INFLUENCE OF LOW-FREQUENCY FLUCTUATIONS ON SUPERCONDUCTING TUNNELING

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The effects of low-frequency fluctuations on the current-voltage characteristics and the magnetic dependences of the critical Josephson junction current are investigated. It is shown that low-frequency noise in the circuit leads to the appearance of a resistive state and to a non-oscillating dependence of the tunnel superconducting current on the magnetic field strength. The experimental results are compared with the theory developed for this case.

1. INTRODUCTION

SUPERCONDUCTING tunneling, or the Josephson effects[1], has been investigated in detail both theoretically and experimentally[2]. Numerous experiments on these effects are described by the Josephson theory[1], which considers the phenomenon in the absence of fluctuations. In individual experiments[3], however, there was observed the so-called anomalous behavior of the superconducting tunnel current (STC), viz., weak oscillations and a monotonic decrease in magnetic fields, which could not be explained within the framework of the Josephson theory[1].

Theoretical investigations of the role of fluctuations in Josephson tunneling[4,5], carried out in recent times, have cast some light on this question. The point is that the fluctuation changes in the phase difference φ of the ordering parameters of two superconductors forming the tunnel junction can be comparable in magnitude with φ in the absence of fluctuations. Under these conditions, a noticeable voltage appears on the barrier at a given current, and the current junction goes over from the superconducting into the resistive state. The resistance of the contact in the resistive state depends essentially on the ratio of the Josephson binding energy $\xi = \hbar I_0/2e$ (I_0— STC amplitude) to the fluctuation energy, which for white noise can be characterized by a certain effective temperature.

The flow of STC at a finite voltage on the barrier, observed experimentally with relatively high-resistance Pb-I-Pb junctions[6] and later also in Sn-I-Sn[7] (near the critical temperature) agrees with the main idea of the theory developed in[4,5]. In the experiments[6] it was also noted that the non-oscillating dependence of the STC on the magnetic field is realized with higher-resistance samples. As was shown recently[8], such a dependence can be observed in the case of strong thermal fluctuations. In[6] it was suggested that the occurrence of the resistive state and the non-oscillating dependence of the STC on the magnetic field have the same nature. In this connection, it was proposed to introduce an adjustable fluctuation source to observe both effects in a single sample.

In principle, the investigation of the influence of fluctuations on the Josephson characteristics can be carried out in several ways:

1) change the resistance of the sample, using, for example, high pressure[9] or natural aging of the junction[10];

2) investigate the behavior of the contacts in a wide temperature interval up to the critical temperature[7];

3) produce relatively high magnetic fields inside the contact[6].

We wish to call attention to the case of low-frequency noise fluctuations, which are practically always present in a real measuring circuit and, as shown in the present paper, lead to anomalous characteristics of the superconducting tunneling.

2. EXPERIMENT

In order to investigate directly the influence of the low-frequency fluctuations on the Josephson tunnel and, a standard noise generator (type G2-12), operating in the frequency range 20—20,000 Hz and making it possible to regulate smoothly the noise voltage, monitored by a voltmeter built into the generator, was connected in series with the sample in the measuring circuit by means of a step-down transformer (winding ratio n = 110). The measuring system made it possible to register the changes of the direct current and of the voltage with accuracy 10^{-8} A and 10^{-7} V, respectively. The current-voltage characteristics were automatically plotted with a two-coordinate electronic potentiometer. To ensure electrostatic screening, the switching block with the measuring circuit were enclosed in a copper housing. The circuit was connected to the metallic cryostat via an electrostatically and magnetically shielded cable. The samples together with the solenoid were placed in a magnetic screen that attenuated the external magnetic field by an approximate factor of 40.

In the experiments we used tunnel junctions of the Sn-I-Sn type[1)], which are convenient from the point of view of investigations in a wide temperature interval, down to critical (for tin $T_c \approx 3.9°$K). The junctions were prepared by vacuum sputtering. The tunnel barrier was an oxide film (Sn_xO_y). For the investigations we chose low-resistance junctions (ρ_N = 0.01—0.02 ohm-mm^2), in which the dependence of the STC on the magnetic field was close to $I_1 \sim (\pi y)^{-1} \sin \pi y$, with a high ratio of the experimentally observed STC to the theoretical one: $I_{exp}/I_{theor} \approx 0.87$—$0.92$, and with dimensions on the order of double the Josephson depth of penetration $2\lambda_j$.

[1)]Three samples of cruciform geometry were placed on one substrate.

The results, which are reported here for a tunnel contact with dimensions 0.45 × 0.14 mm, R_N = 0.34 ohm, λ_j = 0.24 mm, I_{exp}/I_{theor} = 0.92, film thickness ~ 1300 Å, and energy gap 2Δ = 1.23 ± 0.005 MeV, are typical for the investigated samples.

3. RESULTS AND DISCUSSION

The measuring system corresponds to an equivalent circuit consisting of a resistance to the quasi-particle current R[9], having a temperature Θ and connected to an external electric circuit. The external circuit is at a temperature Θ_1 and contains a source of voltage E with internal resistance R_1 and a source of low-frequency noise with effective voltage U_0. To simplify the calculation, the capacitance C and the inductance L_0 of the tunnel junction and the inductance L of the external circuit are neglected[2)] and it is assumed that the thermal fluctuations have the properties of white noise.

The equation for the phase difference is

$$d\varphi / dt + \Omega \sin \varphi = F(t) + F_1(t) + \Omega_0, \qquad (1)$$

where

$$\Omega = \frac{2e}{\hbar} I_1 R_э, \quad \Omega_0 = \frac{2e}{\hbar} E \frac{R}{R+R_1}, \quad R_э = \frac{RR_1}{R+R_1};$$

I_1 is the amplitude of the Josephson current in a magnetic field without noise, F(t) is the stochastic term resulting from the thermal fluctuations:

$$\langle F(t) \rangle = 0,$$

$$\langle F(t) F(t_1) \rangle = 2\left(\frac{2e}{\hbar}\right)^2 R_э \frac{\Theta R_1 + \Theta_1 R}{R+R_1} \delta(t-t_1) = 2D\delta(t-t_1).$$

The low-frequency noise in the circuit is taken into account by the term $F_1(t)$:

$$\langle F_1(t) \rangle = 0, \quad \langle F_1(t) F_1(t_1) \rangle = \Psi(t-t_1), \qquad (2)$$

where the angle brackets denote stochastic averaging, and $\Psi(t - t_1)$ is the correlation function of the low-frequency noise:

$$\Psi(0) = U_0^2 \left(\frac{2e}{\hbar} \frac{R}{R+R_1}\right)^2.$$

It is assumed that the low-frequency fluctuations have the properties of Gaussian noise, i.e., all the correlation functions with odd F_1 are equal to zero, and with even F_1 are expressed in terms of all possible combinations of the function $\Psi(t)$.

The spectrum of the noise $F_1(t)$ can be regarded with sufficient accuracy as rectangular with a maximum frequency $\omega_{max} = 2 \times 10^4$ Hz. Since the characteristic frequencies of the system Ω and Ω_0 greatly exceed ω_{max}, the stochastic averaging $\varphi(t)$ can be carried out in two stages. First one averages over F(t), disregarding the dependence of the slowly-varying function $F_1(t)$ on the time. The resultant expression must then be averaged over $F_1(t)$.

The method developed by Ivanchenko and Zil'berman[4] yields the following expression for the dependence of the voltage across the junction on the source emf:

$$V = \frac{\hbar D}{4e} \Big\langle 1 \Big/ \int_{-\pi/2}^{\pi/2} dx\, I_0(2z\cos x) \exp\{-(\pi - 2x)[z_0 + z_1(t)]\} - 1 \Big/ \int_{-\pi/2}^{\pi/2} dx\, I_0(2z\cos x) \exp\{(\pi + 2x)[z_0 + z_1(t)]\} \Big\rangle_{F_1(t)}$$

$$= \frac{\hbar D}{4e} \int_{-\infty}^{\infty} dy \Big[1 \Big/ \int_{-\pi/2}^{\pi/2} dx\, I_0(2z\cos x) \exp\{-(\pi - 2x)(z_0 + y)\} - 1 \Big/ \int_{-\pi/2}^{\pi/2} dx I_0(2z\cos x) \exp\{(\pi + 2x)(z_0 + y)\} \Big] \langle \delta(z_0 - y) \rangle_{F_1(t)}, \qquad (3)$$

where

$$z = \Omega / D, \quad z_0 = \Omega_0 / D, \quad z_1(t) = F_1(t) / D,$$

$\langle\ \rangle_{F_1(t)}$ denotes averaging over the low-frequency fluctuations, and $I_0(z)$ is a Bessel function of zero order.

After averaging over $F_1(t)$ with allowance for (2), the relation (3) takes the form

$$V = \frac{\hbar D}{8\sqrt{\pi} e\gamma} \int_{-\infty}^{\infty} dy \Big[1 \Big/ \int_{-\pi/2}^{\pi/2} dx\, I_0(2z\cos x) \exp\{-(\pi - 2x)(z_0 + y)\} - 1 \Big/ \int_{-\pi/2}^{\pi/2} dx\, I_0(2z\cos x) \exp\{(\pi + 2x)(z_0 + y)\} \Big] \exp\{-y^2/4\gamma^2\}, \qquad (4)$$

where $\gamma = U_0/\sqrt{2}\, I_1 R_1$.

If the binding energy of the Josephson junction greatly exceeds the energy of the thermal fluctuations ($z \gg 1$), then the influence of the thermal fluctuations on the Josephson current can be neglected. In this case expression (4) simplifies to:

$$V = \frac{I_1 R_э}{2\gamma\sqrt{\pi}} \Big[\int_{1-\alpha}^{\infty} dx \sqrt{(\alpha + x)^2 - 1} \exp\Big\{-\frac{x^2}{4\gamma^2}\Big\} - \int_{1+\alpha}^{\infty} dx \sqrt{(\alpha - x)^2 - 1} \exp\Big\{-\frac{x^2}{4\gamma^2}\Big\} \Big], \quad \alpha = \frac{\Omega_0}{\Omega}. \qquad (5)$$

From Eq. (1) we can obtain the parametric dependence of the total current through the junction on the source emf:

$$I = [E - V(E)] / R_1. \qquad (6)$$

As seen from Fig. 1, the presence of noise voltages (currents) in the circuits leads to the occurrence of a resistive state. Calculation of the curves for the comparison was based in Eqs. (5) and (6). The resistance $R_e \approx R$ in (5) was determined experimentally from the initial slope of the I-V characteristics in a magnetic field of ~ 50 G, and was equal to ~ 2 ohms. This is in satisfactory agreement with the value calculated by the formula of Larkin and Ovchinnikov[11]

$$R = R_N \frac{\mathrm{ch}^2(\Delta/2\Theta_1)}{[1 + (\Delta/8\Theta)\ln(\Theta_1/eV_0)^2]}, \qquad (7)$$

where V_0 is the voltage across the barrier in the absence of fluctuations. From (7) we see that the external resistance R is a function of the voltage across the barrier. A similar relation is observed experimentally, although it is masked to a considerable degree by the contribution made to the resistance R by the destruction of the weak residual STC by the thermal fluctuations. In practice it is difficult to separate the true internal resistance, and furthermore R also changes with U_0 in the presence of noise, owing to the dependence of the resistance on the voltage across the barrier. In the theor-

[2)]Since the main contribution to the resistive state is made in our experiment by the low-frequency noise with $\omega_{max} \sim 10^4$ Hz and $1/\sqrt{C(L+L_0)} \gg \omega_{max}$, this approximation is quite good.

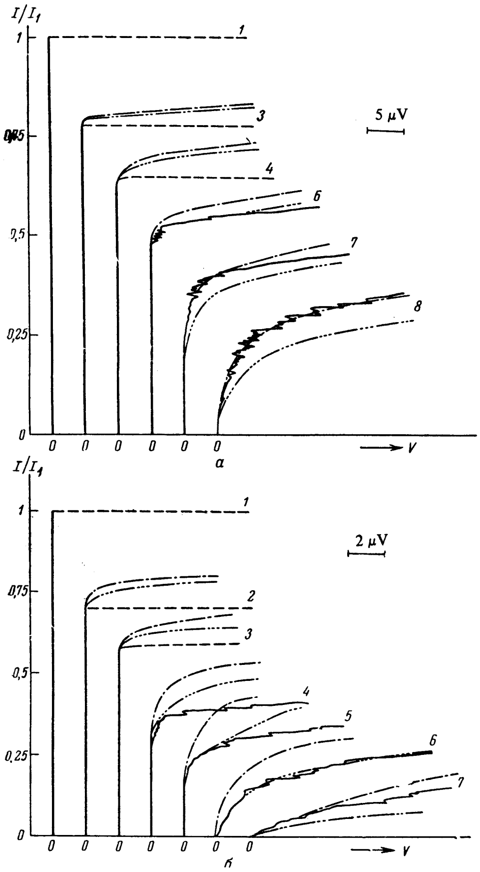

FIG. 1. Influence of low-frequency noise voltages on the initial sections of the current-voltage characteristics: a—H = 0, b—H = 0.35 Oe. The solid lines correspond to the experimental curves (the dashed lines show the break on the single-particle characteristic). The calculated curves are designated by dash-dot lines: ___ . ___ —for R_e = 2 ohms, ___ . . ___ —R_e = 4 ohms, ___ . . . ___ —for R_e = 6 ohms. The numbers at the curves correspond to different noise voltages: 1—U_0 = 0, 2—U_0 = 0.33 V, 3—U_0 = 0.045 V, 4—U_0 = 0.073 V, 5—U_0 = 0.091 V, 6—U_0 = 0.109 V, 7—U_0 = 0.136 V, 8—U_0 = 0.168 V. T = 1.5°K.

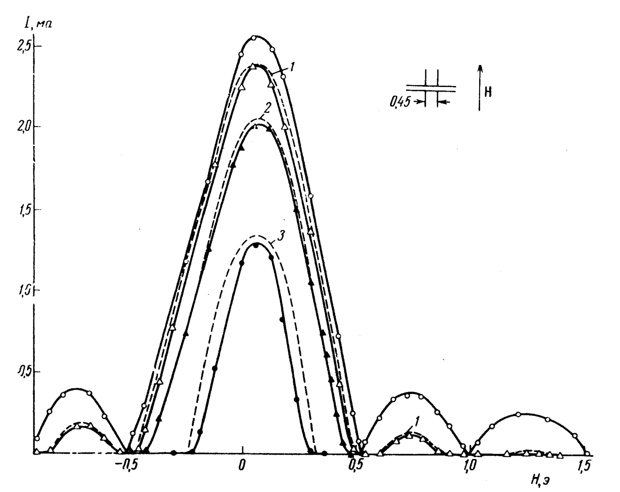

FIG. 2. Influence of low-frequency fluctuations on the magnetic dependence of the critical Josephson current. The solid curves are drawn through the experimental points and the dashed curves are calculated. ○—U_0 = 0, △ and curve 1—U_0 = 0.02 V, ▲ and curve 2—U_0 = 0.045 V, ● and curve 3—U_0 = 0.109 V. Temperature 1.5°K.

etical analysis, the dependence of R on the voltage across the junction and on the noise voltage was disregarded. This is apparently one of the main reasons for the observed discrepancy between the experimental and calculated curves. Better results, as can be seen from Fig. 1, can be obtained by variation of R.

The magnetic dependence of the STC at the lowest registered voltage on the barrier, for different noise levels, is shown in Fig. 2. It is clearly seen that the presence of even weak noise leads to a faster damping of the amplitude of the oscillations.

The STC oscillations in magnetic fields decrease with increasing noise voltage and disappear completely in the case of strong noise. The characteristic I(H), plotted in the absence of noise, differed somewhat from the law $(\pi y)^{-1}\sin\pi y$ (y—magnetic flux expressed in flux quanta). Therefore to calculate the corresponding theoretical curves we chose the experimental I(H) curve at $U_0 = 0$. The use of the experimental curve for $I_1(y)$, in place of $I_1 = I_0(\pi y)^{-1}\sin\pi y$, improved the agreement between the experimental and the theoretical curves. It should be noted that in individual cases there were observed distorted characteristics I(H), connected with the "capture" of the magnetic flux.

In the presence of low-frequency fluctuations, the amplitude of the distorted I(y) dependence usually decreased. In individual cases, however, characteristic changes in the shapes of the I(H, U_0) curves were observed, due apparently to redistribution of the flux inside the tunnel structure. It is impossible to reconcile the curves in the presence of noise with the theoretical ones, even if one uses for I(y) the corresponding experimental curve without noise. After heating the contact above T_c and subsequent lowering of the temperature to T = 1.5°K, the captured flux vanished, and the magnetic characteristics were reproduced fully in the form given in Fig. 2.

The result of the influence of the noise voltage on the resonant "step" of the current-voltage characteristics is illustrated in Fig. 3. In our case the "step" is not small, and perturbation theory is not applicable for the determination of the form of the step in the absence of noise[12]. Therefore in the calculation of the theoretical I(V) dependence at a given U_0 we used the real current-voltage characteristic in the absence of noise, which was averaged by the same method as Eq. (3) over the low-frequency noise. It is seen from the figure that an increase of U_0 leads to an increase of the slope of the step, and consequently leads to a broadening of the radiation line. In the case of large noise voltages, the step on the I-V characteristic disappears completely. We note that certain systematic discrepancies between

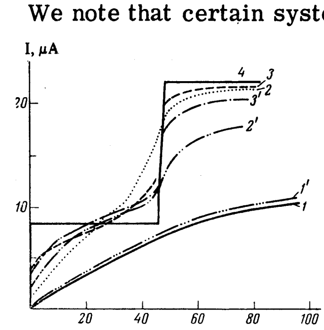

FIG. 3. Influence of low-frequency fluctuations on the resonant step. Different curves correspond to different noise voltages: curves 1 and 1′—U_0 = 4.54 mV, curves 2, 2′—U_0 = 1.91 mV, curves 3, 3′—U_0 = 0.91 mV, curve 4—U_0 = 0. Theoretical curves—2′, 3′, 4′. Temperature 1.5°K, magnetic field 1.90 Oe.

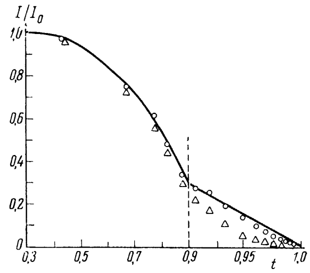

FIG. 4. Influence of low-frequency fluctuations on the temperature dependence of the STC amplitude. Solid line—theoretical curve [13]; O—experimental points for $U_0 = 0$, Δ—experimental points for $U_0 = 0.021$ V.

the calculated curves and the experimental ones at voltages above the step voltage go in a direction opposite to that in the initial section of the I-V characteristic (Fig. 1). One of the causes of the discrepancy is apparently the insufficiently exact description of the shape of the step.

4. CONCLUSION

Our investigation has demonstrated the important role played by low-frequency fluctuations in Josephson tunneling.

There are grounds for assuming that the appearance of fluctuation effects in different experiments is qualitatively similar[3)]. However, when an attempt is made to compare quantitatively the theory with experiment, it is necessary to take into account the possible appearance of induced low-frequency fluctuating noise. A simple estimate of the effective noise temperature Θ_e for the curves obtained in[6] yield $\Theta_e > 300$°K. This result, which at first glance is not understandable, can apparently be explained if account is taken of the induced low-frequency noise. Allowance for the influence of the fluctuations is important not only in the study of the current-voltage and magnetic characteristics of superconducting tunnel junctions, but also in investigation of the $I_0(\Theta)$ temperature dependences. The point is that in the vicinity of T_c the influence of the fluctuations is always appreciable. It is possible that this can explain the discrepancy between the temperature curves obtained theoretically by Ambegaokar and Baratoff[13] and obtained experimentally by Shigi et al.[14]. In any case, our investigation (see Fig. 4) indeed confirms this point of view. It is seen from Fig. 4 that the experimental curve near T_c drops below the theoretical one[13].

In conclusion we note that the experimental and theoretical curves shown in Figs. 1—3 are in good correlation with one another. The discrepancy between the theoretical and experimental curves can be connected with several causes. The most significant apparently are the following: the dependence of the quasiparticle resistance on the voltage across the contact and on the noise voltage, the insufficient screening of the measuring system, and the non-Gaussian character of the noise. Allowance for the influence of the non-Gaussian character of the fluctuations calls for a rather laborious investigation of the output voltage of the G2-12 generator. However, since the noise source in this generator is an ohmic resistance, it can be assumed that the degree of deviation of the noise from Gaussian is insignificant.

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