EFFECT OF PRESSURE ON LATTICE OSCILLATIONS AND ELECTRON-PHONON INTERACTION IN SUPERCONDUCTORS

N. V. ZAVARITSKIĬ, E. S. ITSKEVICH, and A. N. VORONOVSKIĬ

Institute of Physics Problems, U.S.S.R. Academy of Sciences;
Institute of High-pressure Physics, U.S.S.R. Academy of Sciences

Submitted November 3, 1970

Zh. Eksp. Teor. Fiz. **60**, 1408–1417 (April, 1971)

The tunnel effect in lead, tin, and indium under pressure is investigated. The pressure dependence of the gap width Δ_0 and of the frequencies of the phonon spectrum state density singularities ω_i is obtained for the three metals. It is found that the quantity $d \ln \omega_i/dP$ depends on ω_i and decreases somewhat with increasing ω_i. The data obtained and also the amplitude measurements of the quantity dV/dI for lead are employed to determine the pressure-induced change of the function $g(\omega)$, which describes electron-phonon interaction. In all the metals studied it is found that a decrease of the function $g(\omega)$ under pressure is primarily the result of the variation of the lattice oscillation spectrum. The variation of Δ_0, T_c, $2\Delta_0/kT_c$, and of the electron state density near the Fermi surface, induced by pressure, can be derived on the basis of the decrease of $g(\omega)$.

ACCORDING to modern ideas, by virtue of the electron-phonon interaction, the electron and phonon systems in a metal are coupled.[1] This coupling can be investigated in depth by using an external action on the system of electrons and phonons, for example by varying the lattice parameters under hydrostatic pressure. Whereas at present there are data on the change of the electronic structure under pressure,[2] the information concerning the analogous change of the characteristics of the phonons is exceedingly scanty. Such information is obtained mainly from indirect measurements, for example from the temperature dependence of the thermal expansion of crystals.

The present investigation was undertaken for the purpose of obtaining data on the change of the spectrum of the oscillations of the lattice under pressure, and to ascertain the contribution of the electron-phonon interaction to different characteristics of superconductors. The objects chosen for the investigation were lead, tin, and indium—typical nontransition metals. The investigation method used the tunnel effect, which makes it possible to obtain sufficiently complete information on the spectrum of the lattice vibrations.

In 1968 we published the results of an investigation of the tunnel characteristics of lead under hydrostatic pressures up to 15 kbar.[3] In those investigations we determined the pressure-induced change of the gap Δ_0 and a series of frequencies of the phonon spectrum ω_i. The obtained data made it possible to verify the Geĭlikman and Kresin theory of superconductors with strong coupling.[4] Similar measurements under pressures up to 3.5 kbar were made by Franck and Keller,[5] who used solid helium to obtain hydrostatic pressure, and also by Galkin et al.,[6] who used our procedure of applying pressure.

The appearance of experimental data on the influence of hydrostatic compression on the magnitude of the gap and on the phonon frequencies in lead has given rise to a number of theoretical papers,[7-9] using mainly the results of McMillan,[10] who obtained an analytic expression for the temperature T_c of the transition into the superconducting state on the basis of the equations of Éliashberg.[11] In the present paper we submit additional data obtained by investigation of lead and also tin[1)] and indium. The results are analyzed within the framework of the theory of Migdal and Éliashberg.[1,11]

As is well known (see for example [12]), it is possible to obtain from tunnel measurements the relation

$$\frac{dI_{sn}}{dV}\Big/\frac{dI_{nn}}{dV}=\mathrm{Re}\frac{|\omega|}{[\omega^2-\Delta^2(\omega)]^{1/2}}\approx 1+\mathrm{Re}\frac{\Delta^2(\omega)}{2\omega^2},$$

where $\Delta(\omega)$ is the magnitude of the gap in the superconductor spectrum; I_{sn} and I_{nn} are the currents through the junction at the voltage V; the indices s and n pertain respectively to the superconducting and the normal states of the metal.

According to the relations obtained by Éliashberg,[11] the dependence of Δ on the energy ω is determined by the following system of equations, which are expressed in integral form[12] in terms of a function $g(\omega)$ describing the electron-phonon interaction:

$$\begin{aligned}&\Delta(\omega)=\varphi(\omega)/Z(\omega),\quad \Delta(\Delta_0)=\Delta_0,\\ &\varphi(\omega)=\int_{\Delta_0}^{\omega_c}\mathrm{Re}\frac{\Delta(\omega')}{[\omega'^2-\Delta^2(\omega')]^{1/2}}d\omega'\int g(\omega'')O^+(\omega\omega'\omega'')d\omega''-U_c,\\ &Z(\omega)=1-\frac{1}{\omega}\int_{\Delta_0}^{\omega_c}\mathrm{Re}\frac{\omega'}{[\omega'^2-\Delta^2(\omega')]^{1/2}}d\omega'\int g(\omega'')O^-(\omega\omega'\omega'')d\omega'',\\ &O^{\pm}=\frac{1}{\omega'+\omega''+\omega+i\delta}\pm\frac{1}{\omega'+\omega''-\omega-i\delta},\quad \delta\to 0.\end{aligned}\tag{1}$$

Here $U_C(\omega_c)$ is the term that takes into account the Coulomb interaction, $U_C(\omega_e) = U_C^*$; ω_e is the end point of the phonon spectrum;

$$g(\omega)=\Big(\int_S\frac{d^2p}{v_F}\Big)^{-1}\int_S\frac{d^2p}{v_F}\int_{S'}\frac{d^2p'}{(2\pi\hbar)^3v_{F'}}\cdot\sum_\nu q_\nu^2(pp')\delta(\omega-\omega_{p-p'_\nu})\frac{\hbar}{2MN\gamma^2\omega_{p-p'_\nu}}$$

1)Preliminary data on tin were reported in [3c].

where $\mathcal{V}$ is the volume, ν is the lattice vibration mode, v_F the electron Fermi velocity, p the momentum, M the mass of the lattice atom, and $q_\nu(pp')$ the matrix element of the pseudopotential; the integration is carried out over the Fermi surface.

$g(\omega)$ is frequently presented in the form $g(\omega) = \alpha^2(\omega)F(\omega)$, where $F(\omega)$ is the density of states of the lattice vibrations, $\alpha^2(\omega)$ is the effective function of the electron-phonon interaction, which varies smoothly with the energy. As shown by Morel and Anderson,[13] all the singularities of $F(\omega)$ are reflected in the function $\mathrm{Re}\,\Delta^2(\omega)$. Moreover, it has turned out[12,14] that with the aid of the system of equations (1) and the experimentally determined dependence of dI/dV on V it is possible to reconstruct the function $g(\omega)$ and to determine from it the character of variation of $F(\omega)$.

The function $g(\omega)$ plays the principal role in many effects due to the change of the properties of the electrons of the normal metal as a result of the electron-phonon interaction. Thus, the renormalization coefficient $Z = 1 + \lambda$, where λ is the averaged parameter of the electron-phonon interaction, given by

$$\lambda = 2\int \frac{g(\omega)}{\omega} d\omega, \qquad (3)$$

determines, for example, the change of the effective mass, of the density of states, and of the electron velocity near the Fermi surface.

1. The object of the investigation was a system of Al-Al_2O_3-S films (S—investigated superconductor) prepared in the usual manner.[3b] The film thickness was $\sim 10^{-5}$ cm. The aluminum was oxidized in an atmosphere of dry air, the oxidation time being about 60 sec.

During the course of the experiment, using the scheme described in [3b, 15], we registered the quantities dV/dI and d^2V/dI^2 as functions of the voltage V. All the measurements were performed in a cryostat cooled on the outside with helium, making it possible to lower the measurement temperature to 1°K by pumping off the helium with a forevacuum pump.

For the experiments under pressure, we selected samples in which: 1) the film surface was specular, 2) the resistance at 300°K was 50–100 ohms; 3) there were no parasitic singularities on the d^2V/dI^2 characteristics, 4) all the singularities of $F(\omega)$ were sufficiently pronounced on these characteristics.

The hydrostatic pressure was produced in a chamber with fixed pressure.[3b, 16] The pressure was calculated from the change of T_c of a tin wire located in the high-pressure chamber, using the formula $\delta T_c = -4.95 \times 10^{-5} P + 3.9 \times 10^{-10} P^2$ (P is in bars). The possible error in the pressure was 3–4%.

A. Lead. Figure 1 shows plots of $dV/dI = f_1(V)$ and $d^2V/dI^2 = f_2(V)$ for tunnel junctions at normal and high pressures. The curves were used to reconstruct the function $g(\omega)$. In this operation we used the tunnel characteristics of the samples, which coincided with the initial characteristics in all details after the pressure was removed. The $f_2(V)$ curves were used to determine the displacements of the main singularities of the phonon spectrum under pressure. In the measurement interval (up to 15 kbar) the displacement of the singularities was proportional to the applied pressure. The quantitative data are listed in Table I, where ω_T and ω_L are the

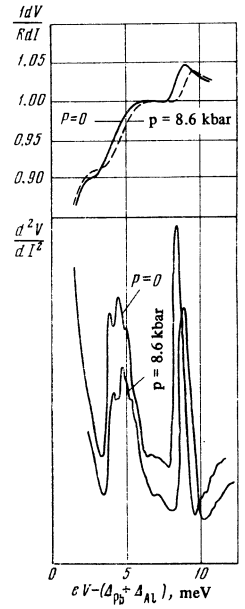

FIG. 1. Sample plots of the quantities $R^{-1}dV/dI$ and d^2V/dI^2 as functions of V for lead at P = 0 and P = 8.6 kbar.

energies of the maximum $F(\omega)$ due to the transverse and longitudinal modes of the oscillations, ω_e is the end point of the spectrum, and ω_1T, ω_2T, and ω_3T are singularities on the $F(\omega)$ function near ω_T, and are manifest in the tunnel characteristics.

As seen from Table I, the quantities $d\omega/dP$ obtained by different authors agree better for ω_L than for ω_T. This is connected with the larger half-width of the ω_T maximum on the $F(\omega)$ and $d^2V/dI^2 = f_2(V)$ curves. Additional measurements and an improvement of the procedure used to calculate the experimental curves have made it possible to refine the value of $d\omega_T/dP$ given in [3a]. In all the subsequent calculations we use the quantity $d\omega_T/dP = 4.0 \times 10^{-6}$ meV/bar, which has a large statistical weight. From among the data of Table I, in our opinion, the least reliable are the results of [5], since the measurements there were carried out only up to 3.3 kbar at 2°K.

B. Tin. Figure 2 shows plots of d^2V/dI^2 against V for tin at pressures 0 and 9 kbar. The arrows locate the the singularities whose displacement under pressure was investigated. Their values do not differ from those obtained earlier by Rowell and Kopf[18] at normal pressure. A comparison of these singularities with the dispersion curves given by Rowe[19] shows that the singularity at 3–5 meV belongs apparently to the transverse acoustic branches, that with 15 meV energy belongs to the optical branches, and the singularity at 17.4 meV belongs to the end of the spectrum. The displacements of all the singularities under pressure are listed in Table II. The numerical values were obtained by averaging six independent experiments. The measurements for tin and indium were carried out in the region up to 11 kbar, where all the characteristics are fully reversible.

C. Indium. Figure 3 shows plots of the d^2V/dI^2 characteristics for indium at P = 0 and P = 9.5 kbar. The arrows show the locations of the investigated singularities. The locations of the singularities at P = 0 coincide with the data of [18]. A comparison of the singularities with the results of the reconstruction of $g(\omega)$[14b] shows that the singularities at 3.5–7 meV

Table I

Measured quantity X	Energy, meV	$\frac{dX}{dP}\cdot 10^6$, meV·bar^{-1}	$\frac{d\ln X}{dP}$, 10^{-6} bar^{-1}	$\frac{d\ln X}{d\ln\gamma^*}$	$\frac{dX}{dP}\cdot 10^6$, $\frac{meV}{bar}$ from [5c]	$\frac{dX}{dP}\cdot 10^6$, $\frac{meV}{bar}$ from [6]
$2\Delta_0$	2.71	−21.6±2	−8		−27±2.5	−22.2±1
ω_{1T}	3.80	32±7	8.4	−3.7		26±4
ω_{2T}	4.45	36±5	8.0	−3.5		33±4
ω_{3T}	4.9	32±7	6.5	−2.9		33±4
ω_T	4.45	40±4	9.0	−3.94	24±3	
ω_L	8.45	60±6	7.1	−3.1	59±6	64±4
ω_e	9.9	70±10	7.1	−3.1		

Notes: 1) ω_T corresponds to the center of gravity of the envelope of the curve $d^2V/dI^2 = f_2(V)$ at $T > T_c$(Al).

2) In the calculation of $d\ln X/d\ln\gamma^*$ the compressibility $d\ln\gamma^*/dP$ was taken from Table IV.

3) From the measurements of the thermal expansion $d\ln\Theta_D/d\ln\gamma^* = -2.84$ [17].

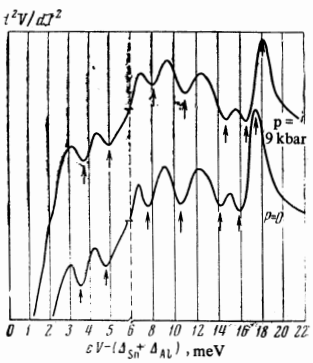

FIG. 2. Sample plots of the d^2V/dI^2 characteristics for tin in arbitrary units at P = 0 and P = 9 kbar. The scale changes at 6 meV.

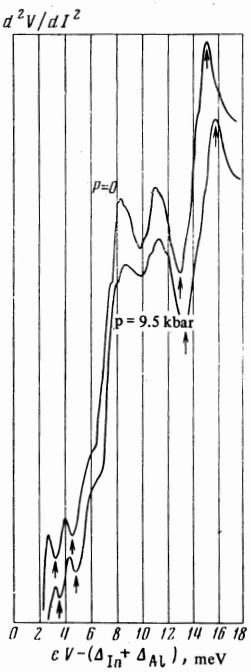

FIG. 3. Sample plot of the d^2V/dI^2 characteristic for indium in arbitrary units at P = 0 and P = 9.5 kbar.

apparently belong to the maximum of F(ω) due to the transverse acoustic branches, the singularity at 12.8 meV belongs to the maximum of F(ω) due to the longitudinal oscillations, and that at 15.4 meV belongs to the end point of the spectrum. The displacements of all the singularities under pressure are given in Table III. The numerical values were obtained by averaging five independent experiments under the assumption, just as in the case of the other metals, that the displacements of the singularities under pressure are linear.

2. Let us determine the change of the electron-phonon interaction, or g(ω), under pressure.

A. Lead. Amplitude measurements of dV/dI as a function of V make it possible to reconstruct the g(ω) of this metal under pressure. The construction was carried out for P = 0 and P = 10 kbar by selecting the amplitude of g(ω) until the values of Δ_0 and the dependence $\mathrm{Re}\,\Delta^2(\omega)/2\omega^2$ (3 meV < ω < 12 meV), calculated from relation (1), agreed with the same values determined from the experimental data. Figure 4 shows the functions g(ω) at P = 0 and P = 10 kbar. The reproducibility of the reconstructed curve is ~0.05. In the absence of pressure, g(ω) coincides with the previously published values.[14] The shifts of the main singularities of g(ω) along the ω axis coincides with the data of Table I. The pressure produces, besides shifts of the maxima of g(ω), also a certain decrease of g(ω), amounting to ~10% at P = 10 kbar.

In a previous attempt by Franck, Keller, and Wu to reconstruct the function g(ω) under pressure,[22] the effect of decreasing g(ω) was not observed. In our opin-

Table II

Measured quantity X	Energy, meV	$\frac{dX}{dP}\cdot 10^6$, meV·bar^{-1}	$\frac{d\ln X}{dP}\cdot 10^6$, bar^{-1}	$\frac{d\ln X}{d\ln\gamma^*}$	Measured quantity X	Energy, meV	$\frac{dX}{dP}\cdot 10^6$, meV·bar^{-1}	$\frac{d\ln X}{dP}\cdot 10^6$, bar^{-1}	$\frac{d\ln X}{d\ln\gamma^*}$
$2\Delta_0$	1.2	−18±1	−15.0		ω_4	10.6	42±5	4.0	−2.21
ω_1	3.5	22±4	6.3	−3.5	ω_5	14.2	55±7	3.9	−2.16
ω_2	4.8	22±10	4.6	−2.6	ω_6	15.9	67±7	4.2	−2.32
ω_3	7.6	50±20	6.5	−3.6	ω_e	17.4	70±14	4.0	−2.21

Notes. 1) From measurements of the thermal expansion $d\ln\Theta_D/d\ln\gamma^* = -2.27$ [17].

2) From the Mossbauer effect under pressure $d\ln\Theta_D/d\ln\gamma^* = -2.34 \pm 0.20$ [20].

3) From the data of Galkin et al. [21], $d(2\Delta_0)/dP = 18.5 \pm 1.0$.

Table III

Measured quantity X	Energy, meV	$\frac{dX}{dP}\cdot 10^{6}$, meV·bar^{-1}	$\frac{d\ln X}{dP}\cdot 10^{6}$, bar^{-1}	$\frac{d\ln X}{d\ln\gamma^{*}}$
$2\Delta_0$	1.04	−14.5±0.5	−14.2	
ω_1	3.25	+20±3	5.0	−2.7
ω_2	4.6	26±4	5.7	−2.4
ω_3	12.8	48±5	3.7	−1.6
ω_e	15.1	75±10	5.0	−2.1

Notes. 1) From measurements of the thermal expansion $d\ln\Theta_D/d\ln\gamma^{*} = -2.48$ [17].
2) From the data of Galkin et al. [21b], $d(2\Delta_0)/dP = -14.3 \pm 1.3$.

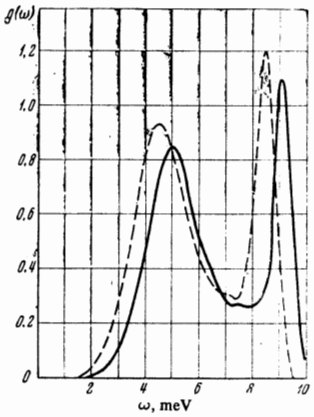

FIG. 4. Reconstructed function g(ω) for lead at P = 0 (dashed curve) and P = 10 kbar (solid curve).

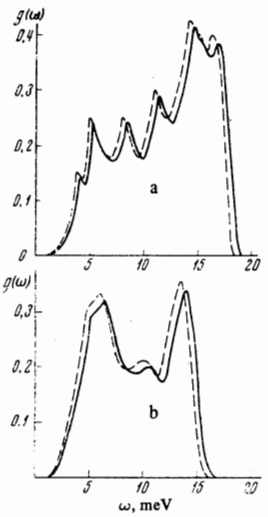

FIG. 5. The function g(ω) for tin (a) and indium (b) at P = 0 (dashed curve) and P = 10 kbar (solid curve).

ion, this is connected primarily with the narrower interval of pressure values in which the experimental data were obtained. Thus, at the maximum pressure of 3.3 kbar attained in [22] the decrease of g(ω) should be ~3%, which is close to the possible reconstruction error (0.03 in accordance with McMillan's program[14a]). The change of Δ_0 under pressure in [22] could be obtained only by assuming a noticeable increase of the Coulomb interaction under pressure. Our calculations show that if this method is used and the value $(d\ln\Delta_0/dP)_{exp}$ is correct, then a patent disparity is obtained between the calculated and experimental relations $\mathrm{Re}\,\Delta^2(\omega)/2\omega^2$ for $V \gg \Delta_0$. Starting from this, we assumed in the reconstruction of g(ω) that the Coulomb interaction remains unchanged under pressure (U_C = const in relation (1)).

B. Tin. For tin and indium, only Δ_0 was calculated from relation (1). If the calculation of Δ_0 at P = 10 kbar for tin is based on the g(ω) determined at P = 0 in [14b] and shifted along the ω axis in accordance with the data of Table II, then we obtain $(d\ln\Delta_0/dP)_{calc} = -12 \times 10^{-5}$ bar^{-1}, which differs noticeably from the measurement results. To eliminate this disparity, it is necessary to assume in the calculations either that g(ω) decreases under pressure or that the role of the Coulomb interaction, i.e., the quantity U_C, increases. Assuming, just as in the case of lead, that pressure produces mainly a change of g(ω), we found that the function g(ω) at P = 10 kbar decreases by 4% (Fig. 5a).

C. Indium. Analogous calculations for indium have shown that besides the shift of g(ω) along the ω axis, it is necessary, in order to describe the observed quantity $(d\ln\Delta_0/dP)_{exp} = -14.2 \times 10^{-6}$ bar^{-1}, to assume that g(ω) decreases 5% at 10 kbar (see Fig. 5b). In the calculation of the amplitude change of g(ω) under pressure for tin and indium it was assumed that $U_C^{*} = 0.12$. The error in the absolute magnitude of g(ω) under the foregoing assumptions did not exceed ~1%. The complete program for the reconstruction of g(ω) under pressure was not carried out for these metals, since it followed from the similar character of all the singularities on the $d^2V/dI^2 = f_2(V)$ curves at P = 0 and P = 10 kbar (Figs. 2 and 3) that no significant change in the form of the g(ω) curve occurs under pressure.

The change of g(ω) under pressure does not contradict the existing theoretical notions. Thus, from an analysis of relation (2), Trofimenkoff and Carbott[8] have established that if the shift of the oscillation spectrum of the lattice under pressure is by a factor β, then

$$g(\beta\omega) = \beta^{-2}Bg_0(\omega); \tag{4}$$

here B is a coefficient that takes into account the average change of the square of the matrix element of the pseudopotential under pressure. The change of g(ω) at a pressure of 10 kbar, calculated by us for lead from the data of [8], is 12–13%, which is in perfectly satisfactory agreement with the results obtained from the reconstruction of g(ω).

Table IV

Investigated quantity, 10^{-6} bar^{-1}	Substance		
	Lead	Tin	Indium
$d\ln\gamma^{*}/dP$ [17]	−2.28	−1.8	−2.39
λ	1.49	0.79	0.82
$d\lambda/dP$	−28±2	−6.3±0.7	−8±1
$d\ln\lambda/dP$	−18.8	−8	−9.0
$d\ln Z/dP$	−11±1	−3.5±0.4	−4.4±0,4
$d\ln N_0/dP$	−8.3±1.5 [23]	−3.6±0.5 [24]	−3.6±0.7 [24]

On the basis of the results, we calculated the averaged parameter of the electron-phonon interaction λ from relation (3). The change of the phonon spectrum under pressure leads to a decrease of this parameter. Data on the $\lambda(P)$ dependence are given in Table IV.

3. Let us discuss the pressure dependence of T_c and of the ratio $2\Delta_0/kT_c$. According to the BCS classical theory, the ratio $2\Delta_0/kT_c$ is a universal constant for all superconductors. From the point of view of this theory, one should expect the quantities $d \ln \Delta_0/dP$ and $d \ln T_c/dP$ to coincide. Actually, $d \ln \Delta_0/dP < d \ln T_c/dP$ for all the superconductors investigated by us (see Table V). The most noticeable difference is observed for lead. As was shown by us earlier,[3a] in this case the difference between the quantities $d \ln \Delta_0/dP$ and $d \ln T_c/dP$ can be fully explained with the aid of the results obtained by Geĭlikman and Kresin[6] for superconductors with strong coupling. Namely, they found that actually

$$\frac{2\Delta_0}{kT_c} = 3.52\left(1 + 5.3\frac{T^2}{\omega^2}\ln\frac{\omega}{T_c}\right), \qquad (5)$$

where ω is the frequency of the phonons that interact mainly with the electrons. It follows from this relation that for all metals with $2\Delta_0/kT_c > 3.52$ one should expect $2\Delta/kT_c$ to decrease under pressure. Owing to the large ratio $T_c/\Theta_D \sim 0.1$ this effect is most strongly pronounced for lead. However, even for tin and indium, metals having $T_c/\Theta_D \sim 0.02$, this effect cannot be neglected. The reason is that the summary value of the electron-phonon interaction, as seen from (1) and (3), receives a particularly large contribution from the low-energy vibration modes. Thus, the maximum of $g(\omega)/\omega$ is located at 4.5 meV for lead and at ~6 meV for tin and indium. Substituting these quantities in (5), we obtain values of $(d \ln \Delta_0/dP)_{calc}$ (Table V) which are in good agreement with $(d \ln \Delta_0/dP)_{exp}$.

Starting from the system of equations (1), McMillan[10] obtained an analytic expression for T_c of superconductors:

$$T_c = \frac{\Theta_D}{1.45}\exp\left\{-\frac{1.04(1+\lambda)}{\lambda - U_c^*(1+0.62\lambda)}\right\} \qquad (6)$$

(the term describing the Coulomb interaction is for most metals $U_C^* \approx 0.11$–0.13). Let us differentiate it and substitute then $d \ln \omega/dP$ from Tables I–III and λ, $d\lambda/dP$ from Table IV. As a result we obtain the values of $d \ln T_c/dP$ listed in Table V. As seen from Table V, formula (6) can be used to calculate the change of T_c of superconductors under pressure only in the first approximation, especially in the case of lead.

Thus, remaining within the framework of the theory of the electron-phonon superconductivity mechanism, we can explain the entire aggregate of parameters characterizing the variation of the superconducting properties of lead, tin, and indium under pressure. From the point of view of this result, we can understand the success attained in the explanation of the growth of T_c of films of these metals by taking into account the change of only the phonon spectrum in these films.[15]

Table V

Quantity	Lead	Tin	Indium
	10^{-6} bar^{-1}		
$(d \ln \Delta_0/dP)$ exp	−8±0.8	−15±1	−14±1
$d \ln T_c/dP$ from [7]	−5.3±0.1	−13.3±0.3	−12.6±0.6
$(d \ln \Delta_0/dP)$ calc from (5)	−9.5	−14.8	−13.7
$(d \ln T_c/dP)$ calc from (6)	−11.0±1.5	−14.5±1.5	−17.0±2

4) The obtained data enable us to determine the role of different factors also in the change of the state of the electrons or the normal metal with changing lattice parameters.

As was shown by Migdal,[1] the electron-phonon interaction leads to a renormalization of the quantities characterizing the electronic spectrum. In particular, the new density of states of the electrons near the Fermi surface is $N(0) = ZN_{BS}(0)$, where $Z = 1 + \lambda$ and $N_{BS}(0)$ is the density of states of the band structure. Therefore, when considering the influence of pressure on the electronic characteristics, it is necessary to take into account the effect of pressure also on the renormalization coefficient Z. The change of Z under pressure is given in Table IV.

The density of the electronic states on the Fermi surface N(0), determined from the electronic specific heat or from the temperature dependence of the critical magnetic field of the superconductor, was measured under pressure for the metals considered in the present article (see Table IV). As seen from Table IV, the quantities $d \ln N(0)/dP$ and $d \ln Z/dP$ agree satisfactorily for all the metals investigated by us. This allows us to conclude that $N_{BS}(0)$ changes more weakly than Z. Consequently, the change of the electron-phonon interaction determines mainly the dependence of the density of states of the electrons of lead, tin, and indium on the lattice parameters. This effect plays a predominant role in comparison with the change of the Fermi surface of the electrons.

5. The data given in Tables I–III on the shift of the phonon spectrum indicate that in all the investigated metals there is apparently a tendency for $d \ln \omega_i/dP$ to increase with decreasing ω_i of the corresponding singularity of the spectrum. Let us present some qualitative considerations making it possible to understand this effect. As is well known, the presence of electrons plays an essential role in the formation of the dispersion curves of the lattice vibration spectrum. The electrons are responsible for presence of long-range forces in the metal and, as a consequence, for the relatively large values of $d \ln \omega_i/dP$ and $d \ln \omega_i/d\gamma$. However, the relative contributions of the electrons to different vibration modes are different. Usually, the higher the frequency the smaller this contribution. Consequently, all of the effects due to the presence of electrons in the metal should be smaller, particularly the quantity $d \ln \omega_i/dP$. Obviously, this question also lends itself to a rigorous analysis, but this problem is beyond the scope of the present article.

The authors are grateful to P. L. Kapitza and L. F. Vereshchagin for interest and support, and to the members of the computation group of the Institute of Physics Problems for collaboration with the calculations.

[1] A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys.-JETP **7**, 996 (1958)].

[2] N. B. Brandt, E. S. Itskevich and N. Ya. Minina, Usp. Fiz. Nauk **104**, 439 (1971) [Sov. Phys.-Usp. **14**, No. 3 (1971)].

[3] N. V. Zavaritskiĭ, E. S. Itskevich and A. N. Voronovskiĭ, a) ZhETF Pis. Red. **7**, 271 (1968) [JETP Lett. **7**, 211 (1968)]; b) Tr. V. Bakurianskogo kollokviuma (Proc. Fifth Bakuriani Colloquium), 2, Tbilisi, 1969, p. 314; c) Proc. LT 11, 2, St. Andrews, 1968, p. 725.

[4] B. T. Geĭlikman and V. Z. Kresin, Fiz. Tverd. Tela **7**, 3294 (1965) [Sov. Phys.-Solid State **7**, 2659 (1966)].

[5] I. P. Franck and W. J. Keller. a) Phys. Lett. **25A**, 624 (1967); b) Phys. Rev. Lett. **20**, 379 (1968).

[6] A. A. Galkin, V. M. Svistunov, A. P. Dikiĭ, and V. N. Taranenko, Zh. Eksp. Teor. Fiz. **59**, 77 (1970) [Sov. Phys.-JETP **32**, 44 (1971)].

[7] R. E. Hodder, Phys. Rev. **180**, 530 (1969).

[8] R. N. Trofimenkoff and J. P. Carbotte, Phys. Rev. **131**, 1136 (1970).

[9] M. A. Conlthard, J. Phys. Proc. Sol. St. Phys. **3**, 820 (1970).

[10] W. L. McMillan, Phys. Rev. **167**, 331 (1968).

[11] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960); **39**, 1437 (1960) [Sov. Phys.-JETP **11**, 696 (1960); **12**, 1000 (1961)].

[12] P. J. Scalapino, J. R. Schrieffer, and J. W. Wilkins, Phys. Rev. **148**, 263 (1966).

[13] P. Morel and P. W. Anderson, Phys. Rev. **125**, 1203 (1966).

[14] W. L. McMillan and J. M. Rowell. a) Phys. Rev. Lett. **14**, 516 (1965); b) Tunneling in Strong Coupling Superconductor, ed. R. D. Parks, Superconductivity **1**, New York (1969), p. 561.

[15] N. V. Zavaritskiĭ, Zh. Eksp. Teor. Fiz. **57**, 752 (1969) [Sov. Phys.-JETP **30**, 412 (1970)].

[16] E. S. Itskevich, A. N. Voronovskiĭ, A. F. Gavrilov, and V. A. Sukhoparov, Pribory i Tekhn. Eksperim. No. 6, 16 (1966).

[17] R. Y. Boghton, J. L. Olsen, and C. Palmy. Pressure Effects on Superconductors, 1969 (Preprint).

[18] J. M. Rowell and L. Kopf, Phys. Rev. **137**, A907 (1965).

[19] J. M. Rowe, Phys. Rev. **163**, 547 (1967).

[20] V. N. Panyushkin and F. F. Voronov, ZhETF Pis. Red. **2**, 153 (1965) [JETP Lett. **2**, 97 (1965)].

[21] A. A. Galkin, V. M. Svistunov, and A. P. Dikiĭ, Phys. Stat. Sol. a) **30**, K107, K107 (1968); b) **35**, 421 (1969).

[22] J. P. Franck, W. J. Keller, and T. M. Wu, Solid St. Comm. **7**, 483 (1969).

[23] M. Garfinkel and D. E. Mapother, Phys. Rev. **122**, 459 (1961).

[24] I. V. Berman, N. B. Brandt, and N. I. Ginzburg, Zh. Eksp. Teor. Fiz. **53**, 124 (1967) [Sov. Phys.-JETP **26**, 86 (1968)].

Translated by J. G. Adashko
155