ON THE THEORY OF ULTRASONIC ABSORPTION AND AMPLIFICATION IN SEMICONDUCTORS IN A MAGNETIC FIELD

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A theory of electron absorption (EA) and amplification (EAM) of ultrasound in semiconductors with an arbitrary degree of degeneracy in a transverse magnetic field is developed. The case is considered of sufficiently low frequencies, when interaction between the electrons and sound can be described "classically" in terms of variable fields created by the wave, and when the hydrodynamic expression is used for the electron current density. Expressions for the EA and EAM coefficients, and also the corresponding corrections to the sound velocity, are obtained for the case in which the sound moves in the direction of the drawing electric field as well as in the Hall direction. It is demonstrated that the condition for sound amplification in degenerate semiconductors has the form $(\mathbf{v}_d^* \cdot \mathbf{q})/q > v_s$ in all cases, where $\mathbf{q}$ and v_s are respectively the wave vector and the sound velocity, and $\mathbf{v}_d^*$ is a certain effective drift velocity, which may differ from the mean electron drift $\mathbf{v}_d$ in magnitude as well as in direction. In particular, in a quantized magnetic field, $\mathbf{v}_d^*$ may be antiparallel to $\mathbf{v}_d$, i.e., sound amplification by an oppositely moving electron beam should be possible. It is also shown that the magnitude of $\mathbf{v}_d^*$ as a function of the magnetic field strength may undergo "giant" quantum oscillations, even under conditions when the ordinary Shubnikov electric conductivity oscillations are small. A physical explanation of the predicted effects is presented.

1. THE effect of a magnetic field on the EA and EAM of ultrasound in semiconductors in the "hydrodynamic" case $ql \ll 1$ (l is the mean free path of the electron) has been considered in a number of researches. In [1–3], the EA coefficient of ultrasound has been found for a transverse unquantized magnetic field in semiconductors with an arbitrary degree of degeneracy and an arbitrary dependence of the relaxation time of the momentum of the electrons $\tau(\epsilon)$ on their energy ϵ. In [4–7], the EA and EAM coefficients are calculated for ultrasound in a transverse unquantized magnetic field in semiconductors under the assumption that the relaxation time of the momentum of electrons τ does not depend on their energy. The EA of ultrasound in a Hall current was considered in [8] for the same conditions, and the EA of ultrasound in both drift and Hall current in a layered medium in [9]. The fundamental results of [4–7] (and also [9]) is that the Maxwell relaxation time increases significantly in a strong magnetic field, when $\omega_H\tau \gg 1$ (ω_H is the cyclotron frequency), while the drift velocity of the electrons (and consequently the threshold of sound amplification) does not depend on the magnetic field. This leads to the result that a smaller drift velocity is needed to obtain optimal amplification than in the absence of a magnetic field. In [10] the results of [4–8] were generalized to the case of an arbitrary dependence $\tau(\epsilon)$.

However, under real experimental conditions, when one can observe a marked decrease in the drift velocity for optimal amplification[11, 12] (InSb samples at low temperatures in magnetic fields of the order of tens of kOe) the quantizing of the energy of the electrons in the magnetic field can be substantial. The theory of EA of ultrasound in a quantizing magnetic field in semiconductors, which is also valid in the hydrodynamic case $ql \ll 1$, was developed in [13]. However, only the case of a longitudinal magnetic field is considered, and specific calculations are carried out for $ql \gg 1$, $\omega\tau \gg 1$ (ω is the frequency of the wave) applicable to "gigantic" quantum oscillations of Gurevich, Skobov, and Firsov (GSF).[14]

In the present research, we consider the EA of ultrasound in a transverse, generally quantized magnetic field both by the drift and the Hall currents and by the "partial" internal currents, which flow in the Hall direction with the Hall contacts disconnected. A physical explanation will be given for a number of interesting features of sound amplification associated with the $\tau(\epsilon)$ dependence, and also with the presence of the quantizing magnetic field.

2. In the present work, we shall assume the sound wavelength to be sufficiently large in comparison with the de Broglie wavelength λ_e of the electron ($q\lambda_e \ll 1$), so that the interaction of the sound with the electrons can be described "classically," by representing the sound wave as some external field.[1)] Also, only the hydrodynamic case $ql \ll 1$, $\omega\tau \ll 1$ will be considered, when the observation of the cyclotron and geometric resonances in sound absorption (see for example, the review[15]) and also of the gigantic oscillations of GSF[14] is impossible. Moreover, we shall completely neglect the heating of the electron gas by the sound wave, assuming the symmetric part of the distribution function (or the diagonal part of the density matrix) to be a Fermi distribution with temperature T equal to the lat-

[1)] In qunatizing magnetic fields, when the region of localization of the electron becomes smaller than λ_e, this approximation will be even better satisfied.

tice temperature[2)] and with a chemical potential depending on the time and coordinates with the frequency of the wave (the latter is connected with the periodic change in the electron concentration in the field of the wave, see [17]). Under these conditions, a phenomenological expression can be used for the electron current density. This expression contains the static electrical conductivity and the diffusion coefficient corresponding to the local electron concentration.

For simplicity, the direction of the wave vector $\mathbf{q}$ is so chosen that the sound propagation is described as a one-dimensional problem. We direct the x axis of our system of coordinates along $\mathbf{q}$, and the z axis along the magnetic field $\mathbf{H}$, which is assumed to be perpendicular to $\mathbf{q}$. With reference to the pulling field $\mathbf{E}_0$ (the sample is connected to the voltage source), we shall consider two situations: (a) $\mathbf{E}_0 \parallel x$ and (b) $\mathbf{E}_0 \parallel y$. In the Hall direction, the sample can either be open circuited (the corresponding quantities will be identified below with the subscript 0) or short circuited (subscript s) in both cases.

In accord with what has been said above, the set of fundamental equations has the form (see [3, 17])

$$\rho\frac{\partial^2 u}{dt^2}=c\frac{\partial^2 u}{\partial x^2}+\beta\frac{\partial E_x}{\partial x}+\Lambda\frac{\partial n}{\partial x}, \tag{1}$$

$$\frac{\partial E_x}{\partial x}=\frac{4\pi e}{\varepsilon_0}(n-n_0)+\frac{4\pi\beta}{\varepsilon_0}\frac{\partial^2 u}{\partial x^2}, \tag{2}$$

$$e\frac{\partial n}{\partial t}+\frac{\partial j_x}{\partial x}=0, \tag{3}$$

$$j_x=\sigma_{xx}\left(E_x-\frac{\Lambda}{e}\frac{\partial^2 u}{\partial x^2}\right)+\sigma_{xy}E_y-eD_{xx}\frac{\partial n}{\partial x}, \tag{4}$$

$$j_y=-\sigma_{xy}\left(E_x-\frac{\Lambda}{e}\frac{\partial^2 u}{\partial x^2}\right)+\sigma_{xx}E_y+eD_{xy}\frac{\partial n}{\partial x}. \tag{5}$$

Here ρ, c, β, ϵ_0 and Λ are respectively the density, elastic modulus, piezomodulus, dielectric permittivity and the deformation potential constant for the crystal under study, u is the mechanical displacement in the wave, E_x and E_y are the components of the total local electric field, $e < 0$, n and n_0 are the charge, local and equilibrium concentrations of electrons, respectively, j_x and j_y the components of the electric current density, and σ_{xx}, σ_{xy}, D_{xx}, D_{xy} the corresponding components of the electrical conductivity and diffusion coefficient tensors, which depend on H. In the case of degenerate semiconductors, and also in quantizing magnetic fields, the diffusion coefficient generally depends on n, while the dependence $\sigma(n)$ does not reduce to simple proportionality (see [18]).

Let us first consider the case (a) ($\mathbf{E}_0 \parallel x$, sound amplification by the drift current of the electrons). For the reasons already discussed in [4-9], we shall neglect the transverse field and current variables (including the Hall current). In accord with this, we linearize the problem by setting $n = n_0 + n_\sim$, $E_x = E_0 + E_\sim$, $E_y = E_H$, where $n_\sim$ and $E_\sim$ are quantities of first order of smallness in the amplitude of the sound wave, and E_H is the Hall field.

[2)]This approximation for InSb at low temperatures, under the conditions of sound amplification, is less substantiated (see [16]). However, in the case of most interest to us in the present research, normly, of a strongly degenerate semiconductor, it is insignificant.

In zeroth order, we find the Hall field E_H. It is equal to zero if the sample is short-circuited in the Hall direction, and equal to $E_0\sigma_{xy}^{(0)}/\sigma_{xx}^{(0)}$ if the sample is open-circuited ($\sigma_{xx}^{(0)} \equiv \sigma_{xx}(n_0)$, $\sigma_{xy}^{(0)} \equiv \sigma_{xy}(n_0)$ and so forth). In the first order in the sound amplitude, we find the dispersion equation of the wave, whence we obtain the following expression for the coefficient of EA (EAM) of sound and the corrections to its velocity:

$$\alpha_e=(\eta+\chi)q\frac{\omega\tau_M(1-\mathbf{qv}_d^*/qv_s)}{\omega^2\tau_M^2(1-\mathbf{qv}_d^*/qv_s)^2+(1+q^2r_D^2)^2}, \tag{6}$$

$$\frac{\Delta v_s}{v_s}=\frac{1}{2}\frac{\eta[\omega^2\tau_M^2(1-\mathbf{qv}_d^*/qv_s)^2+q^2r_D^2(1+q^2r_D^2)]-\chi(1+q^2r_D^2)}{\omega^2\tau_M^2(1-\mathbf{qv}_d^*/qv_s)^2+(1+q^2r_D^2)^2} \tag{7}$$

Here η and χ are respectively the constants of electromechanical coupling through the piezoeffect and the deformation potential, $\tau_M \equiv \epsilon_0/4\pi\sigma_{xx}^{(0)}$ and $r_D = \{\epsilon_0 D_{xx}^{(0)}/4\pi\sigma_{xx}^{(0)}\}^{1/2}$ are the Maxwell relaxation time and the Debye radius in the magnetic field, v_s is the sound velocity and $\mathbf{v}_d^*$ some effective drift velocity. In the given case ($\mathbf{E}_0 \parallel x$)

$$\mathbf{v}_d^*=\mathbf{v}_d r_a, \tag{8}$$

where $\mathbf{v}_d$ is the observed mean value of the electron drift velocity, defined as $\mathbf{v}_d = \mathbf{j}^{(0)}/en_0$, and r_a is a correction factor (see [18]). In the case of shorted Hall contacts,

$$r_a\equiv r_{as}=\frac{d\ln\sigma_{xx}^{(0)}}{d\ln n_0}, \tag{9}$$

and in the case of an open circuit

$$r_a\equiv r_{a0}=\frac{1}{2}\frac{d\ln(\sigma_{xx}^{(0)2}+\sigma_{xy}^{(0)2})}{d\ln n_0}. \tag{10}$$

The scalar product $\mathbf{v}_d\cdot\mathbf{q}$ is positive, when $\mathbf{E}_0$ is antiparallel to $\mathbf{q}$ ($e < 0$) and changes sign upon change in sign of $\mathbf{E}_0$.

In the case (b) ($\mathbf{E}_0 \parallel y$), proceeding in similar fashion, we also obtain expressions (6) and (7) for α_e and $\Delta v_s/v_s$ where, however, the velocity $\mathbf{v}_d^*$ has a different form:

$$\mathbf{v}_d^*=[\mathbf{v}_d\mathbf{h}]r_b. \tag{11*}$$

Here $\mathbf{h}$ is a unit vector in the direction of the magnetic field and the correction factor r_b is equal to

$$r_b\equiv r_{bs}=\frac{\sigma_{xy}^{(0)}}{\sigma_{xx}^{(0)}}\frac{d\ln\sigma_{xy}^{(0)}}{d\ln n_0}, \tag{12}$$

in the case of shorted Hall contacts, and

$$r_b\equiv r_{b0}=\frac{\sigma_{xx}^{(0)}\sigma_{xy}^{(0)}}{\sigma_{xx}^{(0)2}+\sigma_{xy}^{(0)2}}\frac{d\ln(\sigma_{xy}^{(0)}/\sigma_{xx}^{(0)})}{d\ln n_0}. \tag{13}$$

in the case of open circuit. As is seen from (6), the criterion for sound amplification (change of sign of α_e) has the form

$$\mathbf{v}_d^*\mathbf{q}/q>v_s. \tag{14}$$

Thus we have succeeded in expressing the sought-after quantities (α_e and $\Delta v_s/v_s$) in terms of the static characteristics of the crystal. We again emphasize that, since the explicit form of the components of the tensors of electrical conductivity and the diffusion coefficients, and also the coupling between them, has nowhere been used, Eqs. (6)–(14) are valid for any powerful and, in

*$[\mathbf{v}_d\mathbf{h}] \equiv \mathbf{v}_d \times \mathbf{h}$.

particular, quantizing magnetic fields. Finally, we note that the acousto-electric effects considered form a class of kinetic phenomena which are determined not only by the kinetic coefficients, but also by their derivatives with respect to the concentration of the electrons (i.e., with respect to the Fermi energy in the degenerate case). As we see, this leads to a number of interesting features of these effects in degenerate semiconductors, for example, to a significant increase in the amplitude of the Shubnikov oscillations of the corresponding quantities in the quantizing field, to the possibility of sound amplification in an opposing stream of electrons and so on.

3. We first consider the case of classical (nonquantizing) magnetic fields. In this case, as is well known,

$$\sigma_{xx}^{(0)}=\frac{e^2n_0}{m}\left\langle\frac{\tau}{1+\omega_H^2\tau^2}\right\rangle,\quad \sigma_{xy}^{(0)}=\frac{e^2n_0}{m}\left\langle\frac{\omega_H\tau^2}{1+\omega_H^2\tau^2}\right\rangle, \qquad (15)$$

where m is the effective mass of the electron, ω_H the cyclotron frequency, and the symbol $\langle\dots\rangle$ denotes the usual averaging in the theory of kinetic coefficients. For nondegenerate semiconductor $r_{as} = r_{a0} = 1$, $r_{bs} = \sigma_{xy}^{(0)}/\sigma_{xx}^{(0)}$, $r_{b0} = 0$, and we quickly obtain all the results of [4-8, 10].

In the case of a degenerate semiconductor in the presence of the $\tau(\epsilon)$ dependence, as is easy to see from (9), (10), (12), (13), and (15), the correction factors r_{as} and r_{a0} generally differ from unity, and r_{b0} is not equal to zero. In the following, we limit ourselves to the consideration of the cases (as) – $E_0 \parallel x$, the Hall contacts shorted, and (b0) – $E_0 \parallel y$, the Hall contacts open, when the effects considered below are most clearly evident. For these cases we get, for $\tau(\epsilon) \sim \epsilon^{\nu}$,

$$r_{a0}=1+\frac{2}{3}\nu\frac{1-\omega_H^2\tau^2}{1+\omega_H^2\tau^2}, \qquad (16)$$

$$r_{b0}=\frac{2}{3}\nu\frac{\omega_H\tau}{1+\omega_H^2\tau^2}$$

(for H = 0, Eq. (16) gives the result of [18]).

Thus, the effective drift velocity v_d^*, which enters into the criterion for sound amplification (14), differs essentially both in direction and in absolute value, from the observed mean drift velocity of the electrons v_d. For example, in the case (b0), the electron drift is generally absent in the direction of propagation of the sound (the drift velocity $v_d \perp q$). Nevertheless, electron amplification of the sound can take place if the relaxation time of the momentum of the electrons depends on the energy ($\nu \neq 0$). Here, depending on the scattering mechanism (different sign of ν and, consequently of r_{b0}), the sound can be amplified, being propagated in the direction of the Hall field E_H or antiparallel to it.[3)]

In the case (as) for $\omega_H\tau \gg 1$ and impurity scattering ($\nu = 3/2$), $r_{as} \to 0$, i.e., amplification of sound becomes impossible. In principle, in the case (as), the factor r_{as} can be negative upon suitable selection of the scattering mechanism (i.e., of the value and sign of ν). Here it is necessary that $d\sigma^{(0)}/dn_0 < 0$, i.e., that the local electrical conductivity fall off at small changes of the concentration with increase in the concentration (for H = 0, it is required that $\nu < -3/2$ here). In accord with (14) and Eqs. (8), (9), this would mean that for sound amplification, it is necessary to reverse the direction of the electron drift; that is, we would have the interesting effect of sound amplification by an opposing stream of electrons. However, for the well-known mechanisms of scattering, this does not result—for them $r_{as} \geq 0$. It is shown that the possibility of sound amplification by an opposing stream of electrons appears in the case of quantizing magnetic fields (see below).

4. In the case of quantizing fields, we use the results of the classical work of Adams and Holstein,[20] in which the components of the electrical conductivity tensor are expressed in terms of the density of states in the magnetic field and the corresponding constants, which characterize the scattering mechanism.[4)] These expressions were obtained in the Born approximation upon satisfaction of the inequality $\zeta \gg T$ (for a degenerate semiconductor, ζ is the Fermi level, T the temperature in energy units) and $\hbar\omega_H \gg T$, $\omega_H\tau \gg 1$ (the Landau quantization condition).

1) The quasiclassical case $\hbar\omega_H \ll \zeta$ (below the Fermi level there are many Landau zones). According to [20]

$$\sigma_{xy}^{(0)}=e^2n_0/m\omega_H \qquad (18)$$

and does not depend on the scattering mechanism. For $\sigma_{xx}^{(0)}$ an explicit expression is obtained in [20] only for the scattering from acoustic phonons in the interaction through the deformation potential. Limiting ourselves to this case, we have

$$\sigma_{xx}^{(0)}=\frac{\sigma_{xy}^{(0)}}{\omega_H\tau}\left[1+\frac{5}{2}\frac{g_1}{g_0}+\frac{3}{2}\left(\frac{g_1}{g_0}\right)^2\right], \qquad (19)$$

where g_0 and g_1 are the "smooth" and "oscillating" parts of the density of states $g = g_0 + g_1$, which, in the case under consideration ($\hbar\omega_H \ll \zeta$), can be written approximately in the form

$$g_0=\frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\zeta^{1/2}, \qquad (20)$$

$$\frac{g_1}{g_0}=\left(\frac{\hbar\omega_H}{\zeta}\right)^{1/2}\left(\frac{1}{2\sqrt{\delta}}-\sqrt{\delta+1}\right).$$

Here $\delta = \zeta/\hbar\omega_H - (N_{max} + 1/2)$ and N_{max} is the number of saturated Landau zones, the bottom of which is still found below the Fermi level. Following [20], we shall assume that $\delta_{min} \leq \delta \leq 1$, $\delta_{min} = (\omega_H\tau_c)^{-1} \ll 1$, where τ_c is some characteristic time of the order of (or smaller than) the relaxation time of the momentum of the electrons.

[3)] A similar result was obtained in [19] for the case $ql \gg 1$ (for a nondegenerate semiconductor).

[4)] The main shortcoming of [20] is the appearance of non-physical divergences in the expressions obtained for the intersection of the Landau levels with the Fermi level. In this same work, however, it is shown that these divergences can be avoided by assigning a finite width to the bottom of the Landau zone, say $\sim\hbar/\tau$. In subsequent works on this theme (see, for example, [21,22]), the divergences are removed by various self-consistent methods. However, the expressions obtained in these works for the components of the electrical conductivity tensor are very cumbersome, while the formulas of [20] are comparatively simple; furthermore, with account of the finite width of the levels, the basic dependences are described with qualitative correctness. Therefore, we limit ourselves in the present work to the use of the expressions of Adams and Holstein [20] keeping in mind the qualitative nature of the description of the phenomena.

All the sought-after quantities—r, v_d^* and so on—are considered as functions of the magnetic field in the limits of a single oscillation $\delta_{min} \le \delta \le 1$, because all the oscillations are identical in approximations (20), (21). In order to emphasize the features of the considered phenomena, in quantizing magnetic fields, we consider the case $(\hbar\omega_H/\zeta)^{1/2}\delta_{min}^{-1/2} \ll 1$, when the ordinary Shubnikov oscillations of the kinetic coefficients themselves can be neglected.

With the help of Eqs. (9), (13), (18)–(21), we obtain the result that the factors r_{as} and r_{b0} change within the following limits over the range of a single period:

$$-\frac{5}{12}\left(\frac{\zeta}{\hbar\omega_H}\right)^{1/2}\cdot\delta_{min}^{-1/2} \leqslant r_{as} \leqslant -\frac{5}{6}\left(\frac{\zeta}{\hbar\omega_H}\right)^{1/2}, \quad (22)$$

$$\frac{5}{12}\left(\frac{\zeta}{\hbar\omega_H}\right)^{1/2}\frac{\delta_{min}^{-3/2}}{\omega_H\tau} \leqslant r_{b0} \leqslant \frac{5}{6}\left(\frac{\zeta}{\hbar\omega_H}\right)^{1/2}\frac{1}{\omega_H\tau},$$

i.e., $\delta^{-1/2} \gg 1$. Thus r_{as}, r_{b0} and the effective drift velocity v_d^* (and, consequently, the threshold electric field for sound amplification), can undergo "gigantic" quantum oscillations with increase in the magnetic field. Here, as has already been said, the quantum oscillations of the electrical conductivity in our case, and consequently, of the drift velocities, are quite imperceptible. Physically, such a difference in the amplitude of the oscillations of the velocities v_d and v_d^* is associated with the fact that the former is determined by the electrical conductivity itself, $\sigma_{xx}^{(0)}$, and the latter by its derivative with respect to the concentration (i.e., with respect to the Fermi energy). Inasmuch as the density of states depends strongly on the energy near the bottom of the Landau zone, the derivative $(d/d\zeta)(g_1/g_0)$ in a degenerate semiconductor can be large in absolute value, which also explains (see (19)) the "gigantic" oscillations of $d\sigma_{xx}^{(0)}/dn_0$ and, consequently, of the velocity v_d^* (i.e., the sound amplification threshold).

Correspondingly, as is easy to see from (6) and (7), the sound absorption coefficient α_e and the electron correction to its velocity $\Delta v_s/v_s$ in the presence of a constant electric field $\mathbf{E}_0$ will also undergo "gigantic" quantum oscillations.[5)]

Furthermore, it is seen from (19)–(21) that the derivative $d\sigma_{xx}^{(0)}/dn_0 \ll 0$ over the range of the entire oscillation $\delta_{min} \le \delta \le 1$. This leads to the result that the correction factor r_{as} is everywhere negative.[6)] i.e., there is possible sound amplification by the opposing stream of electrons (sign $(\mathbf{v}_s\cdot\mathbf{q}) < 0$).

Finally, as is seen from (8), (11) and (22), (23), for $\delta = \delta_{min}$ in the case (as) and especially in the case (b0), the effective drift velocity v_d^* is significantly larger in absolute value, not only than the observed drift velocity of the electrons, but also than the drift velocity which would occur for the given value of the electric field $\mathbf{E}_0$ in the absence of a magnetic field. Thus, there is an absolute lowering of the sound amplification threshold for definite values of the magnetic field. This fact can facilitate the observation of the noted oscillations of the amplification threshold over the oscillations of the sound intensity at the output of the crystal, if the field E_0 is the slightest bit below threshold.

2) Essentially quantum case $\hbar\omega_H > \frac{2}{3}\zeta$ (all the electrons are in the first Landau zone). For $\sigma_{xy}^{(0)}$, in accord with [20], we have the expression (18) as before. The very cumbersome and unwieldy expressions for $\sigma_{xx}^{(0)}$ for the various scattering mechanisms are given in [20]. It is possible to write them in the form of the following single formula:

$$\sigma_{xx}^{(0)} = \tfrac{1}{2}(\tfrac{3}{2})^{2\nu/3}\sigma_{xy}^{(0)}(\omega_H\tau)_q^{-1}F(\delta)(H/H_q)^{1/2-\nu}, \quad (24)$$

where H_q is the value of the magnetic field, above which (below the Fermi level) there remains a single Landau zone (i.e., for which $\hbar\omega_H = \frac{2}{3}\zeta$), $(\omega_H\tau)_q = \omega_H\tau|_{H=H_q}$; $\delta = (H_q/H)^3$ has the same meaning as in the classical case (for $N_{max} = 0$), and $F(\delta)$ is some positive, slowing changing function in magnitude of the order of unity. The explicit form of the expression $F(\delta)$ depends on the scattering mechanism and can easily be found from a comparison of Eq. (24) with the corresponding formulas (4.8), (5.4) of [20] and Table 3 from this same work.

By using (18) and (24), and the general expressions (9), (10) and (12), (13), it is easy to get expressions for r and v_d for the different cases. For example, for the case (as), we have

$$r_{as} = -2\left(1 - \frac{d\ln F(\delta)}{d\ln\delta}\right), \quad (25)$$

for all the scattering mechanisms considered in [20], it is seen that $|d \ln F(\delta)/d \ln \delta| \ll 1$. Thus, it is seen that even in this quantum limit, sound amplification by an opposing electron stream is possible.

Sound amplification for sound propagating in the Hall direction is also possible when the current in this direction is absent (case (b0)). The amplification condition in this case is

$$v_H(1 - r_{as}) > v_s, \quad (26)$$

where $v_H = cE_0/H$ is the Hall velocity.

Thus, in contrast with the case of classical magnetic fields (see (17)), the sound amplification condition can be satisfied here even when the "bare" (in the absence of a magnetic field) relaxation time of the momentum of the electrons does not depend on the energy (for $\nu = 0$).

5. As was shown above, the sound amplification criterion (10) differs essentially from the "Cerenkov" condition $\mathbf{v}_d\cdot\mathbf{q}/q > v_s$ (or its generalization $f_0\mathbf{v}_d\cdot\mathbf{q}/q > v_s$, where f_0 is some factor that reflects the effect of trapping[23]). For an explanation of this, we consider the motion of electron bunches under the action of electric and magnetic fields. In case (a), we take the simplest situation, in which the magnetic field is absent or the Hall contacts are closed—then the "ohmic" part of the current in the direction of the x axis is $j_x = \sigma_{xx}(n)E_x$. We further assume that, as the result of fluctuations or any sort of action within the range Δx of an arbitrary x_0, a region of slightly increased electronic concentration instantly appears—the "electron bunch," $n(x_0) = n_0 + \Delta n$, $|\Delta n| \ll n_0$. At the first instant

[5)] Of, course, it should be kept in mind that the theory just developed is valid only if the inequalities $|\alpha_e| \ll q$, $|\Delta v_s| \ll v_s$ are satisfied, since we have restricted ourselves in the solution of the dispersion equation to the first terms of the expansion in terms of the electromechanical coupling constants η and χ.

[6)] In the region of smearing out of the bottom of the Landau zone $\delta < \delta_{min}$, not considered by us, the derivative $d\sigma_{xx}^{(0)}/dn_0^0$ and r_{as} naturally change sign.

of time, the change in the electric current at the point x_0 will be

$$\Delta j = \Delta\sigma_{xx}E_0 = \frac{\partial\sigma_{xx}^{(0)}}{\partial n_0}\Delta n E_0 = e\Delta n v_d r_{as}. \quad (27)$$

If $\partial\sigma_{xx}^{(0)}/\partial n_0 > 0$, the current in the considered range Δx increases, while outside this range, it remains as before. This leads to the result that, on the boundary of the region Δx, facing in the direction of the electron drift, the electrons begin to attach themselves, and to flow away from the opposite boundary. Thus, the maximum electron concentration (the electron bunch) under the action of the electric field E_0 will have a tendency to move in the direction of the electron drift. If now $\partial\sigma_{xx}^{(0)}/\partial n_0 < 0$, then it is easy to show by a similar argument that the electrons will attach themselves on the boundary of the region Δx facing in a direction opposite to the direction of the electron drift, and the electron bunch will move counter to the electron flow.

It is seen from (27) that the translation velocity of the bunch is initially $v_d r_{as} = v_d$ and only in the special case $\tau(\epsilon) = \text{const}$, ($\nu = 0$, $r_{as} = 1$) does it coincide with the mean drift velocity of the electrons, v_d. Formally, all that has been pointed out follows directly from the equation of current continuity: if the effect of diffusion is neglected in that equation (the diffusion dissipates the bunch uniformly on all sides) and if the front of the bunch is assumed to be sufficiently steep in the sense of satisfying the inequality $(v_d\tau_M/\Delta L) \gg 1$, where L is the thickness of the "front" of the bunch, then Δn is a function only of the "Riemann" coordinate $(x - v_d^* t)$. Since both the EA and the EAM of ultrasound for $ql \ll 1$ are associated with the formation of a stimulated space charge wave and with a phase shift of the maxima of the electron density relative to the corresponding minima of the electron potential energy in the wave, it is quite natural that EA goes over into EAM when $\mathbf{v}_d \cdot \mathbf{q}/q > v_s$, i.e., when the translation velocity of the electron bunches under the action of the electric field E_0 alone becomes comparable with the sound velocity. The mean drift velocity of the electrons in this case can differ considerably from the sound velocity. In particular, for $r_{as} < 0$ (the case of a quantizing magnetic field), upon satisfaction of the amplification condition, it is necessary that the electrons drift toward the sound.

In case (b0), we consider the motion of the electron bunch along the x axis. This motion is determined by the play of two currents that compensate one another in equilibrium—the "Lorentz" current (proportional to $\sigma_{xy}^{(0)}$) and the "ohmic" current, which is produced by the Hall field E_H (proportional to $\sigma_{xx}^{(0)}$). If the Lorentz current grows more rapidly than the ohmic one ($\sigma_{xy}^{(0)}$ as a function of n_0 increases more rapidly than $\sigma_{xx}^{(0)}$, i.e., $(\partial/\partial n_0)(\sigma_{xy}^{(0)}/\sigma_{xx}^{(0)}) > 0$) upon increase in the concentration, then an analysis similar to that carried out above shows that the bunch will move in the direction of the Lorentz force.

If $(\partial/\partial n_0)(\sigma_{xy}^{(0)}/\sigma_{xx}^{(0)}) < 0$, then the bunch will move in the opposite direction. It is easy to show that the velocity of motion of the bunch at the initial instant in the given case is $\mathbf{v}_d^* = [\mathbf{v}_d\mathbf{h}]r_{b0}$, i.e., it also coincides with the effective drift velocity, which enters into the amplification condition. The sign of the derivative $(\partial/\partial n_0) \times (\sigma_{xy}^{(0)}/\sigma_{xx}^{(0)})$, and consequently, r_{b0}, is determined by the scattering mechanism. Thus, depending on the type of scattering, the sound will be amplified when traveling either in the direction of the Lorentz force or in the opposite direction. This example, in which the drift of the electrons in the direction of the sound propagation is generally absent, especially clearly illustrates the fact that sound amplification for $ql \ll 1$ is connected not with the drift of individual electrons but with the motion of the electron bunches. Consequently, the sound amplification could be obtained in full if the bunches were forced to move with supersonic velocity, not with the help of the electric field E_0, but in any fashion.

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