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SCATTERING OF PARTICLES BY MAGNETIC INHOMOGENEITIES IN A STRONG MAGNETIC FIELD

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An equation describing the scattering of particles by random magnetic pulsations in a strong magnetic field is derived. Approximate solutions to this equation are found for a number of cases including that of a non-uniform field. The time intervals and distances over which isotropization of the particle beam occurs are estimated. A consequence of the scattering anisotropy is that the time for the scattering of particles through an angle $\vartheta \sim \pi$ may considerably exceed the time for scattering through an angle $\vartheta \sim 1$. The scattering of cosmic ray particles in interplanetary space is considered on the basis of the obtained results.

1. FORMULATION OF THE PROBLEM

INVESTIGATION into the multiple scattering of particles in a magnetic field with random inhomogeneities is important for the theory of turbulent plasmas, for the problem of cosmic ray propagation in interplanetary space and for many other phenomena. This problem has been solved in different approximations by Dolginov and one of the authors of this paper in[1,2], by Tverskoĭ in[3,4] and by Vernov and his co-workers, using a numerical method, in[5].

In the present report we consider the scattering of particles in a mildly turbulent plasma located in a strong magnetic field. The energy of the scattered particles is large compared to the energy of the plasma particles and their density is small. We assume that the spectrum of the magnetic pulsations is known and that it is statistically isotropic. The higher order cyclotron harmonics, which exert a strong influence on the scattering when the angle between the particle momentum and the magnetic field is close to $\pi/2$, are taken into account. Scattering at such angles may prove to be extremely small if the magnetic turbulence spectrum decreases with increase in the wave number. As a result, the distance over which the scattering of the particle beam through an angle $\vartheta \sim \pi$ (isotropization) occurs, may be made considerably larger than the distance over which the particles are scattered through an angle $\vartheta \sim 1$. The slow variation of the large-scale magnetic field in space may increase this distance still further.

We determine in this paper the form of the particle distribution function for small angles and for angles close to $\pi/2$, and estimate the distance over which isotropization occurs. The results of the computation are used to explain the observed scattering of cosmic ray particles emanating from the sun.

2. THE KINETIC EQUATION

To solve the problem of the multiple scattering of particles in a strong magnetic field, we use the equation obtained in[1]:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}-\mathbf{H}_0\mathbf{D}\right)F(\mathbf{r},\mathbf{p},t)$$
$$=\int\limits_0 D_\alpha B_{\alpha\beta}(\mathbf{r},\Delta\mathbf{r}(\tau),\tau)\exp\left\{-\Delta\mathbf{p}(\tau)\frac{\partial}{\partial\mathbf{p}}\right\}D_\beta F(\mathbf{r},\mathbf{p},t-\tau)d\tau. \qquad (1)$$

Here, H_0 is the intensity of the regular field,*

$$\mathbf{D}=\frac{e}{mc}\left[\mathbf{p},\frac{\partial}{\partial\mathbf{p}}\right]$$

—the momentum rotation operator, and $B_{\alpha\beta}(\mathbf{r}, \mathbf{x}, \tau)$—the random magnetic field correlation tensor. The argument $\mathbf{r}$ describes the slow variation in space of the square of the random field, $\mathbf{x}$ and τ—the attenuation in space and time of the correlation between the components of the random field; $\Delta\mathbf{r}(\tau)$ and $\Delta\mathbf{p}(\tau)$—the change in the coordinates and momentum of the particle in the regular field $\mathbf{H}_0(\mathbf{r})$, which may be assumed uniform over distances of the order of the Larmor radius.

If the regular field H_0 is sufficiently strong, so that the perturbation of the motion of the particle by the random field in a time interval of the order of the cyclotron period is small, then the distribution function $F(\mathbf{r}, \mathbf{p}, t)$ may be averaged over the angle of rotation. As a result, the left hand side of Eq. (1) to the zeroth order with respect to the small ratio of the Larmor radius $R = cp/eH_0$ to the variation scale L of H_0, takes the form[6]

$$\frac{\partial F}{\partial t}+v\cos\vartheta\frac{\partial F}{\partial z}-\frac{1}{2}(\nabla\mathbf{h})v\sin\vartheta\frac{\partial F}{\partial\vartheta}, \qquad (2)$$

where $\mathbf{h}$ is the unit vector in the direction of $\mathbf{H}_0$, and z is the coordinate measured along the line of force of $\mathbf{H}_0$. The last term in (2) describes the change in the angle between the momentum and the direction of $\mathbf{H}_0$ in the inhomogeneous field due to the conservation of the adiabatic invariant $p_\perp^2/H = \text{const}$.

We assume that the magnetic pulsations, which are described by the tensor $B_{\alpha\beta}$, arose as a result of certain linear oscillations of the plasma. In that case, they can be regarded as the superposition of harmonics with random phases and some definite dispersion law $\omega(\mathbf{k})$. For a statistically isotropic turbulence, the

*$[\mathbf{p}, \partial/\partial\mathbf{p}] \equiv \mathbf{p}\times\partial/\partial\mathbf{p}$.

tensor $B_{\alpha\beta}$ can be written in the form

$$B_{\alpha\beta}(\mathbf{r}, \Delta\mathbf{r}(\tau), \tau) = \int B(k)\left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right) e^{i[\mathbf{k}\Delta\mathbf{r} - \omega(\mathbf{k})\tau]} d^3k. \quad (3)$$

The function B depends on the absolute magnitude of k and on **r**, which describes the weak spatial dependence of the turbulence spectrum. The quantity $\Delta\mathbf{r}(\tau)$ has the form

$$\Delta\mathbf{r}(\tau) = \mathbf{h}v_\parallel\tau + \frac{\mathbf{v}_\perp}{\Omega}\sin\Omega\tau + \frac{[\mathbf{h}\mathbf{v}_\perp]}{\Omega}(1 - \cos\Omega\tau), \quad (4)$$

where $v_\parallel$, $\mathbf{v}_\perp$ are the components of the velocity of the particle parallel and perpendicular to h; $\Omega = eH_0/mc$, being the relativistic mass of the particle.

Substituting in (1) the quantities (3) and (4) and the corresponding expression for $\Delta\mathbf{p}(\tau)$, and averaging over the angle defining the direction of the vector $\mathbf{v}_\perp$, we obtain the required equation:

$$\frac{\partial F}{\partial t} + v\cos\vartheta\frac{\partial F}{\partial z} - \frac{1}{2}(\nabla\mathbf{h})v\sin\vartheta\frac{\partial F}{\partial\vartheta} = \frac{1}{2}\left(\frac{e}{mc}\right)^2\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}b(\vartheta)\sin\vartheta\frac{\partial F}{\partial\vartheta}. \quad (5)$$

The coefficient $b(\vartheta)$ is given by the integral

$$b(\vartheta) = \int_0^\infty d\tau\int d^3k\,B(k)\left[\left(1 - \frac{k_\perp^2}{k^2}\sin^2\varphi\right)\cos\Omega\tau + \frac{k_\perp^2}{k^2}\sin\varphi\cos\varphi\sin\Omega\tau\right]\exp\{ik_\parallel v_\parallel\tau + ik_\perp R_\perp[\sin(\Omega\tau - \varphi) + \sin\varphi] - i\omega(k)\tau\}, \quad (6)$$

where $R_\perp = v_\perp/\Omega$ and φ is the angle between k and $\mathbf{v}_\perp$.

Let us suppose, for the sake of definiteness, that the magnetic pulsations are produced by small amplitude Alfvén waves. In that case,

$$\omega(\mathrm{k}) = u_A k_\parallel - i\gamma, \quad (7)$$

where u_A is the Alfvén velocity ($u_A \ll v$) and γ is a small imaginary part. We choose the spectral function B(k) in a form corresponding to a spectrum that decreases at large k according to the power law:

$$B(k) = A_\nu/(k_0^2 + k^2)^{\nu/2+1}; \quad (8)$$

Here,

$$A_\nu = \frac{\langle H_1^2(r)\rangle\,\nu\Gamma(\nu/2)}{4\pi^{3/2}L_c^{\nu-1}\Gamma(\nu/2 - 1/2)} \quad (\nu > 1), \quad (9)$$

$\langle H_1^2(r)\rangle$ is the mean square of the random field and $k_0^{-1} = L_c$—the correlation length. For such B(k), the magnetic energy density per unit interval of the wave numbers is, for $k \gg k_0$, proportional to $k^{-\nu}$. According to experimental data given in[7-11], for interplanetary magnetic field the exponent ν assumes in different parts of the spectrum values ranging from 1 to 3.8. Henceforth, we shall consider the case $R \ll L_c$ when the scattering of the particles is determined by just this power-law part of the spectrum.

Substituting the quantities (7) and (8) in the integral (6) and using the well-known formulas for the Bessel functions, we express $b(\theta)$ for $R \ll L_c$ in the form of a sum

$$b(\vartheta) = \frac{2\pi^2 R_\perp^\nu A_\nu}{|v_\parallel - u_A|}\sum_{n=0}^\infty\left\{\left[(n+1)\left(n - \frac{\nu}{2} - 1\right) + \alpha_n^2\right]\times\int_0^\infty\frac{J_n^2(x)x\,dx}{(x^2+\alpha_n^2)^{\nu/2+2}} + (n+1)\left(\frac{\nu}{2}+2\right)\alpha_n^2\int_0^\infty\frac{J_n^2(x)x\,dx}{(x^2+\alpha_n^2)^{\nu/2+3}}\right.$$
$$-(n+1)\left(\frac{\nu}{2}+2\right)\alpha_n^2\int_0^\infty\frac{J_{n+2}^2(x)x\,dx}{(x^2+\alpha_n^2)^{\nu/2+3}} \quad (10)$$
$$\left.+\left[(n+1)\left(n+\frac{\nu}{2}+3\right)+\alpha_n^2\right]\int_0^\infty\frac{J_{n+2}^2(x)x\,dx}{(x^2+\alpha_n^2)^{\nu/2+2}}\right\} + \beta(\vartheta),$$

where

$$\beta(\vartheta) = 2\pi\int dk_\parallel dk_\perp\frac{k_\parallel^2 k_\perp B(k)J_1^2(k_\perp R_\perp)}{k^2[\gamma - ik_\parallel(v_\parallel - u_A)]} \quad (11)$$

In all the summands of formula (10), except the last, the passage to the limit $\gamma \to 0$ has been carried out. However, such an approximation turns out to be too crude for the calculation of $\beta(\vartheta)$.

For $v_\perp \ll |v_\parallel - u_A|$, the main contribution to the sum over n is made by the term containing $J_0^2(x)$. Putting $J_0^2(x) \approx 1$, discarding all the other terms, and integrating with respect to x, we have

$$b = 2\pi^2 A_\nu|v_\parallel - u_A|^{\nu-1}/(\nu+2)\Omega^\nu + \beta(\vartheta). \quad (12)$$

In the opposite limiting case $v_\perp \gg |v_\parallel - u_A|$, it is necessary to take into account all the terms of the series (10). Since the large values of x are the important ones in the integral, we may use the asymptotic expressions for $J_n(x)$ and substitute for the square of the cosine, its average value which is ½. Summing the series, we obtain

$$b = \frac{2\pi^{3/2}A_\nu\Gamma(\nu/2 + 3/2)\zeta(\nu+1)|v_\parallel - u_A|^\nu}{\Gamma(\nu/2+2)\Omega^\nu v_\perp} + \beta(\vartheta), \quad (13)$$

where $\zeta(x)$ is the Riemann function. Apparently, for an arbitrary relation between $v_\perp$ and $|v_\parallel - u_A|$, sufficient accuracy will be given by some simple interpolation formula which will go over in the two limiting cases into (12) and (13).

The summand $\beta(\vartheta)$ describes the scattering of the particles moving along the magnetic field with the velocity u_A of the waves, i.e., the particles which are in Cerenkov resonance with the waves. In the approximation $\gamma \to 0$, the effective time of interaction of such particles with the waves turns out to be infinitely large, and the width of the resonance—zero. But in order to calculate the isotropization time for the particles, we must allow for a finite width of the Cerenkov resonance. One of the causes guaranteeing this width is the attenuation of the Alfvén waves with the decrement[12]

$$\gamma(\mathrm{k}) = \frac{k_\parallel^2}{2}\left(\frac{\eta}{\rho} + \frac{c^2}{4\pi\sigma}\right), \quad (14)$$

where η and σ are the coefficients of viscosity and electrical conductivity and ρ is the density of the plasma. Substituting (14) into (11) and evaluating the integral, we obtain for $\gamma_0^2 R^2/|v_\parallel - u_A|^2 \gg 1$

$$\beta(\vartheta) = \frac{10\pi\Gamma(\nu/2 - 1/2)A_\nu L_c^{\nu-1}}{\nu\Gamma(\nu/2)\gamma_0}, \quad (15)$$

and for $\gamma_0^2 R^2/|v_\parallel - u_A|^2 \ll 1$ and $\nu < 3$

$$\beta(\vartheta) = \frac{2\pi C_\nu A_\nu\gamma_0 R_\perp^{\nu-1}R^2}{|v_\parallel - u_A|^2}. \quad (16)$$

Here γ_0 denotes the value of $\gamma(\mathbf{k})$ when $k_\parallel = R^{-1}$,

$$C_\nu = \int_0^\infty dx\int_0^\infty dy\frac{xy^2J_1^2(x)}{(x^2+y^2)^{\nu/2+2}}$$

is a constant of the order of unity.

The second cause of the broadening of the Cerenkov resonance is the scattering of the particles as a result of which their longitudinal velocity varies in the vicinity of the value $v_{\parallel} = u_A$ according to the law

$$v_{\parallel}(\tau) = u_A \pm v\theta\tau, \tag{17}$$

and the particles get out of resonance with the wave. The angular velocity $\dot{\theta}$ of rotation of the momentum of a particle by the random field is roughly equal to $e\overline{H}/mc$, where $\overline{H}$ is the mean field of the waves with wave vectors of the order of the reciprocal of the Larmor radius of the particle or larger, i.e., those waves which effectively scatter particles. Formula (17) describes the action of the same magnetic inhomogeneities on the particle during the time τ. It is precisely because of this that $(v_{\parallel} - u_A)$ increases linearly with τ, and not with $\sqrt{\tau}$, as in the case of diffusion in angle space. In the case of the spectral function (8), we have

$$\dot{\theta} \approx \frac{e\sqrt{\langle H_1^2\rangle}}{mc}\left(\frac{R}{L_c}\right)^{(\nu-1)/2}. \tag{18}$$

In accordance with (17), we shall have for not too large τ

$$\Delta r_{\parallel}(\tau) = u_A\tau \pm {}^1/_2 v\dot{\theta}\tau^2, \tag{19}$$

whereas $\Delta r_{\perp}(\tau)$ is given as before by the last two terms of formula (4). The time of resonance interaction of a particle with the waves is defined as the time of interaction of the particle with a region of the field the dimensions of which are of the order of R. We find this time with the aid of (18) and (19):

$$\tau_0 \approx \sqrt{\frac{R}{v\dot{\theta}}} = \frac{1}{\Omega}\sqrt{\frac{H_0}{\langle H_1^2\rangle^{1/2}}}\left(\frac{L_c}{R}\right)^{(\nu-1)/4}. \tag{20}$$

For a rough estimation of the term $\beta(\vartheta)$ under the conditions when the collision width plays the dominant role, we may use (15) and (16), writing, in place of γ_0 in these formulas, the value of $1/\tau_0$ given by (20).

As a result, the equation takes the form (5) when averaged over the angle of cyclotron rotation of the particles, $b(\vartheta)$ being then given by the formulas (12)–(16). These results have been obtained for isotropic distribution of the wave vectors of the random field. If all the **k** were directed along $\mathbf{H}_0$, the quantity $b(\vartheta)$ would, for any $v_{\perp}$ and $v_{\parallel} - u_A$, be given by (12) while the term containing $\beta(\vartheta)$ would not appear at all. Such a case has been considered by Tverskoĭ[4].

Notice that Eq. (5) does not describe the motion of the particles across the lines of force of H_0. Such a motion arises in the subsequent orders with respect to the small ratio R/L. Also, Eq. (5) does not include terms describing the acceleration of the particles. These terms are $(u_A/v)^2$ times smaller than those corresponding to scattering. Thus, Eq. (5) is suitable for the description of the motion of the particles the velocity of which is much larger than the velocity of the magnetic pulsations during the time $\Delta t \ll \tau_a$, where τ_a is the time interval during which the energy of the particles changes appreciably. The process of isotropization of the particles, which is a more rapid process than the acceleration process, is investigated below with the aid of this equation.

3. MULTIPLE SCATTERING OF PARTICLES IN A MAGNETIC FIELD

It is possible to obtain an analytical solution to Eq. (5) for angles satisfying the conditions $\vartheta \ll 1$ or $|\cos\vartheta| \ll 1$. Setting in the first of the specified regions $\sin\vartheta \approx \theta$, $\cos\vartheta \approx 1$, we transform Eq. (5) for the stationary case into the form

$$\frac{\partial F}{\partial z} - \frac{1}{2}(\nabla \mathbf{h})\vartheta\frac{\partial F}{\partial\vartheta} = \frac{1}{l(z)}\frac{1}{\vartheta}\frac{\partial}{\partial\vartheta}\vartheta\frac{\partial F}{\partial\vartheta} + \frac{1}{\vartheta}\delta(\vartheta)\delta(z - z_0), \tag{21}$$

where a point source has been added to the right hand side. The quantity $l(z)$ is determined from (9) and (12):

$$l(z) = \frac{4(\nu+2)\Gamma(\nu/2 - {}^1/_2)}{\nu\Gamma(\nu/2)\sqrt{\pi}}\frac{H_0^2}{\langle H_1^2\rangle}\left(\frac{L_c}{R}\right)^{\nu-2} L_c. \tag{22}$$

It has the meaning of the mean free path a particle with respect to scattering into an angle of the order of unity. The mean free path depends upon the momentum of the particles and upon the field intensities $\langle H_1^2\rangle$ and H_0 according to the law $l \sim p^{2-\nu}H_0^{\nu}/\langle H_1^2\rangle$. For $\nu > 2$, the mean free path decreases with increase in the energy of the particle. This is explained by the fact that as the Larmor radius increases, the scattering inhomogeneities increase both in scale and in number.

The solution of Eq. (21) may be obtained by slightly generalizing the method used in[1]. We give the final result:

$$F(z, \vartheta) = \frac{H_0(z)}{\pi H_0(z_0)\overline{\vartheta^2}(z)}\exp\left\{-\frac{\vartheta^2}{\overline{\vartheta^2}(z)}\right\}, \tag{23}$$

where

$$\overline{\vartheta^2}(z) = 4\int_{z_0}^{z}\frac{H_0(z)\,dz'}{H_0(z')\,l(z')}. \tag{24}$$

The solution (23) is valid provided $\overline{\vartheta^2}(z) < 1$.

If the dependence of H_0 and l on z can be neglected, then the mean square of the scattering angle (24) increases linearly with distance from the source. If H_0 and H_1 vary with distance according to one and the same power law $H_0 \sim H_1 \sim (z_0/z)^{\alpha}$, while the shape of the spectrum (the quantities ν and L_c) remains unchanged, then for $z \gg z_0$

$$\overline{\vartheta^2}(z) = \frac{4z_0}{[\alpha(\nu-1)+1]\,l(z_0)}\left(\frac{z}{z_0}\right)^{\alpha(\nu-2)+1}. \tag{25}$$

Let us apply these results to the scattering of low energy solar particles in interplanetary space. According to the Parker model[13], in the region inside the earth's orbit, $\alpha \approx 2$ for the large-scale magnetic field H_0. According to measurements made on Mariner-4[10] the spectrum exponent $\nu = 1.5 \pm 0.2$. If the larger part of the magnetic inhomogeneities is generated near the sun and then transported into interplanetary space by the solar wind, then H_1 should vary roughly linearly with H_0. This conjecture is confirmed for the region outside the earth's orbit by measurements[10]; when the distance from the sun increased from 1 AU to 1.43 AU, the value of $\langle H_1^2\rangle$ decreased by a factor of 2.4 and H_0^2—by a factor of 2.5. Notice that according to Parker's model, the field should decrease rather more slowly. As the plasma is radially dispersed the transverse—with respect to the scattering direction—scale of the inhomogeneities should increase linearly with distance, but small-angle scattering is determined by the longitudinal dimension of the inhomogeneities which remains approximately constant.

Substituting into (25) $\alpha = 2$ and $\nu = 1.5$, we find that $\overline{\vartheta^2}$ does not depend on z. If then $\overline{\vartheta^2} < 1$, the particles generated on the sun should arrive on earth in the form of a corpuscular stream of large anisotropy. Such anisotropic streams of protons of energy of several MeV and, lasting many hours, have been recorded more than once in experiments by Vernov and his collaborators[14]. The possibility that the situation considered above was approximately realized in these cases is not to be ruled out. It should be noted that the condition $\overline{\vartheta^2} < 1$ is not fulfilled at that level of magnetic pulsations that is cited in[10]. However, that cannot be a decisive argument against the considered model since the level of pulsations fluctuates strongly even from day to day.

If the source of the particles operates for a long time, i.e., if a stationary formulation of the problem is possible, then there will exist in the region of applicability of the solution (23), together with the anisotropic stream, isotropized particles which, having undergone large-angle scattering at points far away from the source, had diffused back to the source. But a simple estimation shows that the number of such particles will be small. For example, if l = const., then in the region $r \ll l$, where the solution (23) is applicable, the density of the isotropic background will be roughly $(r/l)^2 \approx \theta^4 \ll 1$ times the density of the corresponding anisotropic stream given by (23). The separation of the anisotropic current from the cosmic ray plasma background is fully within the reach of modern experimental means and is confidently carried out in experiments[14].

We should note also that the value for the spectrum exponent $\nu = 1.5$ cannot be considered as having been finally established. According to data obtained by Burlaga and Ness[11,15], the principal part of the small-scale inhomogeneities of the magnetic field in interplanetary space is due to tangential and rotational discontinuities. The spectrum exponent has then the value $\nu = 2$ and the formula (25) leads to the dependence $\overline{\vartheta^2} \sim z$ – the same as when the particles are scattered in a homogeneous medium.

In the considered region of space, where the small-angle approximation is applicable, it is easy to obtain the solution of the nonstationary problem as well. Let, for example, the source be time independent and not monoenergetic: $Q = f(t)\varphi(v)\delta(z - z_0)\delta(\vartheta)/\vartheta$. The kinetic equation, in the same approximation as for (21), takes the form

$$\frac{1}{v}\frac{\partial F}{\partial t} + \frac{\partial F}{\partial z} - \frac{1}{2}(\nabla \mathbf{h})\vartheta\frac{\partial F}{\partial \vartheta} = \frac{1}{l(z)}\frac{1}{\vartheta}\frac{\partial}{\partial \vartheta}\vartheta\frac{\partial F}{\partial \vartheta} + \frac{1}{\vartheta}\delta(\vartheta)f(t)\varphi(v)\delta(z - z_0). \tag{26}$$

Its exact solution may be written as

$$F = \frac{f(t - (z - z_0)/v)\varphi(v)H_0(z)}{\pi H_0(z_0)\overline{\vartheta^2}(z)}\exp\left\{-\frac{\vartheta^2}{\overline{\vartheta^2}(z)}\right\}, \tag{27}$$

This may readily be verified by a direct substitution. We note a few properties of this solution.

Even for an instantaneous outburst in the source ($f(t) = \delta(t)$), a detector will record a finite duration of the outburst if the particle source is not monoenergetic. The recorded duration of the flare will be

$$\Delta t = \frac{z - z_0}{v^2}\Delta v, \tag{28}$$

where Δv is the spread in the velocities of the particles in the source. As $\Delta v \to 0$, we obtain $\Delta t \to 0$. This result is connected with the use of the small-angle approximation in which all terms of the order of and higher than ϑ^2 are discarded. In fact, the duration of the instantaneous monoenergic outburst as recorded by a detector will be

$$\Delta t \approx \frac{z - z_0}{v}\overline{\vartheta^2}.$$

It is determined by the fact that the longitudinal velocities of the particles are confined within the limits: from v to a value of the order of $v(1 - \overline{\vartheta^2})$. The duration Δt has the order of the ϑ^2-terms which were neglected in the equation in comparison with the time of flight of the particles from the source to the detector.

We note also that the distribution of the particles according to their velocities at a fixed moment of time, recorded by the detector and proportional to the quantity

$$f\left(t - \frac{z - z_0}{v}\right)\varphi(v),$$

does not coincide with the spectrum in the source.

Let us now consider scattering in the region of angles where the inequality $x = |\cos\vartheta| \ll 1$ holds. In this region, the first term in (13) is small and it decreases as $v_\parallel \to u_A$ while the second term peaks at $v_\parallel = u_A$. Therefore, the quantity $b(\vartheta)$ has a minimum at some value $x = x_0$ which may be determined from (13) and (16):

$$x_0 \approx (\gamma_0/\Omega)^{1/(\nu+2)}, \tag{29}$$

where, for simplicity, we have neglected u_A in comparison with v. If x_0 is of the order of unity, then the minimum of $b(\vartheta)$ is not deep and does not exert any significant influence on the scattering. But if $x_0 \ll 1$, then the scattering is greatly weakened for angles such that $|\cos\vartheta| \approx x_0$, which leads to a considerable increase in the time of isotropization of the particles. Let us consider this interesting case in greater detail.

For the region $1 \gg x > x_0$ and for H_0 = const, Eq. (5) takes the form

$$\frac{1}{v}\frac{\partial F}{\partial t} \pm x\frac{\partial F}{\partial z} = \frac{1}{l'(z)}\frac{\partial}{\partial x}x^{\nu}\frac{\partial F}{\partial x}, \tag{30}$$

where the signs ± correspond to $\cos\vartheta \gtrless 0$, and the quantity

$$l'(z) = \frac{4(\nu+2)}{(\nu^2-1)\zeta(\nu+1)}\frac{H_0^2}{\langle H_1^2\rangle}\left(\frac{L_c}{R}\right)^{\nu-2}L_c \tag{31}$$

differs from $l(z)$ by a factor of the order of unity.

Let us begin the analysis of Eq. (30) with the case when $\nu = 2$. We neglect the spacial inhomogeneity of the system ($\partial F/\partial z = 0$, l' = const) and follow the filling-up of the region of angles between $x = x_0$ and $x = x_1$ ($x_0 \ll x_1 \ll 1$). Eq. (30) may be written as

$$\frac{\partial F}{\partial \tau} = x^2\frac{\partial^2 F}{\partial x^2} + 2x\frac{\partial F}{\partial x}, \tag{32}$$

where $\tau = vt/l'$. We impose on the distribution function the boundary conditions $F(x_1) = F_1$, $F(x_0) = 0$. The constant $F_1 \approx (2\pi)^{-1}$ if the region $x > x_1$ is filled with particles and the distribution function is normalized to unity. The second condition corresponds to the supposition that particles which reach the boundary $x = x_0$, i.e. the central maximum region of $b(\vartheta)$, are instantly transported to the backward hemisphere of angle space. Such an approximation is sufficient for a rough estimation of the isotropization time.

Solving Eq. (32) with the specified boundary conditions, we have

$$F(x,\tau) = F_1\left(1-\frac{x_0}{x}\right) + \frac{1}{\sqrt{x}}\sum_{n=1}^{\infty} A_n e^{-\left(1/4+\lambda^2\right)\tau} \sin\left(\lambda_n \ln\frac{x}{x_0}\right), \quad (33)$$

where $\lambda_n = n\pi/\ln(x_1/x_0)$. The coefficients A_n are to be found from the initial conditions. The time required to fill up the region $x_1 > x > x_0$, as found from (33), is roughly given by

$$\tau_1 \approx \left(\frac{1}{4}+\frac{\pi^2}{\ln^2(x_1/x_0)}\right)^{-1}. \quad (34)$$

This time varies from zero for a broad resonance $x_0 \approx x_1 \approx 1$ to the value $\tau_1 = 4$ for $x_0 \to 0$. The time τ_0 required to scatter particles through an angle $\vartheta \sim 1$, according to the foregoing results, is roughly equal to unity. If the initial distribution is such that the number of particles for which $\vartheta < \pi/2$ is roughly the same as the number for which $\vartheta > \pi/2$, then the isotropization time $\tau_S \approx \tau_0 + \tau_1$. But if at the initial moment the distribution is considerably anisotropic, for example, if $\vartheta \approx 0$ for all the particles, then the isotropization time will be considerably larger than $\tau_0 + \tau_1$. This is explained by the fact that the "leakage" of particles from the forward to the backward hemisphere of angle space takes place slowly for small x_0. The rate of transition of particles into the backward hemisphere $dN/d\tau$ is obtained by integrating (32) with respect to x:

$$dN/d\tau = -x_0^2 F'(x_0), \quad (35)$$

where the prime denotes differentiation with respect to x. We determine from this the order of magnitude of the isotropization time:

$$\tau_s \approx (2\pi x_0^2 F'(x_0))^{-1}. \quad (36)$$

Let us estimate $F'(x)$ in the quasi-stationary approximation, assuming that $\tau_S \gg \tau_0 + \tau_1$. When $\tau_S \gg \tau \gg \tau_1$, only the first term on the right hand side of (33) remains. This term does not depend on time and yields a quasi-stationary distribution of the particles in the forward hemisphere. Using the indicated value for F, we find the isotropization time:

$$\pi\tau_s = x_0^{-1} \gg 1. \quad (37)$$

The solution (33) and the above estimates for the characteristic times show that during a time interval of the order of $t_S = l'/vx_0$ (at a distance of the order of l'/x_0 from the source) the corpuscular stream has a peculiar structure: the forward hemisphere is almost completely filled with particles while the backward hemisphere contains a small number of particles, and a steep gradient exists at $x = x_0$ in the angular distribution.

For $\nu \neq 2$ the qualitative features of the isotropization process remain the same as for $\nu = 2$. For the isotropization time we obtain the estimate $\tau_S \approx x_0^{1-\nu}$, valid for $x_0^{1-\nu} \gg 1$. The mean free path with respect to scattering through π, according to this estimate and formulas (29) and (31), has the order of magnitude

$$\Lambda \approx l'\tau_s \approx l'(\Omega/\gamma_0)^{(\nu-1)/(\nu+2)}. \quad (38)$$

An additional increase in the mean free path Λ occurs if the regular field H_0 is nonuniform and the particles move in the direction of decreasing intensity. Focusing, arising as a result of the conservation of the quantity $\sin^2\vartheta/H_0$, hinders the penetration of the particles into the backward hemisphere.

The corresponding estimate may be obtained in the following fashion. In a mildly nonuniform field Eq. (5) for the stationary case takes the form

$$x^\nu \frac{d^2F}{dx^2} + (\nu x^{\nu-1} - \theta_1)\frac{dF}{dx} = 0, \quad 1 \gg x > x_0, \quad (39)$$

where $\theta_1 = \frac{1}{2} l' \operatorname{div} \mathbf{h} = \text{const.}$; $\theta_1 > 0$ if the particles move in the direction of decreasing H_0. Let us solve (39) with the same boundary conditions that were used to obtain (33).

For $\theta_1 \ll \nu x_0^{\nu-1}$, we obtain the same result as in the case when $H_0 = \text{const.}$, while for $\theta_1 \gg \nu x_0^{\nu-1}$, we shall have

$$F(x) = F_1\left\{1 - \exp\left[\frac{\theta_1(x^{\nu-1} - x_0^{\nu-1})}{(\nu-1)x_0^{\nu-1}x^{\nu-1}}\right]\right\}. \quad (40)$$

An estimation of the isotropization time yields

$$\tau_s = \frac{x_0^{2-\nu}}{\theta_1}\exp\left[\frac{\theta_1}{(\nu-1)x_0^{\nu-1}}\right]. \quad (41)$$

Consequently, for $\theta_1 \gg \nu x_0^{\nu-1}$, it is necessary to multiply the mean free path Λ given by formula (38) by $(x_0/\theta_1)\exp(\theta_1/(\nu-1)x_0^{\nu-1})$.

Using experimental magnetic spectrum data, we can estimate with the aid of the obtained formulas the mean free path of low energy particles in interplanetary space. However, experimental results obtained by different authors at different times appreciably differ from each other. Using the Mariner-4 data[10] and estimating the collision width of the Cerenkov resonance with the aid of the formulas (20) and (29), we find for protons of energy 1 Mev $x_0 = 0.9$. This means that the weakening of the scattering when $x \approx x_0$ is small in this case. Estimation of the transport mean free path from the formula (22) yields a value of the order of 0.1 AU. Such a value agrees with the experimental data obtained by Vernov et al.[14] on the diffusive propagation of low energy particles in interplanetary space. The transport mean free path computed from the data obtained by Sari and Ness[11] turns out to be greater than 1 AU. If the main contribution to the observed magnetic spectrum is made by discontinuities in the magnetic field, as is proposed in[11], then the theory developed here may prove to be inapplicable since a particle can be scattered at once through a large angle as it passes through a discontinuity.

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