(\mathbf{p})$ depends very strongly on $\mathbf{p}$ and varies over intervals $|\Delta\mathbf{p}|$ that are comparable with "smearing out" of the resonance. The necessity therefore arises for describing this smearing out in detail, which is also done in the present paper. The condition of nonlinearity is obtained here under the additional assumption $k \gg m/k\tau$ (m is the effective mass of the electrons, τ the relaxation time of the momentum, $\hbar = 1$) and has the form

$$\lambda^2(\bar{\varepsilon}\tau)^2 \gg 1, \quad (2)$$

where the Born parameter $\lambda = \delta\epsilon/\bar{\epsilon}$, $\delta\epsilon$ is the energy of interaction of the electron with the sound, and $\bar{\epsilon} = \bar{p}^2/2m$ the mean energy of the electron. Inasmuch as $\bar{\epsilon}\tau \gg 1$ frequently, (2) can be satisfied in the Born limit $\lambda \ll 1$. In this limit, the sound excites only small oscillations of the electron plasma. Therefore, the higher harmonics generated by the nonlinearity turn out to be much less than the fundamental.

1. KINETIC EQUATION FOR ELECTRONS AND ULTRASOUND

The Hamiltonisn of the problem under study can be written down in the following general form:

$$\hat{H} = \hat{H}_e + \hat{H}_{eU} + \hat{H}_{e\varphi} + \hat{H}_{eE} + \hat{H}_{eT}, \quad (3)$$

where $\hat{H}_e$ is the Hamiltonian of the free electrons in the effective-mass approximation, H_{eU} is the Hamiltonian of interaction of the electrons with the ultrasound which is propagating along the x axis, $\hat{H}_{e\varphi}$ is the Hamiltonian of interaction with the screening electric field (the potential of which is $\varphi(x, t)$), and has the spatial and temporal periodicity of the sound wave, $\hat{H}_{eE}$ is the Hamiltonian of interaction with a smoothly changing electric field of intensity $E(x)$,[1)] and $\hat{H}_{eT}$ is the Hamiltonian of interaction with the thermostat. We shall describe the ultrasound as a classical field of elastic displacements $U(x, t)$ (t is the time), which satisfies the equation of elasticity theory with account of the deformation interaction with electrons and of lattice absorption. It will be shown at the end of the paper just how one should make the substitutions in the final formulas in order also to describe the piezoelectric interaction of the ultrasound with the electrons. For simplicity of the cal-

[1)]This can be both an external field and the field appearing in the system itself because of its inhomogeneity.

culations, we shall assume the medium to be isotropic and the electron energy spectrum to be parabolic and single-valleyed. The corresponding generalizations do not contain any matters of principle and are sufficiently clear. In estimates of such materials as n-PbTe and Bi, the multi-valleyed character of the spectrum will be taken into account.

We shall be dealing with the quantum equation of motion for the Wigner density

$$F(\mathbf{x}, \mathbf{p}, t) = \sum_{\varkappa} e^{i\varkappa\mathbf{x}} \langle a^{+}_{\mathbf{p}-\varkappa/2} a_{\mathbf{p}+\varkappa/2} \rangle, \qquad (4)$$

where summation is carried out over all electron momenta $\varkappa$ ($\mathbf{x}$ is the radius vector) and $\langle \ldots \rangle = \mathrm{Sp}\, \hat{S}(t) \times (\ldots)$, $\hat{S}(t)$ is the statistical operator of the entire system, and a^{+}_{p} and a_p are the electron operators of second quantization. The quantum equations are necessary not only to cover the case $k \sim \bar{p}$. Even if $k \ll \bar{p}$, i.e., the "classical" situation, the necessity arises for taking into consideration the finiteness of the transferred momentum (k) in the scattering of the electrons by the ultrasonic phonons. The fact is that the function f(p) calculated here is always a strong function of p_x, changing materially when p_x obtains an increment $\sim m/k\tau \ll \bar{p}$ ($kl \sim k\bar{p}\tau/m \gg 1$). Therefore, the finiteness of k is important for $k \gtrsim m/k\tau$, even if $k \ll \bar{p}$. Differentiating (4) with respect to time, we get in the usual way

$$\frac{\partial F(\mathbf{x},\mathbf{p})}{\partial t} + \frac{\partial \varepsilon(\mathbf{p})}{\partial \mathbf{p}} \frac{\partial F(\mathbf{x},\mathbf{p})}{\partial \mathbf{x}} + e\mathbf{E}\frac{\partial F(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}} = -i\sum_{\mathbf{k}} B(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}} \times \left[F\left(\mathbf{x},\mathbf{p}-\frac{\mathbf{k}}{2}\right) - F\left(\mathbf{x},\mathbf{p}+\frac{\mathbf{k}}{2}\right)\right] + 2\sum_{\mathbf{q}} |C_{\mathbf{q}}| \,\mathrm{Re}[V_{\mathbf{q}}(\mathbf{x},\mathbf{p},\mathbf{p}-\mathbf{q}) - V_{\mathbf{q}}(\mathbf{x},\mathbf{p}+\mathbf{q},\mathbf{p})], \qquad (5)$$

where $\epsilon(\mathbf{p}) = \mathbf{p}^2/2m$, e is the charge on the electron, $\mathbf{k}$ and $\mathbf{q}$ wave vectors,

$$B(\mathbf{k}) = \frac{1}{V}\int d^3x e^{-i\mathbf{k}\mathbf{x}} \left[\Lambda \frac{\partial U(x,t)}{\partial x} + e\varphi(x,t)\right] \qquad (6)$$

is the Fourier transform of the energy of interaction with the sound and the screening field φ, Λ the constant of the deformation potential, and V the volume of the medium.

It is assumed that if the electric field $\mathbf{E}$ changes over the distances $\sim L$, then $Lm/k\tau \gg 1$. This allows us to write down the field component on the left hand side of (5) in classical form. In the problem on the acousto-electric effects, one usually has $L \sim \alpha^{-1}$, where α is the sound absorption coefficient. Then the previous condition takes the form

$$\alpha \ll m/k\tau. \qquad (7)$$

The second sum on the right hand side of (5) describes the interaction of the electrons with the thermostat. If the thermostat is a system of equilibrium acoustic phonons, then

$$V_{\mathbf{q}}(\mathbf{x}, \mathbf{p}, \mathbf{p}-\mathbf{q}) = \sum_{\varkappa} e^{i\varkappa\mathbf{x}} \langle b_{\mathbf{q}} a^{+}_{\mathbf{p}-\varkappa/2} a_{\mathbf{p}-\mathbf{q}+\varkappa/2} \rangle, \qquad (8)$$

where $b_{\mathbf{q}}$ is the operator of second quantization of the phonons of the thermostat. As will be seen from the following, the chief results of the research do not depend on this specialization of the thermostat.

The equation of motion for $V_{\mathbf{q}}(\mathbf{x}, \mathbf{p}, \mathbf{p}-\mathbf{q})$ is put together in similar fashion. In this equation, we carry out a decoupling of the higher distribution functions and limit ourselves to the principal order in the coupling constant with the thermostat, $C_{\mathbf{q}}$. We then obtain

$$\frac{\partial V_{\mathbf{q}}(\mathbf{x},\mathbf{p},\mathbf{p}-\mathbf{q})}{\partial t} + \frac{\partial \varepsilon(\mathbf{p}-\mathbf{q}/2)}{\partial \mathbf{p}} \frac{\partial V_{\mathbf{q}}(\mathbf{x},\mathbf{p},\mathbf{p}-\mathbf{q})}{\partial \mathbf{x}} + e\mathbf{E}\frac{\partial V_{\mathbf{q}}(\mathbf{x},\mathbf{p},\mathbf{p}-\mathbf{q})}{\partial \mathbf{p}} + i[\varepsilon(\mathbf{p}-\mathbf{q}) - \varepsilon(\mathbf{p}) + \omega_{\mathbf{q}}] V_{\mathbf{q}}(\mathbf{x},\mathbf{p},\mathbf{p}-\mathbf{q})$$
$$+ i\sum_{\mathbf{k}} B(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}} \left[V_{\mathbf{q}}\left(\mathbf{x},\mathbf{p}-\frac{\mathbf{k}}{2},\mathbf{p}-\mathbf{q}-\frac{\mathbf{k}}{2}\right) - V_{\mathbf{q}}\left(\mathbf{x},\mathbf{p}+\frac{\mathbf{k}}{2},\mathbf{p}-\mathbf{q}+\frac{\mathbf{k}}{2}\right)\right]$$
$$= |C_{\mathbf{q}}|\left[N_{\mathbf{q}} F(\mathbf{x},\mathbf{p}-\mathbf{q}) - (N_{\mathbf{q}}+1) F(\mathbf{x},\mathbf{p}) - e^{i\mathbf{q}\mathbf{x}} F\left(\mathbf{x},\mathbf{p}-\frac{\mathbf{q}}{2}\right)\int \frac{d^3y}{V} e^{-i\mathbf{q}\mathbf{y}} \times \sum_{\varkappa} F(\mathbf{y},\varkappa) + \int \frac{d^3y d^3z}{V^2} \sum_{\varkappa,\varkappa'} e^{-i\varkappa(\mathbf{y}-\mathbf{x})} e^{-i\varkappa'(\mathbf{z}-\mathbf{x})} F\left(\mathbf{y},\mathbf{p}-\frac{\varkappa'}{2}\right) F\left(\mathbf{z},\mathbf{p}-\mathbf{q}+\frac{\varkappa}{2}\right)\right], \qquad (9)$$

where $N_{\mathbf{q}} = \langle b_{\mathbf{q}} + b_{\mathbf{q}} \rangle$ is the distribution function of the phonons of the thermostat, $\omega_{\mathbf{q}} = sq$ the sound frequency, s the sound velocity.

Let a sound beam of strictly determined frequency $\omega_{\mathbf{k}} = sk$ be introduced into the crystal from outside. In propagation in the nonlinear medium, harmonics can be generated. Therefore, in the medium,

$$U(x,t) = \sum_{k'>0} [U_{k'}(x,t)\exp\{i(k'x - \omega_{k'}t)\} + U_{k'}^{*}(x,t)\exp\{-i(k'x - \omega_{k'}t)\}], \qquad (10)$$

where summation is carried out over the harmonics $k' = k, 2k, 3k, \ldots$. The complex amplitudes of the harmonics $U_{k'}(x, t)$ change little over a wavelength in space and a period of oscillation in time if

$$\alpha \ll k, \qquad (11)$$

which is well satisfied in practice. Amplitudes of the potential of the screening field $\varphi_{k'}(x, t)$ are also introduced, similar to (10). It is clear from the structure of the equations (5) and (9) that $F(\mathbf{x}, \mathbf{p}, t)$ and $V_{\mathbf{q}}(\mathbf{x}, \mathbf{p}, \mathbf{p}-\mathbf{q}, t)$ change in space and time only in connection with the presence of a coherent ultrasonic field. Inasmuch as there are no reflected waves in this field ($k' > 0$), then, in accord with [1], the functions F and $V_{\mathbf{q}}$ can be sought in the form

$$F(x,\mathbf{p},t) = f(x,\mathbf{p},t) + \sum_{k'>0} [F_{k'}(x,\mathbf{p},t)\exp\{i(k'x - \omega_{k'}t)\} + F_{k'}^{*}(x,\mathbf{p},t)\exp\{-i(k'x - \omega_{k'}t)\}], \qquad (12)$$

$$V_{\mathbf{q}}(x,\mathbf{p},\mathbf{p}-\mathbf{q},t) = v_{\mathbf{q}}(x,\mathbf{p},\mathbf{p}-\mathbf{q},t) + \sum_{k'>0} [V^{(+)}_{\mathbf{q}k'}(x,\mathbf{p},\mathbf{p}-\mathbf{q},t) \times \exp\{i(k'x - \omega_{k'}t)\} + V_{\mathbf{q}k'}(x,\mathbf{p},\mathbf{p}-q,t)\exp\{-i(k'x - \omega_{k'}t)\}] \qquad (13)$$

where f, $v_{\mathbf{q}}$, $F_{k'}$, $V^{(+)}_{\mathbf{q}k'}$ are smoothly changing functions of x and t. We note that the function $f(x, \mathbf{p}, t)$ arising in (12) is also the desired distribution function of the electrons with respect to the momentum.

If the conditions (7) and (11) are satisfied, then we can introduce the smooth function of x

$$B(\mathbf{k}, x) = \sum_{|\varkappa| \leqslant \alpha} B(\mathbf{k} + \varkappa) e^{i\varkappa x} \qquad (14)$$

in (5) and (9), in place of $B(\mathbf{k})$, and express it in terms of the amplitudes $U_{k'}(x, t)$ and $\varphi_{k'}(x, t)$. Then, substituting (12) and (13), in (5) and (9), the equation of elasticity theory and the Poisson equation for $\varphi(x, t)$, we put together an equation for the smoothly changing functions, similar to what was done, for example, in plasma theory.[3] The equations thus obtained turn out to be extraordinarily cumbersome, which is not surprising—they describe the propagation of sound of arbitrary frequency and intensity in a nonlinear medium. We limit ourselves to the consideration only of rather high frequencies, when[2)]

$$k^2\tau / m \gg 1. \qquad (15)$$

The requirement (15) means that the momentum of the ultrasonic phonon appreciably exceeds the indeterminacy of p_x because of collisions ($\sim m/k\tau$). This requirement assumes the satisfaction of (1). The intensity of the first harmonic will be assumed to be so small that the Born parameter

$$\lambda \sim k\Lambda |U_k| / \bar{\varepsilon} \ll 1. \qquad (16)$$

We then obtain the following from the equations for the higher harmonics:

$$|U_{2k} / U_k| \sim \lambda \ll 1, \quad |U_{k'}| \lesssim |U_{2k}|, \quad k' = 3k, 4k, \ldots, \qquad (17)$$

i.e., the higher harmonics are relatively small. We also require that

$$k\Lambda |U_{2k}|\tau \ll 1. \qquad (18)$$

The sense of the requirement (18) is that the second harmonic (and higher ones) not only should be relatively small, but should still weakly perturb the equilibrium of the electrons in their momentum. However, for this case, the first harmonic can perturb this equilibrium very strongly, because the requirements (17) and (18) can be compatible with the condition $k\Lambda |U_k| \tau \gtrsim 1$, which is identical with (2). Then the equations for the smooth functions are simplified, and after elimination of v_q and $V^{\pm}_{qk}$, we obtain

$$\frac{\partial f(\mathbf{p})}{\partial t} + \frac{\partial \varepsilon(\mathbf{p})}{\partial p_x}\frac{\partial f(\mathbf{p})}{\partial x} + eE(x)\frac{\partial f(\mathbf{p})}{\partial p_x} = 2\operatorname{Re}\left\{ [\Lambda k U_k - ie\varphi_k] \cdot \right. \qquad (19)$$

$$\left. \times \left[F_k^*\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - F_k^*\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right]\right\} + I[f_0(\varepsilon_p)] - \frac{\Phi(\mathbf{p})}{\tau(\varepsilon_p)},$$

$$eE(x)\frac{\partial F_k(\mathbf{p})}{\partial p_x} + \left\{ - i\left[\omega_k + \Omega(\mathbf{p}, k) + \varepsilon\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - \varepsilon\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right]\right.$$

$$\left. + \tau^{-1}(\mathbf{p}, k)\right\} F_k(\mathbf{p}) = [\Lambda k U_k - ie\varphi_k]\left[f\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - f\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right.$$

$$\left. - R(\mathbf{p}, k) - iQ(\mathbf{p}, k)\right], \qquad (20)$$

$$\frac{\partial |U_k|^2}{\partial t} + s\frac{\partial |U_k|^2}{\partial x} = -\frac{2\Lambda}{\rho s}\frac{1}{V}\sum_{\mathbf{p}} \operatorname{Re}[U_k^* F_k(\mathbf{p})] - \frac{\eta k^2}{\rho}|U_k|^2, \qquad (21)$$

where

$$\varphi_k = \frac{4\pi e}{\epsilon k^2}\frac{2}{(2\pi)^3}\int d^3p F_k(\mathbf{p}), \qquad (22)$$

$$\Omega(\mathbf{p}, \mathbf{k}) = -\frac{1}{\pi}\int d^3q v(|\mathbf{q}|) \frac{m^{-1}\mathbf{k}(\mathbf{p} - \mathbf{q}/2)}{[\varepsilon(\mathbf{p} - \mathbf{q}) - \varepsilon(\mathbf{p})]^2 - m^{-2}[\mathbf{k}(\mathbf{p} - \mathbf{q}/2)]^2}, \qquad (23)$$

$$\tau^{-1}(\mathbf{p}, k) = \frac{1}{2}\int d^3q v(|\mathbf{q}|)\left[\delta\left[\varepsilon_{\mathbf{p}-\mathbf{q}} - \varepsilon_{\mathbf{p}} + m^{-1}\mathbf{k}\left(\mathbf{p} - \frac{\mathbf{q}}{2}\right)\right]\right.$$

$$\left. + \delta\left[\varepsilon_{\mathbf{p}-\mathbf{q}} - \varepsilon_{\mathbf{p}} - m^{-1}\mathbf{k}\left(\mathbf{p} - \frac{\mathbf{q}}{2}\right)\right]\right], \qquad (24)$$

$$R(\mathbf{p}, k) = -\frac{i}{\pi}\int d^3q v(|\mathbf{q}|)\left[\frac{F_k(\mathbf{p} - \mathbf{q})}{\Lambda k U_k - ie\varphi_k}\right]$$

$$\times \frac{m^{-1}\mathbf{k}(\mathbf{p} - \mathbf{q}/2)}{[\varepsilon(\mathbf{p} - \mathbf{q}) - \varepsilon(\mathbf{p})]^2 - m^{-2}[\mathbf{k}(\mathbf{p} - \mathbf{q}/2)]^2} \qquad (25)$$

$$Q(\mathbf{p}, k) = \frac{i}{2}\int d^3q v(|\mathbf{q}|)\left[\frac{F_k(\mathbf{p} - \mathbf{q})}{\Lambda k U_k - ie\varphi_k}\right]\left[\delta(\varepsilon_{\mathbf{p}-\mathbf{q}} - \varepsilon_{\mathbf{p}}\right.$$

$$\left. + m^{-1}\mathbf{k}\left(\mathbf{p} - \frac{\mathbf{q}}{2}\right)\right) + \delta(\varepsilon_{\mathbf{p}-\mathbf{q}} - \varepsilon_{\mathbf{p}} - m^{-1}\mathbf{k}(\mathbf{p} - \mathbf{q}/2))\Big]$$

$$v(|\mathbf{q}|) = \frac{V}{2\pi^2}|C_{\mathbf{q}}|^2 N_{\mathbf{q}}, \quad N_{\mathbf{q}} \gg 1. \qquad (26)$$

Here η is the coefficient of lattice viscosity, ρ and ϵ are the density and the dielectric permittivity of the medium, $I[f_0(\epsilon_p)]$ is the well-known form for the isotropic part of the collision integral,[3)] $\tau(\epsilon_p)$ is the "drift" time of relaxation of the momentum, $f_0(\epsilon_p)$ is the isotropic, and $\Phi(\mathbf{p}) \equiv [f(\mathbf{p}) - f_0(\epsilon_p)]$ the anisotropic part, respectively, of the distribution function $f(\mathbf{p})$.

In the derivation of (19)–(22), it was assumed that the drift velocity $v_{dr} \sim eE\tau_{\bar{\varepsilon}} m \lesssim s \ll \bar{p}/m$, $\omega_q \ll \bar{\epsilon}$ and $\bar{\epsilon}\tau \gg 1$. The degree of degeneracy of the electron gas can be arbitrary. Equations (19)–(22) actually do not depend on the specific character of the thermostat and are suitable for any quasi-elastic scattering mechanism, the probability of which $v(|\mathbf{q}|)$ is a function only of the modulus of the transferred momentum. For certain specific mechanisms, $v(|\mathbf{q}|)$ is given, for example, in [5]. Neglect of the "arrival" in the anisotropic part of the collision integral is justified by considerations similar to those given in [6]. In the given case, these considerations are applicable, since the $\Phi(\mathbf{p})$ computed in Sec. 2 depends sufficiently strongly on p_x and differs from zero only near $p_x = ms \pm k/2$.

In Eq. (20), the quantities Ω, τ^{-1}, R, and Q, which are proportional to $v(|\mathbf{q}|) \sim |C_{\mathbf{q}}|^2$, are kept along with quantities of zero order in $|C_{\mathbf{q}}|$. The fact is that the quantities $|C_{\mathbf{q}}|^2$ and the field component ($\sim$E) remain alone on the left side of (20) at the resonance point $p_x k/m = \omega_k$, and therefore determine the detailed path of $F_k(\mathbf{p})$ in the neighborhood of this point. The behavior of $F_k(\mathbf{p})$ at resonance is very important for the shape of Eq. (19), since summation over k is lacking in the latter (monochromatic sound). The neighborhood of resonance Δp_x, in which the quantities $\sim |C_{\mathbf{q}}|^2$ are important, can be estimated from the condition $\Delta p_x k/m \sim \tau^{-1}$ (it follows from (23) that $\Omega \lesssim \tau^{-1}$). Inasmuch as (1) is satisfied, we then obtain $\Delta p_x \ll \bar{p}$. In this connection, the principal contribution to the integrals (25) and (26) is made by the region of p_x that is far from resonance. Therefore, instead of the exact $F_k(\mathbf{p})$, we have substituted in these equations their values far from resonance $F_k^{(0)}(\mathbf{p})$, which are determined from

$$i\left[\frac{p_x k}{m} - \omega_k\right] F_k^{(0)}(\mathbf{p}) = [\Lambda k U_k - ie\varphi_k][f_0(\varepsilon_{\mathbf{p}-\mathbf{k}/2}) - f_0(\varepsilon_{\mathbf{p}+\mathbf{k}/2})] \qquad (27)$$

We note that the field component in Eq. (20), far from resonance, can be important, even in $v_{dr} \lesssim s$. Actually, for $p_x k/m \sim \tau^{-1}$, we have $eE\partial F_k(\mathbf{p})/\partial p_x \sim eEk\tau F_k(\mathbf{p})/m$, and we can discard this quantity in (20) only if

$$v_{dr} k\tau \ll 1. \qquad (28)$$

If (28) is not satisfied, then it is necessary to solve the

[2)]The opposite limiting case $k^2\tau/m \ll 1$ was considered in [4].

[3)]Besides collisions with the thermostat, interelectronic collisions (frequency ν_{ee}) are also included in $I[f_0(\epsilon_p)]$. These latter collisions are important if $\nu_{ee} \gtrsim \tau^{-1}(\epsilon/\omega_q)^2 \equiv \tau_{en}$ (τ_{em} is the relaxation time of the energy). It is assumed that $\nu_{ee} \ll \tau^{-1}$ and that one can therefore neglect the effect of the interelectronic collisions on the anisotropic part of the collision integral.

differential equation of first order (20), which is entirely possible. In this way, we can take into account the effect of the field on the collisions with the monochromatic phonon flux. Here it is clear that for $v_{dr}k\tau \gg 1$, the "smearing out" of the resonance is principally determined not by the collisions, but by the field $E(x)$. In the given research, however, we limit ourselves only to the case when (28) is satisfied and we can discard the field contribution to (20).

We substitute $F_k(p)$ from (20) in (22) and express φ_k in terms of U_k. Then, substituting $F_k(p)$ and φ_k in (19), we finally obtain

$$2\,\mathrm{Re}\left\{[\Lambda kU_k - ie\varphi_k]\left[F_k^*\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) - F_k^*\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right)\right]\right\} = 2\pi|\tilde{\Lambda}|^2k^2|U_k|^2 \times \{\Delta(\mathbf{p},k)[A(\mathbf{p},k)+\Phi(\mathbf{p}-\mathbf{k})-\Phi(\mathbf{p})] - \Delta(\mathbf{p}+\mathbf{k},k)[A(\mathbf{p}+\mathbf{k},k)+\Phi(\mathbf{p})-\Phi(\mathbf{p}+\mathbf{k})]\}, \tag{29}$$

where

$$A(\mathbf{p},k) = f_0(\varepsilon_{\mathbf{p}-\mathbf{k}}) - f_0(\varepsilon_\mathbf{p}) - R\left(\mathbf{p}-\frac{\mathbf{k}}{2},k\right) + Q\left(\mathbf{p}-\frac{\mathbf{k}}{2},k\right)\tau\left(\mathbf{p}-\frac{\mathbf{k}}{2},k\right) \times [\omega_k + \Omega(\mathbf{p}-\mathbf{k}/2,k) + \varepsilon_{\mathbf{p}-\mathbf{k}} - \varepsilon_\mathbf{p}], \tag{30}$$

$$\Delta(\mathbf{p},k) = \frac{1}{\pi}\,\frac{\tau^{-1}(\mathbf{p}-\mathbf{k}/2,k)}{\{(\omega_k+\Omega(\mathbf{p}-\mathbf{k}/2,k)+\varepsilon_{\mathbf{p}-\mathbf{k}}-\varepsilon_\mathbf{p})^2+\tau^{-2}(\mathbf{p}-\mathbf{k}/2,k)\}}$$

$$\tilde{\Lambda} = \Lambda\left[1+\frac{4\pi e^2}{\varepsilon k^2}\tilde{K}_k^{(0)}(\omega_k)\right]^{-1}, \tag{31}$$

$$\tilde{K}_k^{(0)}(\omega_k) = \frac{2i}{V}\sum_\mathbf{p}\left[\frac{F_k(\mathbf{p})}{\tilde{\Lambda}kU_k}\right]. \tag{32}$$

Substituting $F_k(p)$ in (21) and taking it into account that by definition the first component on the right side of (21) is $s\alpha_e|U_k|^2$, where α_e is the electronic sound absorption coefficient, we get

$$\alpha_e = \frac{2\pi|\tilde{\Lambda}|^2k}{\rho s^2}\,\frac{1}{V}\sum_\mathbf{p}\Delta(\mathbf{p},k)[A(\mathbf{p},k)+\Phi(\mathbf{p}-\mathbf{k})-\Phi(\mathbf{p})]. \tag{33}$$

The function $\tilde{K}_\mathbf{k}^{(0)}(\omega_k)$ can be computed in Eqs. (29) and and (33) by replacing $F_k(p)$ by $F_k^{(0)}(p)$ from (27) in the expressions for it, on the same basis as in the integrals (25) and (26). Then $\tilde{K}_\mathbf{k}^{(0)}(\omega_k)$ transforms into the function $K_\mathbf{k}^{(0)}(\omega_k)$, which was investigated in detail in [7].

Equation (29) differs from the standard collision integral in two respects. First, instead of δ functions in the mathematical sense, there are Δ functions with a finite spreading out in them. Second, the structure of the "statistical factor" is changed—the functions $R(\mathbf{p}, k)$ and $Q(\mathbf{p}, k)$ appear in $A(\mathbf{p}, k)$. These functions are small and can be discarded in $A(\mathbf{p}, k)$ only for $\omega_k\tau \gg 1$. For $\omega_k\tau \lesssim 1$, in the classical case ($k \ll \bar{p}$), we get from the definitions (23)–(26)

$$\Omega \lesssim \left(\frac{k}{\bar{p}}\right)^3\tau^{-1}, \quad R \lesssim \left(\frac{k}{\bar{p}}\right)^3\tau^{-1}f_0'(\varepsilon_p),$$
$$\tau(\mathbf{p},k)\approx\tau(\varepsilon_p), \quad Q(\mathbf{p},k)\approx-\frac{f_0'(\varepsilon_p)}{\tau(\varepsilon_p)}. \tag{34}$$

Substituting (34) in (30) and neglecting quantities $\sim k/\bar{p}$, we obtain

$$A(\mathbf{p},k) \approx -\omega_k f_0'(\varepsilon_p). \tag{35}$$

2. SOLUTION OF THE KINETIC EQUATION FOR ELECTRONS

We shall seek this solution for only the "local" case, when one can neglect the derivatives with respect to x and t in Eqs. (19) in the calculation of both the isotropic $f_0(\epsilon_\mathbf{p})$ and anisotropic $\Phi(\mathbf{p})$ distribution functions. The first requirement is seen to be the more rigorous.[8] It can be written in the form

$$al(\bar{v}/s) \ll 1. \tag{36}$$

Furthermore, we shall neglect the derivative $\partial\Phi/\partial p_x$ in the field component in (19), in spite of the strong dependence of $\Phi(\mathbf{p})$ on the momentum p_x. It follows from the solution obtained later that this is valid upon satisfaction of (28). We average Eq. (19) over the angles and obtain

$$I[f_0(\varepsilon_p)] = \overline{-2\,\mathrm{Re}\left\{\tilde{\Lambda}kU_k\left[F_k^*\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right)-F_k^*\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right)\right]\right\}}^{\theta} + \overline{eE\frac{\partial\Phi(\mathbf{p})}{\partial p_x}}^{\theta} \equiv D(\varepsilon_p), \tag{37}$$

where

$$\overline{(\ldots)}^{\theta} \equiv \frac{1}{2}\int_{-1}^{1} d\cos\theta(\ldots). \tag{38}$$

Equation (37) serves as the definition of the isotropic function $f_0(\epsilon_\mathbf{p})$. We obtain the equation for the anisotropic function $\Phi(\mathbf{p})$ if we subtract (37) from (19). Then

$$\Phi(\mathbf{p}) = D(\varepsilon_p)\tau(\varepsilon_p) - eE\tau(\varepsilon_p)\frac{\partial f_0(\varepsilon_p)}{\partial p_x} + \frac{\pi w(\varepsilon_p)}{2\tau(\varepsilon_p)}\{\Delta(\mathbf{p},k) \times [A(\mathbf{p},k)+\Phi(\mathbf{p}-\mathbf{k})-\Phi(\mathbf{p})] - \Delta(\mathbf{p}+\mathbf{k},k)[A(\mathbf{p}+\mathbf{k},k)+\Phi(\mathbf{p})-\Phi(\mathbf{p}+\mathbf{k})]\}, \tag{39}$$

where

$$w(\varepsilon_p) = 4|\tilde{\Lambda}|^2k^2|U_k|^2\tau^2(\varepsilon_p) \sim \lambda^2(\bar{\varepsilon}\tau)^2 \tag{40}$$

is of the order of magnitude of the left side of the inequality (2).

The equation of finite differences (39) can be solved in the following fashion. We rewrite (39) with the substitutions $\mathbf{p}\to\mathbf{p}-\mathbf{k}$, $\mathbf{p}\to\mathbf{p}+\mathbf{k}$. We then substitute $\Phi(\mathbf{p}-\mathbf{k})$ and $\Phi(\mathbf{p}+\mathbf{k})$, which are defined by the rewritten equations, in (39). For satisfaction of (15), the overlap of the δ functions, displaced along p_x by the amount k, can be small, which allows us to decouple the chain of equations and obtain a closed equation for $\Phi(\mathbf{p})$. The exact condition for the possibility of such a decoupling has the form

$$\pi^2w^2(\varepsilon_p)\tau^{-2}(\varepsilon_p)\Delta(\mathbf{p},k)\Delta(\mathbf{p}\pm\mathbf{k},k) \lesssim \left[\frac{\Lambda k|U_k|}{k^2/m}\Lambda k|U_k|\tau\right]^2 \ll 1. \tag{41}$$

Equation (41) can be compatible with (2) only upon satisfaction of (15).

Computing $\Phi(\mathbf{p})$ in the manner described and taking it into account that $w(\epsilon_\mathbf{p})$ and $\tau(\epsilon_\mathbf{p})$ differ little when $\epsilon_\mathbf{p}$ is changed by an amount $\sim\omega_\mathbf{q}$ or τ^{-1}, we get

$$\Phi(\mathbf{p}) = -eE\tau(\varepsilon_p)\frac{\partial f_0(\varepsilon_p)}{\partial p_x} + \frac{\pi w(\varepsilon_p)\Delta(\mathbf{p},k)}{2\tau(\varepsilon_p)[1+\pi w(\varepsilon_p)\Delta(\mathbf{p},k)\tau^{-1}(\varepsilon_p)]} \times\left[A(\mathbf{p},k) - eE\tau(\varepsilon_{\mathbf{p}-\mathbf{k}})\frac{\partial f_0(\varepsilon_{\mathbf{p}-\mathbf{k}})}{\partial p_x} + eE\tau(\varepsilon_p)\frac{\partial f_0(\varepsilon_p)}{\partial p_x}\right]$$
$$-\frac{\pi w(\varepsilon_p)\Delta(\mathbf{p}+\mathbf{k},k)[A(\mathbf{p}+\mathbf{k},k)+eE\tau(\varepsilon_{\mathbf{p}+\mathbf{k}})\partial f_0(\varepsilon_{\mathbf{p}+\mathbf{k}})/\partial p_x - eE\tau(\varepsilon_p)\partial f_0(\varepsilon_p)/\partial p_x]}{2\tau(\varepsilon_p)[1+\pi w(\varepsilon_p)\Delta(\mathbf{p}+\mathbf{k},k)\tau^{-1}(\varepsilon_p)]}. \tag{42}$$

The quantity $D(\epsilon_\mathbf{p})$ makes a very small contribution $\sim mv_{dr}\Phi/\bar{p}$ to (42) and is therefore omitted. In the quasiclassical case $k \ll \bar{p}$, taking (34), (35) and the definition (31) into account, we rewrite (42) in the form

$$\Phi(\mathbf{p}) = -eE\tau(\varepsilon_p)\frac{\partial f_0(\varepsilon_p)}{\partial p_x} - \frac{msw(\varepsilon_p)f_0'(\varepsilon_p)}{2\sqrt{2m\varepsilon_p}\tau(\varepsilon_p)}\left[1-\frac{eE\tau(\varepsilon_p)}{ms}\right] \times\left\{\frac{[kl(\varepsilon_p)]^{-1}}{[\cos\theta-\cos\theta_+]^2-[1+w(\varepsilon_p)][kl(\varepsilon_p)]^{-2}} - \frac{[kl(\varepsilon_p)]^{-1}}{[\cos\theta-\cos\theta_-]^2+[1+w(\varepsilon_p)][kl(\varepsilon_p)]^{-2}}\right\}, \quad (43)$$

where

$$l(\varepsilon_p) = \frac{\sqrt{2m\varepsilon_p}\,\tau(\varepsilon_p)}{m}, \quad \cos\theta_\pm = \frac{ms}{\sqrt{2m\varepsilon_p}} \pm \frac{k}{2\sqrt{2m\varepsilon_p}}.$$

Substituting $\Phi(\mathbf{p})$ from (43) in (37), we obtain an equation for the determination of $f_0(\epsilon_p)$. The sound contribution to this equation is seen to be negligibly small. To be precise, the perturbation δf_0 of the function f_0 produced by the sound is estimated as

$$\delta f_0/f_0 < \frac{\omega_k \tau e n}{(\varepsilon\tau)}\left[\Lambda k|U_k|\tau\frac{\Lambda k|U_k|}{k^2/m}\right]^2. \quad (44)$$

Thus, at least in the region of wave numbers $\bar{p} \gg k \gg m/k\tau$, monochromatic sound does not lead to a significant heating up of the electron gas.[4)] Here the electron heating is connected only with the action of the electric field. The function calculated in the theory of field heating can be used as $f_0(\epsilon_p)$.

3. CURRENT AND ABSORPTION COEFFICIENT

By knowing the distribution function (42), we can compute various physical quantities, including the electron current density j and the coefficient of electronic absorption α_e. These calculations for j and α_e are carried out to the end in the quasiclassical case $k \ll \bar{p}$. Substituting (43) in the standard expression for the current, and taking (1) into consideration, we obtain

$$j = en\mu E - \frac{em\omega_k}{4\pi}\int_0^\infty d\varepsilon\frac{f_0'(\varepsilon)w(\varepsilon)}{\tau(\varepsilon)}\frac{[1-eE\tau(\varepsilon)/ms]}{\sqrt{1+w(\varepsilon)}}, \quad (45)$$

where n and μ are the concentration and the mobility of the electrons in the field E. The first component is the ordinary current with account of field heating, and the second the acousto-electric current. After linearization with respect to $w(\epsilon)$ (45) transforms into the well known formula of the linear theory.

It is interesting that the nonlinearity leads to the appearance of a square root of the sound flux in the denominator of the expression for the acousto-electric current in (45). This expression differs from the formulas for the current in other nonlinear theories([10, 2] and others). In the theories mentioned, a first power of the sound flux appears in the denominator because of the nonlinearity. For high fluxes, this leads to saturation of the current. In our case, the current saturation is not achieved, inasmuch as j increases as $\sqrt{w}$ for $w > 1$. However, this does not mean that the current can be so very large. The estimate $|j - en\mu E| \sim ens\lambda k/\bar{p} \ll ens$ follows from (45). Thus, under the conditions of the applicability of Eq. (45), the acousto-electric current represents only a small contribution.

The electronic absorption coefficient is obtained by substituting (43) in the general formula (33) and computing the integrals with account of (1). Then

$$\alpha_e = -\frac{\omega_k|\tilde{\Lambda}|^2 m^2}{2\pi\rho s^2}\int_0^\infty d\varepsilon\frac{f_0'(\varepsilon)[1-eE\tau(\varepsilon)/ms]}{\sqrt{1+w(\varepsilon)}}. \quad (46)$$

Neglect of the quantity $w(\epsilon)$ leads to the well known formulas of linear theory. Because of the nonlinearity, the square root of the sound flux again appears, and not the first power of it as was the case obtained previously.[10, 2] The quantity α_e can be significantly decreased with increase in $w(\epsilon)$. Consequently, the nonlinear effects of the given type actually lead to a limitation in the sound amplification. We note that the formal substitution in the final formulas (for example in (45) and (46)) of $[4\pi\beta e/\epsilon k]$ for $|\Lambda|$, where β is the piezomodulus, leads to expressions that are valid for the piezoelectric interaction of electrons with ultrasound. Taking this into account for n-InSb at 77 °K with $n = 10^{14}$ cm^{-3}, $\mu = 6 \times 10^5$ cm^2/V-sec, $m = 1.5 \times 10^{-29}$ g, $\epsilon = 16$, $s = 2 \times 10^5$ cm/sec, $\chi = 4\pi\beta^2/\epsilon\rho s^2 = 1.4 \times 10^{-3}$ at a frequency of 5 GHz, we obtain the result that the nonlinear effects begin ($w \gtrsim 3 \times 10^{-2}$ W/cm^2 (W is the acoustic power flux density).

The multi-valleyed structure of Bi leads to the result that the screening of the sound field cannot play a role (see, for example, [11]). Therefore, one can take the formula of the paper for estimates, without account of screening (i.e., with the substitution $|\tilde{\Lambda}| \to |\Lambda|$). Then, for 4.2 °K, with $n = 10^{18}$ cm^{-3}, $\mu = 10^7$ cm^2/V-sec, $m = 2 \times 10^{-29}$ g, $x = 2 \times 10^5$ cm/sec, $\Lambda = 10$ eV, $\rho = 5$ g/cm^3 at a frequency of 1.6 GHz, we obtain the result that one must have $W \gtrsim 5 \times 10^{-4}$ W/cm^2 for nonlinear effects.

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[4)] In this derivation, it is essential that $\nu_{ee} \ll \tau^{-1}$. Acoustical heating of electrons for $\nu_{ee} \gg \tau^{-1}$ has been estimated in [9].