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DISPERSION OF FIRST SOUND IN SUPERFLUID He^3–He^4 *SOLUTIONS DUE TO DAMPING OF THE NORMAL COMPONENT*

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Sound propagation in narrow channels filled with superfluid He^3–He^4 solutions is studied. Dispersion of the sound velocity is observed and is due to partial damping of the normal component; the dispersion depends on the degree of damping and the He^3 concentration. The obtained dependences of the sound velocity and absorption on the temperature, frequency of oscillation, and He^3 concentration are in satisfactory agreement with the theoretical predictions.

1. INTRODUCTION

THE propagation of first sound in He II with partial damping of the normal component has a number of distinctive features, as has been shown theoretically[1–3] and experimentally.[4–6] The most interesting of these are the dispersion of sound and the dependence of the wave propagation velocity on the degree of damping of the normal component.

Such an effect is usually found experimentally in a system of narrow channels filled with the superfluid, in which the characteristic transverse dimension d of the channel should have the same order of magnitude as the the penetration depth of the viscous wave $\lambda_n (\delta = d/\lambda_n \sim 1)$. Detailed measurements of the velocity and absorption of first sound in He II for various δ [6] have shown excellent agreement with the theoretical predictions.

If $\delta \ll 1$, the first sound transforms into fourth sound, and in the other limiting case $\delta \gg 1$, the usual first sound occurs.

For superfluid He^3–He^4 solutions, a similar set of problems has been considered recently by Karchava and Sanikidze[7] who showed that dispersion of the waves should occur in this case also, depending not only on the degree of damping of the normal component, but also on the concentration of He^3 in the solution. A calculation, carried out on the basis of the equations of hydrodynamics of superfluid solutions with account of the corresponding boundary conditions, made it possible to compute the velocity of propagation and the absorption coefficients of the different wave processes in narrow channels filled with He^3–He^4 solution. In particular, the following expression was obtained for the velocity of first sound:

$$u_{1\delta} = u_1 \left[1 - br_n \frac{(1 - r_{sn}\bar{c})^2}{1 + r_{sn}\bar{c}^2}\right]^{1/2} \times \left\{1 + \frac{3}{8} \frac{[ar_n(1 - r_{sn}\bar{c})^2]^2}{[1 + r_{sn}\bar{c}^2 - br_n(1 - r_{sn}\bar{c})^2]^2}\right\}, \tag{1}$$

where u_1 is the velocity of ordinary first sound in the solution, $r_n = \rho_n/\rho$, $r_{sn} = \rho_s/\rho_n$, $\rho = \rho_s + \rho_n$ is the total density of the liquid, $\bar{c} = (c/\rho)\partial\rho/\partial c$, c the weight concentration of He^3 in the solution, the parameters $a(\delta)$ and $b(\delta)$ take into account the degree of damping of the normal component and depend on the shape of the channel cross section. For plane parallel channels, the expressions for a and b are given in [2], and for cylindrical channels, in [3].

The absorption coefficient for first sound in narrow channels is written down in the form

$$\alpha_{1\delta} = \frac{\omega}{2u_1} \frac{ar_n(1 - r_{sn}\bar{c})^2 (1 + r_{sn}\bar{c}^2)^{1/2}}{[1 + r_{sn}\bar{c}^2 - br_n(1 - r_{sn}\bar{c})^2]^{3/2}} \times \left\{1 - \frac{5}{8} \frac{[ar_n(1 - r_{sn}\bar{c})^2]^2}{[1 + r_{sn}\bar{c}^2 - br_n(1 - r_{sn}\bar{c})^2]^2}\right\}, \tag{2}$$

where $\omega = 2\pi f$ is the frequency of the sound.

It follows from (1) and (2) that the addition of He^3 should intensity the dispersion of the velocity of first sound and lead to a greater increase in the absorption than in the case of pure He^4.

The present work was undertaken with the aim of studying experimentally the features of the propagation of first sound in superfluid He^3–He^4 solutions under the conditions of partial damping of the normal component and of comparison of the results with the theoretical calculations.

2. FEATURES OF THE METHOD

In the given experiments, the damping of the normal component was brought about by means of a system of linear channels formed by thin glass fibers. A special "filter" was used, composed of such fibers,[6] with the mean effective transverse dimension of the channels 2d = 1.85 microns. For comparison with theory, it was assumed that the channels had cylindrical shape and that the quantity 2d was determined in a calibration experiment with pure He^4.

All the measurements were carried out on the same "filter," i.e., for fixed d, while the damping of the normal component was changed by changing the frequency of oscillation. The propagation of the sound was studied in the frequency range from 35 to 115 kHz, while the parameter δ, the characteristic degree of damping of the normal component, changed in the range from ~ 1.0 to ~ 4.0.

The determination of the velocity and the absorption coefficient of the sound was accomplished by a method described previously.[6] Radio pulses were used with the carrier frequency given above, which guaranteed the required monochromaticity of the oscillations. Identical condenser pickups with high sensitivity served as source and receiver. The sound velocity was determined by the time of propagation of the signal, which was measured on the electronic frequency meter ChZ-3, operating so as to measure time intervals. The amplitude of the signal was measured directly on the oscilloscope.

For the determination of the sound velocity, it was necessary to know the exact value of the acoustic path length l traveled by the sound wave. The quantity l was determined in a special calibration experiment with He^4.[6] For the filter used, $l = 22.28$ mm.

We note that in experiments with He^3–He^4 solutions the pulse method of measurement is preferable to the method of continuous oscillations, which is connected to a high mean radiated power. The excess power in the solutions can lead to a nonequilibrium distribution of the impurity particles. For this reason, no rapid cooling or heating was allowed during the recording of the temperature dependence of the propagation time, and particularly the signal amplitude, and the measurements were made after an isothermal hold of 10–20 minutes.

In research with solutions, the question also arises as to the concentration of the impurity particles in the filter channels. This question has been investigated in detail[8] in a study of fourth sound in superfluid He^3–He^4 solutions; it was shown that in narrow channels the concentration of the solution did not differ from the concentration in the macroscopic liquid. Apparently, one is dealing only with such channels for which the liquid has the same temperature of λ transition and density of the normal component as in the bulk.

The experiments were carried out with solutions containing 6.3, 11.0, and 15.2% He^3 (molar concentrations are specified), which were prepared by a mixture of the pure gaseous isotopes in a special apparatus. The error in the determination of the concentration did not exceed 0.1% He^3.

To obtain low temperatures, a cryostat with He^3 was used, in which the low temperatures were obtained by pumping off the He^3 vapor with an adsorption pump with silica gel as the adsorbent. The temperature was determined from the He^3 vapor pressure with an accuracy to within 0.003°K. The sound propagation was studied in the temperature range from 0.5 to 1.8°K.

The error in the determination of the values of the sound speed did not exceed 1.5% and in the determination of the sound absorption coefficient, 6%. The size of the error decreased as the temperature decreases and with decrease in the concentration of He^3 in the solution.

3. VELOCITY OF SOUND

Figure 1 shows a typical temperature dependence of the velocity of first sound in the narrow channels filled with a 6.3% solution (Fig. 1a) and a solution containing 15.2% He^3 (Fig. 1b). In order not to clutter the graph, only the results pertaining to two frequencies have been shown: 35.8 and 71.0 kHz. As follows from the drawing, the sound velocity $u_{1\delta}$ depends significantly on the frequency of oscillation, i.e., an unusual sound dispersion takes place in solutions as in He^4, produced by the fact that only part of the normal component of the liquid is damped in the channels of the "filter." For the 15.2% solution, measurements were not made at frequencies above 71.0 kHz because of the high sound absorption, about which more will be said below. Because of the high damping of the oscillations, it has also not been possible to continue the measurements down to the λ point of the solutions.

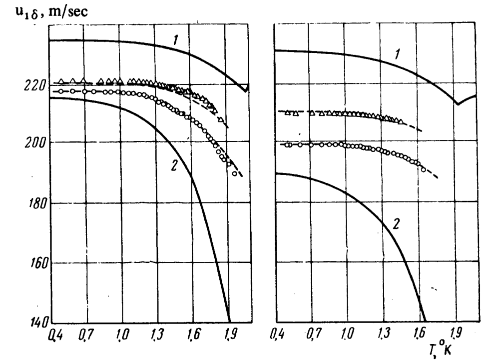

FIG. 1. Temperature dependence of the velocity of first sound in narrow channels filled with superfluid solution a–6.3% He^3; b–15.2% He^3. O–35.8 kHz, Δ–71 kHz, curve 1–ordinary first sound, curve 2–fourth sound.

For comparison, the temperature dependences of pure first and second sound in these same solutions is also given in Fig. 1. One should especially note the fact that in solutions, in contrast with pure He^4, the velocity of fourth sound at low temperatures is not equal to the velocity of first sound, and this difference increases with increase of the He^3 concentration. This is due to two reasons.[8] First, the impurity excitations give a noticeable contribution to the density of the normal component of the solution, so that we have $\rho_n \pm 0$ as $T \rightarrow 0$. Second, the presence of the oscillations of concentration in the wave of fourth sound in the solutions leads to the result that the velocity of the fourth sound depends explicitly on the concentration.[9]

The fact that the quantity u_4 differs appreciably from u_1 in solutions, down to very low temperatures, makes possible the observation of sound dispersion in any temperature range where superfluid solutions exist, which is also shown in Fig. 1. In pure He^4, dispersion is observed only for $T \geq 1.0$°K.[6]

The dashed curves in Fig. 1 show $u_{1\delta}$ as a function of the temperature, computed from Eq. (1). The data on u_1, ρ_n and η_n necessary for the calculation were taken from [10–13]. It follows from the graphs that the experimental results are in excellent agreement with theory, the only exception being some data in the region of high temperatures, where a small difference is observed; nevertheless, this difference lies within the limits of the total error of measurements and of the calculation by Eq. (1).

We also note that in solutions for the frequencies used in the experiment, it was not possible to obtain a regime of fourth sound, while in He^4, the first sound goes over into fourth sound (at these same frequencies) even for $T < 1$°K. This is associated with the fact that, first, the penetration depth of the viscous wave in the solutions is much smaller than the dimensions of the channel down to very low temperatures. Second, the mean free path length of the elementary excitations in

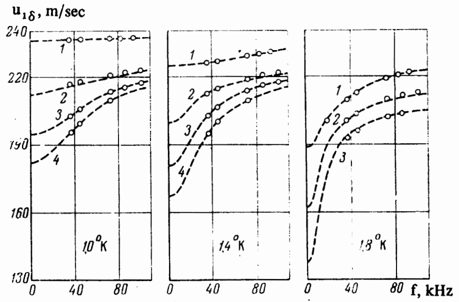

FIG. 2. Dependence of the velocity of sound on the oscillation frequency for various temperatures and He^3 concentrations. Curve 1—He^4, 2—6.3% He^3, 3—11.0% He^3, 4—15.2% He^3.

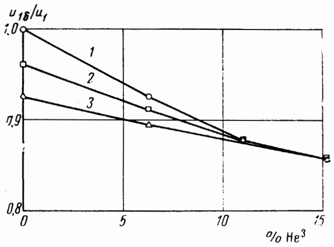

FIG. 3. Concentration dependence of the sound velocity for an oscillation frequency of 35.8 kHz; 1—1.0°K, 2—1.4°K, 3—1.7°K.

the solutions, as estimates from [14] show, also remain smaller than the dimensions of the channel. Thus, in the given experiments, the conditions for the realization of fourth sound in solutions have not been satisfied. For observation of fourth sound under the given conditions, one would need a much lower frequency—less than 1 kHz.

With increase in the oscillation frequency, the penetration depth of the viscous wave decreases and a gradual transition is observed from fourth sound to first, which is shown graphically in Fig. 2. For comparison, the dispersion graphs for pure He^4 are also shown.[6] In accord with the theory given, the sound velocity dispersion in solutions is much greater at higher temperatures, just as in He^4. However, dispersion takes place in solutions even in the low temperature region, for example, at $T = 1.0\,°K$, while the sound velocity at this temperature is no longer dependent on the frequency of oscillation, just as in He^4.

So far as the concentration dependence of the sound dispersion is concerned, it is very lucidly shown in Fig. 3, where the values of $u_{1\delta}/u_1$ are given as a function of the molar concentration of He^3 in the solution for the temperatures 1.0, 1.4, and 1.7°K, for an oscillation frequency of 35.8 kHz. It follows from the graphs that the degree of departure of $u_{1\delta}$ from the velocity of ordinary first sound increases with increase in the concentration He^3 and with increase in the temperature. For concentrations greater than ~ 12% He^3, the curves $u_{1\delta}/u_1$, referring to different temperatures, run together. This is connected with the fact that the temperature dependence of $u_{1\delta}/u_1$ for a given concentration of He^3, in accord with Eq. (1), is determined by two competing factors: the change in the degree of damping of the normal component (parameter b) and the change in the density of the normal component of the liquid. It is shown that for concentrated solutions, these factors compensate one another, and the value of $u_{1\delta}/u_1$ is practically independent of the temperature.

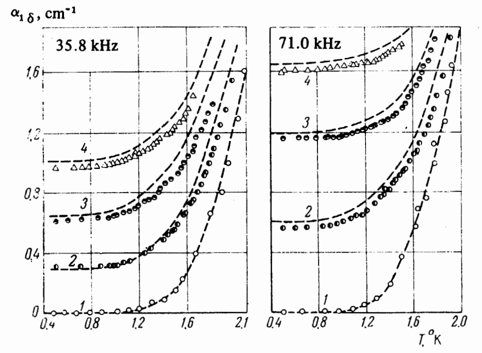

FIG. 4. Temperature dependence of the sound absorption coefficient for oscillation frequencies of 35.8 and 71.0 kHz. 1—He^4, 2—6.3% He^3, 3—11.0% He^3, 4—15.2% He^3.

4. SOUND ABSORPTION

The determination of the absolute values of the sound absorption coefficients in the given experiments was performed by means of auxiliary measurements in He^4, in which the amplitude of the wave radiated into the liquid was determined. Such measurements were carried out for $T = 0.5\,°K$, where the sound absorption in pure He^4 is so small[6] that it does not exceed the errors of measurement of $\alpha_{1\delta}$ and therefore the amplitude of the signal could be taken to be equal to the radiated amplitude. Measurements were carried out in the solutions under the same conditions as in He^4; the amplitude radiated into the liquid in this case was changed relative to pure He^4 only by the decrease in the acoustical impedance of the liquid. Inasmuch as the change in impedance amounted to more than 10%, a special correction for this effect was introduced in the determination of the amplitude radiated into the solution.

Figure 4 shows the temperature dependences of the measured values of the absorption coefficient in different solutions for frequencies of 35.8 and 71.0 kHz; for comparison, the data of [6] on sound absorption in pure He^4 at the same frequencies are also shown. The values of $\alpha_{1\delta}$ computed from Eq. (2) are shown by dashed curves. In the derivation of this relation, only dissipative processes connected with the friction of the normal component with the walls of the channels were taken into account. Bulk mechanisms of dissipation, associated with the viscosity coefficients η, ζ_1, ζ_2, ζ_3, and the thermal conductivity contribute to the sound absorption, as does the process of heat flow through the walls. All these mechanisms were recently considered in detail by Karchava and Sanikidze,[15,16] who showed that the most important of these in the region with strong first sound dispersion is the dissipative process associated with ζ_2. However, as estimates have shown, the contribution of the coefficient ζ_1 to the sound absorption, for the investigated solutions and in the range of frequencies used in these experiments, is smaller by several orders of magnitude than the contribution made by slippage of the normal component relative to the walls. (For purposes of estimation, it was assumed that ζ_1 in the solutions

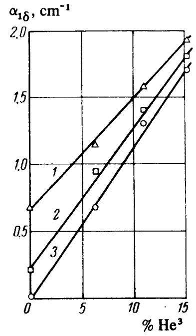

FIG. 5. Dependence of the sound absorption coefficient on the He^3 concentration for a frequency of 71.0 kHz. 1–1.6°K, 2–1.4°K, 3–1.0°K.

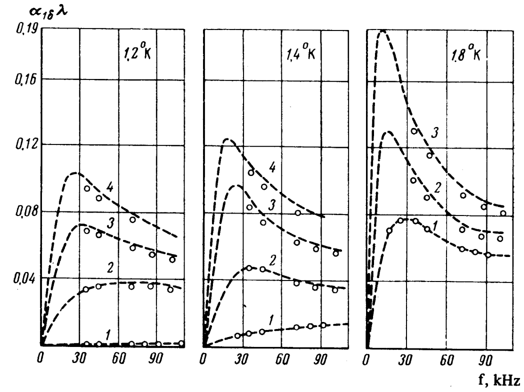

FIG. 6. Sound absorption per wavelength as a function of the frequency of oscillation for various temperatures. Curve 1–He^4, 2–6.3% He^3, 3–11.0% He^3, 4–15.2% He^3.

had the same order of magnitude as the coefficient of first viscosity.)

Thus, one can assume that the absorption of first sound in narrow channels, under the conditions of the experiment, is caused almost entirely by the friction of the normal component on the wall. This fact was also confirmed by the experimental results, which agreed with those calculated from Eq. (2) within the limits of error.

As follows from the graphs in Fig. 4, the sound absorption increases with increase in temperature and increase in He^3 concentration. This is principally explained by the increase in the density of the normal component of the liquid, which increases the degree of its slipping relative to the walls and thus leads to an increase in the absorption. For the same reason, the sound absorption coefficient in solutions differs from the corresponding values in He^4, a difference that is especially large in the low temperature region (below ~1.0° K), where a very large difference is found in the density of the normal components.

The effect of He^3 impurities on the sound absorption in superfluid helium filling narrow channels is shown more graphically in Fig. 5. Although the given data refer to a frequency of 71.0 kHz, the curves have a completely similar form at other frequencies. Moreover, the character of the concentration dependence of $\alpha_{1\delta}$ changes little with temperature.

The presence of sound dispersion is also shown in the behavior of the absorption coefficient. The frequency dependence of the sound absorption coefficient per wavelength $\alpha_{1\delta}\lambda = \mathrm{Im}\, k/\mathrm{Re}\, k$ (k is the wave vector of the first sound) is shown in Fig. 6. As in the case of He^4, in sound propagation in the solutions, the absorption brought about by slippage of the normal component at first increases with increase in the oscillation frequency. It then passes through a maximum and begins to decrease. As is seen from Fig. 6, the values and the positions of the maxima depend both on the temperature and on the He^3 concentration. The magnitude of the maxima increases with increase in temperature and they shift in the direction of lower frequencies, so long as the frequencies are such that $\delta \sim 1$. The value of the maximum of the absorption increases strongly with increase in the concentration of the impurity particles. This is especially clearly seen in the graph corresponding to a temperature of 1.2°K, where the maximum for pure He^4 is very weak and "smeared out," while the maxima grow markedly with increase in He^3 content, because of the greater dispersion.

Thus, in superfluid He^3–He^4 solutions, as in pure He II, the propagation of first sound in narrow channels is characterized, in the case of partial damping of the normal component, by noticeable dispersion of the sound velocity, which depends on the amount of the damping and on the He^3 concentration in the solution. This effect is accompanied by a large increase in the sound absorption, which arises chiefly from the slippage of the normal component.

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