ELECTRON SOUND AND THE SOFT PLASMA BRANCH IN SEMICONDUCTORS

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It is shown that a low-frequency collective oscillation branch should be possible at low temperatures and high carrier densities in the electronic subsystem of a pure compensated semiconductor. The oscillations are similar to electron sound in metals, a phenomenon previously considered by the authors; however, in contrast to metals, in semiconductors the interaction between the carriers involves direct Coulomb interaction between the electrons. Macroscopically, electron sound is a temperature wave with a velocity on the order of the thermal velocity of the heavy carriers. If compensation is not complete and the difference between the number of electrons and holes is sufficiently small, a soft optical branch arises with an activation frequency that is proportional to the difference between the electron and hole concentrations. It is shown that in the hydrodynamic region soft activation frequencies also appear in transverse electromagnetic waves.

1. INTRODUCTION. SECOND SOUND IN A SYSTEM OF CHARGED PARTICLES

Peshkov, who observed experimentally the second sound predicted by Landau in He_2, was apparently the first to note[1] that a similar phenomenon can occur in crystals, namely, collision sound in a gas of phonons, which is perceived macroscopically as a temperature wave. It is important that the sound can exist only in a "window"[2]

$$1/\tau^{V} \ll \omega \ll 1/\tau^{N}, \qquad (1.1)$$

where its attenuation is small[1)]:

$$\omega^{-1}\,\mathrm{Im}\,\omega \sim \omega\tau^{N}, \qquad 1/\omega\tau^{V}. \qquad (1.2)$$

Here $1/\tau^N$ is the frequency of the normal collisions between the quasiparticles, in which both the energy and the quasimomentum are conserved; $1/\tau^V$ is the frequency of collisions in which at least one of the indicated quantities is not conserved. (Among such processes are also collisions with Umklapp.) In crystalline He_4, the existence of a "window" was proved experimentally: Mezhov-Deglin observed hydrodynamic thermal conductivity of thin samples[4], while Ackerman, Bertman, Fairbank, and Guyer observed phonon second sound[5,6].

We have previously shown[3] that a collective branch similar to second sound can also exist in a system of charged quasiparticles—electrons in a metal, if besides the conditions (1.1) (which are discussed for metals in[3] and will be discussed for semiconductors below) there is also satisfied the condition that the number of electrons and holes be equal.

Indeed, frequent normal collisions lead to the occurrence of a drift described by a local-equilibrium distribution function

$$f^{(0)} = f_0\left(\frac{\varepsilon(\mathbf{p}) - \mathbf{pu} - \mu}{T_e(1+\vartheta)}\right), \quad f_0(x) \equiv \frac{1}{e^x+1}, \qquad (1.3)$$

where the wave of second sound can be described as a wave of drift parameters ($\mathbf{u}(\mathbf{r}, t)$, $\delta\mu(\mathbf{r}, t)$, and $\vartheta(\mathbf{r}, t)$). Here $\delta\mu(\mathbf{r}, t) = \mu(\mathbf{r}, t) - \mu_0$ is the local deviation of the chemical potential μ from the stationary value μ_0. In the linear approximation, the function (1.3) leads to an electric current proportional to the drift velocity u:

$$\mathbf{j} = {}^1/_3 e\langle \mathbf{pv}\rangle \mathbf{u} = e(n_- - n_+)\mathbf{u}, \qquad (1.4)$$

which vanishes only if the numbers of electrons and holes are equal[2)]. (An analogous condition can be formulated for open Fermi surfaces[3].) We have used the notation

$$\langle\varphi\rangle \equiv \frac{2}{h^3}\int d\mathbf{p}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\varphi(\mathbf{p}),$$

where the integration is over the main cell in **p**-space and summation over the energy bands is implied.

The presence of drift currents would lead to an appreciable role of longitudinal electric fields, with the produced activation oscillations with high (plasma) frequency, generally speaking, not falling in the "window" (1.1) where hydrodynamic motion is possible. As will be shown below, in semiconductors with small decompensation there can exist soft optical branches of oscillations with activation frequencies proportional to the difference of the electron and hole densities.

According to (1.1), for the existence of sound it is necessary to ensure a sufficiently strong "normal" interaction between the electrons. Unlike in metals, where such an interaction is possible by exchange of thermal phonons, in nondegenerate semiconductors frequent normal collisions can be ensured by direct Coulomb interaction at low temperatures and high carrier densities. The Coulomb mean free path

$$l_{ee}^{N} \approx T_e^2/ne^4 \qquad (1.5)$$

then becomes the smallest of the characteristic lengths. The electronic system turns out to be practically isolated from the lattice (see Sec. 4). A similar situation can arise in pure intrinsic semiconductors with suffi-

1)It is assumed that the velocity of sound V and the average velocity of the quasiparticles v are equal. In the general case (see [3]) we have $\omega^{-1}\,\mathrm{Im}\omega \sim \omega\tau^N\,[1 + (v/V)^2]$, $1/\omega\tau^V$.

2)The importance of such a condition was first pointed out by Gurevich and Shklovskiĭ [7] in an analysis of phonon second sound in semiconductors.

ciently narrow forbidden bands, and can also be obtained by illumination at large lifetimes of the non-equilibrium carriers. (These states are similar to those used in semiconductor lasers[8] or investigations of the exciton transition[9].)

As already mentioned, satisfaction of (1.1) presupposes a small probability of collisions with Umklapp. If the electron or hole groups lie on the boundaries of the Brillouin zone, then the Umklapp is just as probable as normal collisions. But by choosing the biased main cell[3] it is possible to ensure (but now with a new classification of states) remoteness of the boundaries from the extrema of the zone and exponential smallness of the Umklapp probability.

2. ELECTRON SOUND IN A SEMICONDUCTOR

In this section it is assumed that the electron and hole densities are equal with sufficient degree of accuracy, so that the drift current and the associated electric field are negligible. As will be shown below, to this end it is necessary to have $|n_- - n_+|/n \ll 1/\omega\tau^{V}$.

Let us consider a picture typical of a semiconductor, when there are several electron and hole groups, particle transitions between which in interelectron collisions have exponentially low probability (of scale $\exp(-\Delta\epsilon/T)$, where $\Delta\epsilon$ is the band energy). At the same time, owing to the pure normal collisions between the groups, there occurs exchange of momentum and energy and a common drift velocity $\mathbf{u}$ and a temperature ϑ are established. The drift function (1.3) depends on the index of the group a: the oscillations of the chemical potentials $\delta\mu^a$ (and in the case of a nonequilibrium situation the chemical potentials μ_0^a themselves) can be different for different groups.

The problem of electron sound in such a situation, for an arbitrary statistics and spectrum, were considered for an electron-phonon system in[3]. Therefore the expressions of interest to us for the velocity and polarization of the wave of the electron sound can be obtained by omitting the phonon mean values from the corresponding formulas (22) and (23) of[3]. The velocity of sound V_0 is given here by

$$V_0 = [(\hat{a}^{-1})_{\varkappa\varkappa} c_{\varkappa\varkappa}]^{1/2}, \tag{2.1}$$

where the index κ denotes the projection on the direction of the wave vector $\mathbf{k}$,

$$a_{ik} = \langle p_i p_k\rangle - \sum_a \frac{\langle p_i\rangle^a \langle p_k\rangle^a}{\langle 1\rangle^a},$$

$$c_{\varkappa\varkappa} = \sum_a \frac{[\langle p_\varkappa v_\varkappa\rangle^a]^2}{\langle 1\rangle^a} + \left(\langle p_\varkappa v_\varkappa \varepsilon'\rangle - \sum_a \frac{\langle\varepsilon'\rangle^a \langle p_\varkappa v_\varkappa\rangle^a}{\langle 1\rangle^a}\right)^2, \tag{2.2}$$

$$\langle\varepsilon'\rangle^a = \tilde{\varepsilon}^a \left(\sum_b \left[\langle\tilde{\varepsilon}^2\rangle^b - \frac{(\langle\tilde{\varepsilon}\rangle^b)^2}{\langle 1\rangle^b}\right]\right)^{-1/2}, \quad \tilde{\varepsilon}^a = \varepsilon - \mu_0^a.$$

We note that the relative placement of the groups in p-space does not affect V_0.

Let us estimate the velocity V_0, assuming the electron and hole spectra to be isotropic and quadratic. We reckon the quantities ϵ, μ and $\mathbf{p}$ in each group from their values in the corresponding band extrema. Then $\epsilon_a = \pm p^2/2m_a$ and $f_0^a = \exp(\pm(\mu_a - \epsilon_a)/T_e$. (Here and below the upper sign pertains to the electrons and the lower to holes, $\mu_a \equiv \mu_0^a$). We introduce for convenience the density $n_F^a = {}^8/_3\pi(2T_e m_a/\hbar^2)^{3/2}$, corresponding to degeneracy for the given group. Accurate to coefficients of the order of unity under the logarithm sign we have

$$\mu_a \cong T_e \ln\frac{n_F^a(T_e)}{n_a}, \tag{2.3}$$

where $n_a > 0$ is the density of the carriers in the group. The estimates for the mean values entering in (2.2) are of the form

$$\begin{gathered}\langle pv\rangle^a \sim \pm n_a, \quad \langle 1\rangle^a \sim n_a/T_e, \quad \langle p^2\rangle^a \sim m_a n_a,\\ \langle p\rangle^a \sim n_a\sqrt{m_a/T_e}, \quad \langle pv\tilde{\varepsilon}\rangle^a \sim (T_e \mp \mu_a) n_a,\\ \langle\tilde{\varepsilon}\rangle^a \sim \pm(T_e \mp \mu_a) n_a/T_e, \quad \langle\tilde{\varepsilon}^2\rangle^a \sim (T_e \mp \mu_a)^2 n_a/T_e,\\ \langle pv\varepsilon'\rangle^a \sim (T_e \mp \mu_a) n_a \left[\sum_b (T_e \mp \mu_b)^2 n_b/T_e\right]^{-1/2}.\end{gathered} \tag{2.4}$$

As a result

$$a_{ik} \sim \sum_a m_a n_a, \quad c_{\varkappa\varkappa} \sim T_e \sum_a n_a \left(1 + \frac{(T_e \mp \mu_a)^2 n_a}{\sum_q (T_e \mp \mu_b)^2 n_b}\right),$$

$$V_0^2 \sim \frac{T_e}{\sum_a m_a n_a} \sum_a n_a \left(1 + \frac{(T_e \mp \mu_a)^2 n_a}{\sum_b (T_e \mp \mu_b)^2 n_b}\right). \tag{2.5}$$

If it is assumed that the numbers of the carriers in the groups are of the same order, then

$$V_0 \sim \sqrt{T_e/m^*}, \tag{2.6}$$

where m^* is the mass of the heavy carriers.

We note that if the carrier velocities differ strongly, then as a result of the diffusion dispersal of the fast particles from the slow wave (for example, $v_e \gg v_h \sim V$) the attenuation of the sound increases and the "window" (1.1) becomes narrower (see footnote 1).

At large differences of the effective masses, interest may attach to a situation in which a common drift is established, owing to the rapid exchange of quasimomentum, but the exchange of energy is difficult, as is the exchange of particles. In the case of isotropic groups, the equations for the drift parameters then take the form

$$\begin{gathered}\delta\mu^a\langle 1\rangle^a + \vartheta^a\langle\tilde{\varepsilon}\rangle^a - u_\varkappa\langle \mathbf{pv}\rangle^a/3V = 0,\\ \delta\mu^a\langle\tilde{\varepsilon}\rangle^a + \vartheta^a\langle\tilde{\varepsilon}^2\rangle^a - u_\varkappa\langle\mathbf{pv}\tilde{\varepsilon}\rangle^a/3V = 0,\\ \sum_a \delta\mu^a\langle\mathbf{pv}\rangle^a + \sum_a \vartheta^a\langle\mathbf{pv}\tilde{\varepsilon}\rangle^a - Vu_\varkappa\langle\mathbf{p}^2\rangle = 0.\end{gathered} \tag{2.7}$$

From this we get

$$\begin{gathered}\vartheta^a = \frac{u_\varkappa}{3V}\frac{\langle\mathbf{pv}\rangle^a\langle\tilde{\varepsilon}\rangle^a - \langle 1\rangle^a\langle\mathbf{pv}\tilde{\varepsilon}\rangle^a}{(\langle\tilde{\varepsilon}\rangle^a)^2 - \langle 1\rangle^a\langle\tilde{\varepsilon}^2\rangle^a},\\ \delta\mu^a = \frac{u_\varkappa}{3V}\frac{\langle\tilde{\varepsilon}\rangle^a\langle\mathbf{pv}\tilde{\varepsilon}\rangle^a - \langle\mathbf{pv}\rangle^a\langle\tilde{\varepsilon}^2\rangle^a}{(\langle\tilde{\varepsilon}\rangle^a)^2 - \langle 1\rangle^a\langle\tilde{\varepsilon}^2\rangle^a},\\ V^2 = \frac{1}{\langle\mathbf{p}^2\rangle}\sum_a \frac{1}{\langle 1\rangle^a}\left\{\frac{(\langle\mathbf{pv}\tilde{\varepsilon}\rangle^a\langle 1\rangle^a - \langle\mathbf{pv}\rangle^a\langle\tilde{\varepsilon}^2\rangle^a)^2}{\langle 1\rangle^a\langle\tilde{\varepsilon}^2\rangle^a - (\langle\tilde{\varepsilon}\rangle^a)^2} + (\langle\mathbf{pv}\rangle^a)^2\right\}.\end{gathered} \tag{2.8}$$

In order of magnitude, the velocity is equal, as before, to the thermal velocity of the heavy carriers.

3. PLASMA OSCILLATIONS AND TRANSVERSE ELECTROMAGNETIC WAVES IN THE HYDRODYNAMIC REGION

We consider now the more general situation when $\frac{1}{3}\langle\mathbf{pv}\rangle \equiv n_- - n_+ \neq 0$ and the drift current also differs from zero. In this case an important role is played by electric fields, and it is necessary to consider a system of bound equations: the kinetic equation (or the hydrodynamic equations that follow from it) and Maxwell's equations. The connection with the electric field, as we shall show, strongly influences the spectrum of the oscillations, leading to the appearance of an activation frequency and to additional damping, which in some cases is quite appreciable.

As before we assume that inside the system of carriers there is established (locally) a common drift and a temperature, with different chemical potentials for the isolated groups. In the kinetic equation we retain the term with the electric field $e\mathbf{E}\cdot\mathbf{v}\partial f_0/\partial\epsilon$. The complete system of equations with allowance for the dissipative terms is of the form

$$\beta_{ik}u_k = \frac{ien}{\omega} V^2 E_k\left(\delta_{ik}\frac{\delta n}{n} + 1_{ik} i\omega\tau^N\right), \tag{3.1}$$

$$\left(k^2\delta_{il} - k_i k_l - \frac{\omega^2}{c^2}\tilde{\varepsilon}_{il}\right)E_l = \frac{4\pi i\omega}{c^2} j_i^d, \tag{3.2}$$

$$j_i^d = neu_l\left(\delta_{il}\frac{\delta n}{n} + 1_{il} i\omega\tau^N\right), \quad \tilde{\varepsilon}_{il} = \varepsilon_{il} + i\omega\tau^N\left(\frac{\omega_p}{\omega}\right)^2 1_{il}. \tag{3.3}$$

We have introduced here the notation

$$\beta_{ik} = V^2 a_{ik} - c_{ik}, \quad V = \omega/k, \quad \delta n = n_- - n_+.$$

The hydrodynamic equations can be derived from the kinetic equation by the Chapman-Enskog method (see, for example,[10]). Eq. (3.1) follows from the complete system of hydrodynamic equations after exclusion of the drift parameters $\delta\mu^a$ and ϑ. For the proof of the vanishing of the coefficient of β_{ik} which is linear in V, see[3]; a_{ik} and the only nonvanishing element $c_{\kappa\kappa}$ of the matrix c_{ik} are given above (see (2.2)).

In Maxwell's equations (3.2) there is separated a "deformation" current j^d, proportional to u. On the other hand, the part of the current proportional to the field is referred to the dielectric constant $\tilde{\epsilon}_{il}$, which, in addition, contains the lattice part of ϵ_{il}.

All the dissipative terms (proportional to $i\omega\tau^N$) are written out in order of magnitude, and the quantities 1_{ik} that enter in them are numerical coefficients of the order of unity. In (3.1) we have left out for simplicity the usual dissipative hydrodynamic terms that lead to the damping (1.2), which will be taken into account directly in the final results.

In the general case, the dispersion equation that follows from (3.1)—(3.3) is quite complicated. But if the direction of propagation of the wave κ coincides with a principal direction of the crystal, then the longitudinal and transverse branches of the oscillations separate.

For the longitudinal wave, the dispersion equation then takes the form

$$\omega^2 - k^2V_0^2 = \omega_\parallel^2\frac{(1+i\omega\tau^N n/\delta n)^2}{1+i\omega\tau^N(\omega_p/\omega)^2}, \tag{3.4}$$

where

$$\omega_\parallel^2 = \frac{4\pi e^2\langle p_\kappa v_\kappa\rangle^2}{a_\parallel\varepsilon_\parallel}, \tag{3.5}$$

V_0 is the velocity of the electron sound at $n_- = n_+$ (see (2.1), $a_\parallel \equiv a_{\kappa\kappa}$, $\epsilon_\parallel \equiv \epsilon_{\kappa\kappa}$, and the coefficients of order of unity were omitted from the terms containing $\omega\tau^N$.

We present an expression for the dispersion of the longitudinal wave in those cases when the absorption is small (ω^{-1} Im $\omega \ll 1$). Under the condition

$$\omega_p^2\tau^N \ll \omega \ll \frac{\delta n}{n}\frac{1}{\tau^N}, \tag{3.6}$$

where ω_p is the plasma frequency of the heavy carriers, the spectrum takes the form

$$\omega^2 = \omega_\parallel^2 + k^2V_0^2 + i\omega\tau^N\omega_p^2(\delta n/n + \omega_\parallel^2/\omega^2). \tag{3.7}$$

We note that the activation frequency $\omega_\parallel$ (3.5) is smaller, in order of magnitude, by a factor $n/\delta n$ than the plasma frequency ω_p:

$$\omega_\parallel \sim \omega_p\delta n/n, \quad \omega_p^2 = 4\pi ne^2/m^*\varepsilon_\parallel.$$

It is important that under hydrodynamic conditions when $n_- \approx n_+$ the soft-activation branch of the oscillations is due not to the low density of the carriers, but to the mutual compensation of the current and the charge density in the common drift. We note that the activation becomes manifest noticeably only if the very stringent condition $\omega_p\tau^N \ll \delta n/n$ is satisfied; this condition follows from (3.6) with $\omega = \omega_\parallel$.

In the remaining possible cases, the spectrum has an activationless form

$$\omega^2 = k^2V_0^2 + i\omega^2\Gamma, \tag{3.8}$$

where the relative damping Γ is equal to

$$\Gamma \approx \omega\tau^N\left(\frac{\delta n}{n} + (\omega_p\tau^N)^2\right)\frac{\omega_p^2}{\omega^2} \text{ for } \frac{1}{\tau^N}\frac{\delta n}{n},\ \omega_p^2\tau^N \ll \omega \ll \frac{1}{\tau^N},$$

$$\Gamma \approx \frac{1}{\omega\tau^N}\left(\frac{\delta n}{n}\right)^2 \quad \text{for } \frac{1}{\tau^N}\left(\frac{\delta n}{n}\right)^2 \ll \omega \ll \frac{1}{\tau^N}\frac{\delta n}{n},\ \omega_p^2\tau^N,$$

$$\Gamma \approx \omega\tau^N \text{ for } \frac{1}{\tau^N}\frac{\delta n}{n} \ll \omega \ll \frac{1}{\tau^N},\ \omega_p^2\tau^N. \tag{3.9}$$

It is necessary to add to the damping in (3.7) and (3.9) the term $1/\omega\tau^V$; the viscous damping $\omega\tau^N$ (see (1.2)) has already been taken into account in these formulas.

From the foregoing results we see that in the case when $\omega_p\tau^N > \delta n/n$ the damping is small in the "window"

$$\frac{1}{\tau^V},\ \frac{1}{\tau^N}\left(\frac{\delta n}{n}\right)^2 \ll \omega \ll \frac{1}{\tau^N}, \tag{3.10}$$

and when $\omega_p\tau^N < \delta n/n$ the damping is small if $\omega > \omega_p\delta n/n$ and the condition (1.1) is satisfied.

As follows from (3.8) and (3.9), in this case the second-sound wave remains practically a purely temperature wave, although the electric fields may make an appreciable contribution to the damping of the wave. To the contrary, the solution (3.7) describes a soft plasma branch of the oscillations, in which the decisive role is played by longitudinal electric fields. This longitudinal branch of the oscillations can be regarded as a continuation of the high-frequency collective branch of Pines and Schrieffer[11] to the low-frequency region.

Let us consider now the transverse case. Its dispersion law with allowance for the hydrodynamic damping (1.2) is

$$\omega^2 = \omega_\perp^2 + c^2k^2/\varepsilon_\perp + i\omega\tau^N\omega_p^2 + (i\omega\tau^N + i/\omega\tau^V)\omega^2, \tag{3.11}$$

where the activation frequency is

$$\omega_\perp^2 = 4\pi e^2\langle p_\kappa v_\kappa\rangle^2/\varepsilon_\perp a_\perp. \tag{3.12}$$

It is assumed that the wave is polarized along one of the principal axes of the two-dimensional tensors $\epsilon_{\alpha\beta}$

and $a_{\alpha\beta}$ (α and β number the axes that are orthogonal to the propagation direction κ), $\epsilon_\perp$ and $a_\perp$ are the corresponding principal values. The condition of the smallness of the damping of the transverse wave is of the form

$$1/\tau^V,\ \omega_p{}^2\tau^N \ll \omega \ll 1/\tau^N. \tag{3.13}$$

As seen from the foregoing results, the activation frequency for the transverse wave is of the same order as for the longitudinal one: $\omega_\perp \sim \omega_\parallel \sim \omega_p\delta n/n$. In both cases, the activation appears significantly if the condition $\omega_p\tau^N \ll \delta n/n$ is satisfied. If $a_\perp\epsilon_\perp > a_\parallel\epsilon_\parallel$, then the longitudinal and transverse branches intersect, and since $V_0 \ll c$, the point of intersection is close to the limiting frequency of the longitudinal wave $\omega_\parallel$ (see (3.4), (3.5), (3.11), and (3.12)).

In propagation along the principal direction, the longitudinal and transverse branches are independent, and the presence of the point of intersection does not affect the two waves in any manner. However, even a small deviation from the principal direction leads to lifting of the degeneracy and to an appreciable mutual influence of the branches. For the case of intersection of the longitudinal branch with one of the transverse branches (twofold degeneracy) we can easily obtain the spectrum

$$\omega^2 \cong \frac{\omega_\parallel{}^2+\omega_\perp{}^2(k)}{2} \pm \left[\left(\frac{\omega_\parallel{}^2-\omega_\perp{}^2(k)}{2}\right)^2 + \frac{a_\perp}{a_\parallel}\left(\frac{a_{\kappa\alpha}}{a_\perp}+\frac{\varepsilon_{\kappa\alpha}}{\varepsilon_\parallel}\right)^2\omega_\parallel{}^2\omega_\perp{}^2(k)\right]^{1/2}. \tag{3.14}$$

Here $\omega_\perp^2(k) = c^2k^2/\epsilon_\perp + \omega_\perp^2$, α is the direction of polarization of the transverse wave, and the nondiagonal elements $a_{\kappa\alpha}$ and $\epsilon_{\kappa\alpha}$ are small because of the smallness of the deviation of the vector κ from the principal direction.

For an arbitrary propagation direction, the results remain qualitatively in force: there exist three waves with activation frequencies of the order of $\omega_p\delta n/n$, and when $\omega \gg \omega_p\delta n/n$ their spectrum goes over into a line spectrum, the phase velocities of two of the "transverse" waves are close to c, and that of one "longitudinal" wave is close to $V_0 \sim \sqrt{T/m^*}$; there is no intersection of the branches.

It is important that at sufficiently small $\delta n/n$ the longitudinal wave attenuates weakly within the limits of the "window" (1.1), whereas in the case of the transverse wave there should be satisfied the additional condition $\omega_p\tau^N \ll 1$ (see (3.10) and (3.13)). On the other hand, the presence in the hydrodynamic region of transverse waves with soft activation frequencies can be revealed by the reflection of the electromagnetic waves from the surface of the semiconductor. Apparently this is easier to effect experimentally than observation of a temperature longitudinal wave of electron sound.

We present in this connection the reflection coefficient of electromagnetic waves (for the simplest case, when the wave propagates along the principal axis normal to the surface of the metal):

$$R = \left|\left(1-\frac{V_\perp}{c}\right)\Big/\left(1+\frac{V_\perp}{c}\right)\right|^2, \tag{3.15}$$

where $V_\perp = k^{-1}\sqrt{k^2c^2/\epsilon_\perp + \omega^2}$ is the phase velocity of the transverse wave (see (3.11)).

4. POSSIBILITY OF HYDRODYNAMIC SITUATION IN SEMICONDUCTORS

We now return to estimates of the frequencies of the collisions of the carriers with one another and with the lattice, in order to ascertain the feasibility in principle of the existence of the "window" (1.1) of interest to us, in which the hydrodynamics is applicable.

We consider first collisions between electrons and lattice vibrations. Scattering by ionized impurities will be assumed to be sufficiently small in view of the smallness of the concentration of impurities in comparison with the concentration of the carriers. (At the time that the scattering cross sections for electron-impurity and electron-electron interactions are of the same order.)

In scattering by acoustic phonons, two limiting situations are possible, depending on the ratio of the thermal momenta of the electron p_T and of the phonon q_T. (The temperatures of the carriers T_e and of the lattice T_L are assumed to be different, satisfying the conditions $ms^2 \ll T_e \ll \Theta_D$, $ms^2 \ll T_L \ll \Theta_D$, where s is the speed of sound, m is the electron mass, and Θ_D is the Debye temperature.)

When $p_T \gg q_T$ ($T_e \gg T_L^2/ms^2$) the main contribution to the relaxation is made by processes of spontaneous emission of phonons with large nonthermal momentum p_T. Their energy is $p_Ts \sim \sqrt{ms^2T_e} \ll T_e$, i.e., this is almost elastic scattering, leading to a collision frequency

$$\nu^{\mathrm{I}}_{\mathrm{acoust}} \approx \frac{\Lambda^2}{\hbar^4}\frac{m^2T_e}{\rho s},$$

where Λ is the deformation potential and ρ is the density. We note that the same mechanism is responsible for relaxation in metals at high temperatures.

In another case typical of semiconductors, $p_T \ll q_T$ ($T_e \ll T_L^2/ms^2$), only phonons with momenta $q \sim p_T$, and accordingly with energies $sq \ll T_L$, take part in the relaxation. (For processes in which thermal phonons take part, the laws of energy and momentum conservation are incompatible.) There are many such phonons ($N \approx T_L/\hbar\omega \gg 1$), and the principal role is played by the processes of their induced emission and absorption. An additional factor $N \approx T_L/\sqrt{ms^2T_e}$ appears in the collision frequency[12]:

$$\nu^{\mathrm{II}}_{\mathrm{acoust}} \approx \frac{\Lambda^2}{\hbar^4}\frac{m^{3/2}T_LT_e{}^{1/2}}{\rho s^2}.$$

The general expression valid in both limiting cases is

$$\nu_{\mathrm{acoust}} \approx \frac{\Lambda^2}{\hbar^4}\frac{m^{3/2}T_LT_e{}^{1/2}}{\rho s^2}\frac{2}{z^4}\int\limits_0 dy\, y^4 \operatorname{ctg}\frac{y}{2}, \quad z = \sqrt{8\frac{T_ems^2}{T_L{}^2}}.$$

The electron mean free path is equal to

$$l_{\mathrm{acoust}} \approx l_0\begin{cases}\sqrt{ms^2/T_e}, & T_e \gg T_L{}^2/ms^2,\\ ms^2/T_L, & T_e \ll T_L{}^2/ms^2,\end{cases}$$

where $l_0 \approx \hbar^4\rho/m^3\Lambda^2 \sim 10^{-3}$ cm for $\Lambda \sim 2$ eV, $m \sim 10^{-27}$ g, and $\rho \sim 5$ g/cm^3.

The interaction with optical phonons with activation energy $\Omega_0 \gg T_L$, T_e leads to the following reciprocal energy relaxation time[12]:

$$\nu_{\mathrm{opt}} \approx \frac{\Lambda^2g^2m^{3/2}T_e{}^{1/2}}{\hbar^2\rho\Omega_0}\left\{e^{-\Omega_0/T_L}\sqrt{\frac{\Omega_0}{T_e}} + e^{-\Omega_0/T_e}\left(\frac{\Omega_0}{T_e}\right)^{5/2}\right\}.$$

The expression for the momentum relaxation frequency differs only in the degree of the factor Ω_0/T_e of $\exp(-\Omega_0/T_e)$, namely 3/2 in lieu of 5/2.

The carrier recombination time in semiconductors is usually quite large[13]. We shall therefore not discuss the different recombination mechanisms.

A comparison of the presented formulas with the expression for the interelectron mean free path (1.5) leads to the conclusion that at low temperatures of the lattice and of the electrons, the electron-lattice mean free paths can be made much larger than the electron-electron ones, provided the electron concentration is high. For example, at $T_e \sim T_L \sim 1°K$ and $n \sim 10^{14}$ we have $l_{acoust} \sim 10^{-4}$ and $l_{ee} \sim 10^{-6}$ cm.

The use of a pure Coulomb cross section in the estimate of the electron-electron interactions is justified by the fact that at densities that are sufficiently large but smaller than $n_F(T)$, the Coulomb diameter $\rho_\perp = e^2/T_e$, the Debye length $(T_e/4\pi ne^2)^{1/2}$, and the thermal de Broglie wavelength of the carriers $\lambda_T = \hbar/\sqrt{mT_e}$ do not differ very strongly from one another, and consequently neither the Rayleigh nor the diffraction factors should greatly change the cross section in the region of interest to us. The Coulomb logarithm is also small, and it can be assumed that it is compensated for by the factor due to the dielectric constant of the lattice.

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