# *MULTIPLE SCATTERING OF SLOW PARTICLES*

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The angular distribution of particles penetrating through a layer of matter is calculated under the assumption that the potential of the interaction between the particles and the target atoms is of the form $r^{-2}$. It is shown that the theoretical angular distribution is close to the experimental one for protons having an energy of several times ten keV.

WE consider in the present paper multiple scattering of slow particles in a medium. Particles are assumed to be slow if they have an energy such that the potential of the interaction between the particles and the target atoms can be approximated by a potential in the form $r^{-2}$.

In the general form the angular distribution of the particles passing through a layer of matter d was found in[1,2] and is written in the form

$$f(\theta,d)=\frac{1}{2\pi}\sum_0^\infty(l+{}^1/_2)\exp\left\{-N\int_0^d\sigma_l\,dz\right\}P_l(\cos\theta), \qquad (1)$$

where

$$\sigma_l=2\pi\int_0^\pi[1-P_l(\cos\alpha)]\frac{d\sigma(\alpha)}{d\Omega}\sin\alpha\,d\alpha, \qquad (2)$$

$P_l(\cos\alpha)$ are Legendre polynomials, $d\sigma(\alpha)/d\Omega$ is the differential cross section for the scattering of a particle by an angle $\alpha$, $\Omega$ is the solid angle, and N is the number of particles per unit volume of the target.

The differential cross section for the scattering of a particle by a potential of the type $r^{-2}$ equals, according to[3],

$$\frac{d\sigma(\alpha)}{d\Omega}=\frac{4{,}1e^2a_TZ_1Z_2(M_1+M_2)}{32\pi M_2E\sin^3(\alpha/2)}=\frac{2\sqrt{2}A}{(1-x)^{3/2}}, \qquad (3)$$

where $Z_1e$, $M_1$, and $Z_2e$, $M_2$ are the change and mass of the incoming particle and the target atom, respectively, E is the energy of the particle, $x = \cos\alpha$, and $a_T$ is the Thomas-Fermi radius.

To find $\sigma_l$ we use the identity

$$\frac{1}{1-t}-\frac{1}{(1-2xt+t^2)^{1/2}}=\sum_0^\infty[1-P_l(x)]t^l,\quad t<1. \qquad (4)$$

Multiplying (4) by $2\pi d\sigma(\alpha)/d\Omega$ and integrating with respect to x we obtain

$$2\pi\sum_0^\infty t^l\int_{-1}^1[1-P_l(x)]\frac{d\sigma(x)}{d\Omega}dx=\sum_0^\infty\sigma_lt^l$$

$$=2\pi\int_{-1}^1\left(\frac{1}{1-t}-\frac{1}{(1-2xt+t^2)^{1/2}}\right)\frac{2\sqrt{2}A\,dx}{(1-x)^{3/2}}=16\pi A\frac{t}{(1-t)^2} \qquad (5)$$

From the expansion of the right-hand side of formula (5) in powers of t it follows that

$$16\pi A\frac{t}{(1-t)^2}=16\pi A\sum_0^\infty lt^l. \qquad (6)$$

If we compare the coefficients of identical powers of t in (5) and (6), then we can find the value of $\sigma_l$:

$$\sigma_l=\frac{2.05e^2a_TZ_1Z_2(M_1+M_2)}{M_2E}l. \qquad (7)$$

Thus, substituting (7) in (1), we obtain the angular distribution of the particles experiencing multiple scattering in a layer of matter:

$$f(\theta,d)=\frac{1}{2\pi}\sum_0^\infty(l+{}^1/_2)e^{-\bar{p}l}P_l(\cos\theta),$$

where

$$\bar{p}=N\int_0^d\sigma_1\,dz$$

and $\sigma_1$ is the transport scattering cross section ($l = 1$).

The series (8) can be summed[1)]. Indeed, denoting $e^{-\bar{p}} = t$, we get

$$f(\theta,d)=\frac{1}{2\pi}\sum_0^\infty(l+{}^1/_2)t^lP_l(\cos\theta)=\frac{1}{2\pi}\left(t\frac{d}{dt}+\frac{1}{2}\right)\sum_0^\infty t^lP_l(x)$$

$$=\frac{1}{2\pi}\left(t\frac{d}{dt}+\frac{1}{2}\right)(1-2xt+t^2)^{-1/2}=\frac{1}{4\pi}\frac{1-t^2}{(1-2xt+t^2)^{3/2}} \qquad (9)$$

To calculate $\bar{p}$ it is necessary to know the dependence of the energy loss on the energy when the particle is slowed down in the medium. For a wide range of particle velocities, such a dependence is known only for protons[4]. In the energy range of interest to us ($E < 100$ keV) the energy lost by protons in a medium can be represented in the form

$$-dE/dz=\alpha y(1-\beta y)\ [\text{эв}/\text{Å}], \qquad (10)$$

where y is a dimensionless quantity numerically equal to the velocity of the proton in units of $10^8$ cm/sec; the values of the coefficients $\alpha$ and $\beta$ for different solid substances are listed in the table.

The value of $\bar{p}$ calculated with the aid of these data turns out to be

$$\bar{p}=\frac{2.56\cdot10^4e^2a_TZ_1Z_2(M_1+M_2)}{\alpha M_2}N\left[\frac{1}{y_1}-\frac{1}{y_0}+\beta\ln\frac{y_0(1-\beta y_1)}{y_1(1-\beta y_0)}\right] \qquad (11)$$

where $\gamma_0$ is the velocity of the particle prior to striking the target and

$$y_1=\frac{1}{\beta}\left[1-(1-\beta y_0)\exp\left(\frac{1{,}6\cdot10^{-20}\alpha\beta d}{M_1}\right)\right] \qquad (12)$$

is the velocity of the particle after leaving the target.

Figure 1 shows a comparison of the theoretical (solid curves) and experimental[5,6] data on the scattering of protons in certain substances. The theoretical curves were calculated from formula (9) with allowance for (11) and (12). We see that for slow protons one observes good agreement between the experimental data and the theory in the entire measured range of

[1)]This possibility was pointed out by O. B. Firsov.

| Z | | α | β | Z | | α | β | Z | | α | β |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | Be | 7.4 | 0.137 | 25 | Mn | 6.4 | 0.065 | 47 | Ag | 7.7 | 0.083 |
| 6 | C | 8.1 | 0.124 | 26 | Fe | 7.3 | 0.076 | 50 | Sn | 6.1 | 0.090 |
| 13 | Al | 7.5 | 0.134 | 27 | Co | 7.3 | 0.060 | 51 | Sb | 6.5 | 0.111 |
| 14 | Si | 2.9 | 0.080 | 28 | Ni | 9.7 | 0.092 | 79 | Au | 7.3 | 0.065 |
| 22 | Ti | 12.3 | 0.145 | 29 | Cu | 8.0 | 0.081 | 83 | Bi | 7.2 | 0.106 |
| 24 | Cr | 7.9 | 0.077 | 32 | Ge | 8.5 | 0.129 | | | | |

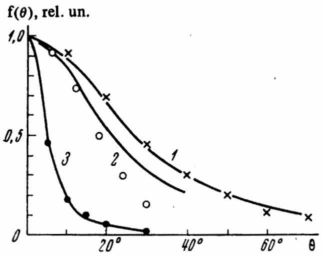


FIG. 1. Scattering of protons in matter: curve 1–5.8 keV, Si, 760Å [5]; 2–158.5 keV, Cu, 6000Å [6]; 3–13.9 keV, Bi, 200Å [5]. Solid curves–theoretical.

scattering angles (0–70°). The difference between the calculated and measured scattering probabilities for protons with energy 158.5 keV can apparently be attributed to the fact that at such a high energy the approximation of the interaction potential by a potential of the type $r^{-2}$ is inadequate.

Figure 2 shows the experimental data on the scattering of protons in matter, averaged over the parameter p. In the averaging we used the results on the scattering of hydrogen ions $H^+$ in Ni, Si, Bi, Fe (2.5–30 keV)[5], in Cu (~160 keV)[6], and in Ni (15–90 keV[7]. The solid curves represent the theoretical values. We see that in the range of variation of $\bar{p}$ from 0.05 to 0.2 the theoretical and experimental data coincide within the limits of the measurement errors. For $\bar{p} > 0.3$, the experimental values are smaller than the theoretical ones. This may be connected with the fact that large values of $\bar{p}$ correspond to low energies and large thicknesses of matter. In this case the free paths for certain particles may turn out to be smaller than the thickness of the foil and such particles will not be registered.

In conclusion, the author considers it his pleasant duty to thank O. B. Firsov for valuable advice and discussions.

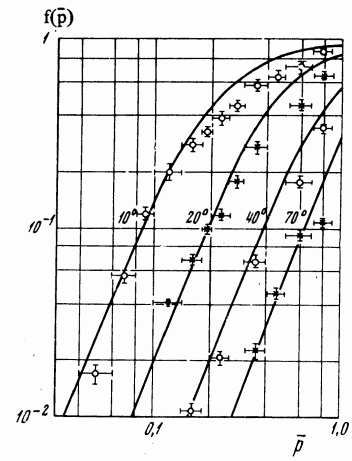


FIG. 2. Comparison of theoretical and experimental data on the scattering of protons in matter for different θ. Solid curves–theoretical.

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