EFFECTS OF FIELDS IN RESONANT INTERACTION OF OPPOSING WAVES IN A GAS. I

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A semiclassical theory of resonance interaction between two opposing waves of identical frequency in a gas is developed. The intensity of one of the waves is assumed small, and that of the other may be arbitrary. The shape of the absorption line of the weak wave in the presence of the strong one is determined. When the influence of a strong field on the polarization of the medium is taken into account, the shape of the absorption line of the weak wave is found to be qualitatively different from that found when the population effects are considered. The weak-wave absorption coefficient in the middle of the line then does not tend to zero with increasing field strength, but tends to a constant value. This and other phenomena are interpreted as level-splitting effects in a strong and rapidly oscillating field. The main contribution to absorption of a weak wave is not from those atoms whose velocities satisfy the resonance condition in the absence of a strong field, but rather those atoms for which the Doppler shift is compensated by level splitting. A new parameter β is defined and depends only on the level relaxation constants. It yields the contribution of splitting effects to weak-wave absorption when both variation of the level population and polarization of the medium are taken into account. The contribution of the coherent effects is maximal for $\beta = 1$ (equal relaxation constants for both levels). The parameter β is undoubtedly a characteristic quantity in problems pertaining to the interaction of several fields in a gas. Coherent effects can be neglected and balance equations for the population can be employed if $\beta \ll 1$ (the relaxation constants are very different for the two levels). The results of the theory are extended to the case of oppositely traveling waves with different frequencies. The effect of collisions on the shape of the absorption line is considered. Application of the theory to the problems of stability of generation of a certain mode in nonlinear-absorption lasers and to spectroscopic investigations is considered.

INTRODUCTION

M UCH attention is presently being paid to theoretical and experimental investigations of the nonlinear interaction of strong optical fields with moving atoms and molecules (for example,^[1-4]). Interest in such problems is due to the fact that most of them are directly connected with the construction of the theory of gas lasers and serves as a basis for the vigorously developing highresolution spectroscopy using gas lasers.

In the present paper we consider the interaction of two opposing waves of identical frequency, the intensity of one of which is assumed to be small. Certain problems in the interaction of opposing waves of different intensity were considered in the construction of the theory of gas lasers with ring resonators— see^[5]. The formulation and the solution of the problems in these papers had a specific character, namely, they considered the self-consistent problem for a medium and a field in a ring resonator, where principal attention was paid to problems of stability, competition, intensity, etc. of the opposing waves in the generator. In addition, these problems were considered with a number of limitations with respect to the field and with respect to the relaxation constants.

Principal attention has been concentrated on such a spectroscopically important question as the absorption line shape of a weak wave in the presence of a strong one traveling in the opposite direction. Interest in this question is connected with the formation of a narrow dip in the center of the absorption line of the weak opposing wave. This was observed experimentally by Yu. A. Matyugin and by one of the authors (V. Ch.) and was reported in^[6]. The formation of the dip in the center of the line can be used to construct a fully automatically frequency-stabilized laser with nonlinear absorption and with low resonator Q^[7]. Basov et al.^[8] observed narrow resonances in the absorption of a weak opposing wave on the vibrational-rotational transitions of the SF_6 molecule. In^[9] a Zeeman absorption cell was used with a discharge in pure neon both for spectroscopic research and for frequency stabilization. Detailed investigations of narrow resonances in SF₆ were reported quite recently^{(10]}. The results of ^{(p-10]} have shown that</sup></sup>this method is highly efficient for spectroscopic research and for frequency stabilization.

Outwardly, the formation of the dip at the center of the absorption line is similar to the formation of the Lamb dip in the field of a standing wave^[2]. The explanations of these phenomena are different, however. In our case, in view of the weakness of one of the waves, only one dip is produced in the velocity distribution of the atoms. The width of this dip depends on the intensity of the strong wave, and its position depends on the frequency of the detuning Ω relative to the center of the line. This dip lies in the vicinity kv = Ω , where k is the wave number and v is the projection of the velocity of the atom on the direction of propagation of the strong wave. The weak opposing wave will interact with atoms having kv' = $-\Omega$. If Ω is much larger than the width of the dip, then the weak wave does not "feel" the presence of the strong field and consequently the number of atoms with which it interacts does not change in the presence of the strong field. Therefore the absorption coefficient is equal to the non-saturated value. When $\Omega = 0$ both waves interact with the same atoms. This causes the number of atoms with which the weak wave interacts to decrease appreciably, corresponding to a decrease of the absorption.

In^[9,10] the parameters of the dip were considered under the assumption that the influence of the strong field reduces only to a change in the velocity distribution of the atoms. Such a consideration, however, is insufficient, since it is necessary to take into account the influence of the strong field not only on the population difference, but also on the polarization of the medium. A combined account of the influence of these factors on the absorption line shape is the main content of the present paper. Unlike the case of two opposing waves of arbitrary intensity, the solution of our problem can be written in analytic form, making it possible to reveal the role of the characteristic parameters that determine the interaction of the two waves. We have separated the parameter

$$\beta = \gamma / \Gamma$$
, $2 / \gamma = 1 / \gamma_1 + 1 / \gamma_2$,

where Γ is the half-width of the line, and $1/\gamma_1$ and $1/\gamma_2$ are the lifetimes of the upper and lower levels, respectively. It determines the relative contribution of the effects of the population and the polarization, due to the strong field. A rigorous analysis of the indicated phenomena leads to qualitatively different results than $in^{(9,10)}$. For example, the absorption coefficient of the weak wave at the center of the line ($\Omega = 0$) no longer tends to zero with increasing intensity of the strong field, but tends to a constant value that depends only on the parameter β . This result was interpreted as the effect of the splitting of the levels in a strong rapidly oscillating field. In this case the main contribution to the absorption of the weak wave is given by atoms with essentially nonzero velocities.

In Sec. 1 we present a derivation of the line shape with allowance for only the population effects. In Sec. 2, which is the principal one, we present a rigorous analysis of the problem. The results are generalized to include the case of interaction of opposing waves with different frequencies. We investigate the influence of collisions. In Sec. 3 we give a physical interpretation of the results and explain the meaning of the parameter β . In the last section we consider briefly certain applications of the results. We consider the question of obtaining stable generation at one mode in lasers with nonlinear absorption at large saturation parameters. Spectroscopic applications are discussed.

1. POPULATION EFFECTS

Before we proceed to an exact solution of the problem, we present an analysis that takes into account the influence of a strong field on the population difference. This will help subsequently in the analysis of the exact solution and in separating the effects connected with the population and with the level splitting.

Assume that a strong wave with amplitude 2E, propa-

gates in the positive z direction, and a weak wave with amplitude $2E_{\rm p}$ propagates in the negative direction. The probability of the transition of an atom from level 1 to level 2 under the influence of a weak field $E_{\rm r}$ is

$$W = \frac{2\Gamma}{\gamma_i} (E_{-}d)^2 \frac{1}{(\Omega + kv)^2 + \Gamma^2}.$$
 (1)

Here $\Gamma = (\gamma_1 + \gamma_2)/2$ is the half-width of the line, γ_1 and γ_2 are the widths of the upper and lower levels, v is the projection of the velocity of the atom on the z axis, and d is the matrix element of the dipole moment of the atom in units of \hbar . The transition probability, averaged with a Maxwellian distribution

$$f_{0}(v) = (\sqrt[]{\pi v_{0}})^{-1} \exp \{-v^{2} / v_{0}^{2}\},\$$

has for $kv_0\gg\Gamma$ the value

$$W_{0} = \frac{2\sqrt{\pi} (E_{-}d)^{2}}{kv_{0}\gamma_{1}} \exp\left\{-\frac{\Omega^{2}}{(kv_{0})^{2}}\right\}$$

In the presence of a strong field, the population difference between levels 1 and 2, normalized to unity, will take the form

$$\Delta n^{(0)} = f_0(v) \left[1 - \frac{\Gamma^2 \chi}{(kv - \Omega)^2 + \Gamma^2 (1 + \chi)} \right], \qquad (2)$$

where $\chi = 4(\mathbf{E}_{\star}d)^2/\gamma\Gamma$ is the saturation parameter. We now average W with the distribution (2):

$$\overline{W} = \int_{-\infty}^{\infty} dv \,\Delta n^{(0)} W = \int_{-\infty}^{\infty} dv \frac{1}{\sqrt{\pi} v_0} \exp\left\{-\frac{v^2}{v_0^2}\right\}$$
$$\times \left[1 - \frac{\Gamma^2 \chi}{(kv - \Omega)^2 + \Gamma^2(1 + \chi)}\right] \frac{2\Gamma}{\gamma_1} (E_-d)^2 \frac{1}{(\Omega + kv)^2 + \Gamma^2} \cdot (3)$$

Carrying out the integration under the condition $kv_0 \gg \Gamma \sqrt{1+\chi}$ and changing over to the absorption coefficient of the weak wave, we obtain

$$\frac{a'}{\alpha_0} = \left(1 - b \frac{\tilde{\Gamma}^2}{\Omega^2 + \tilde{\Gamma}^2}\right) \exp\left\{-\left(\frac{\Omega}{kv_0}\right)^2\right\},\tag{4}$$

where¹⁾ α' is the absorption coefficient of the weak field in the presence of the strong one, α_0 is the unsaturated absorption coefficient at the line center

 $(\alpha_0 = 4\pi^{3/2} d^2 N/kv_0, N = N_1 - N_2$ is the unsaturated population difference),

$$b = \frac{\chi}{1 + \chi + \sqrt{1 + \chi}}, \quad \tilde{\Gamma} = \Gamma \frac{1 + \sqrt{1 + \chi}}{2}$$

Expression (4) describes the absorption line shape of the weak opposing wave in the presence of the strong one. At the center of the line there is a dip of dispersion form with width $2\tilde{\Gamma}$ and depth b. It is interesting to note that at the center of the line the absorption coefficient of the weak opposing wave is equal to

 $\alpha' = \alpha_0 (1 + \chi)^{-1/2}$, which coincides with the absorption coefficient of the strong wave.

At small saturation parameters $\chi \ll 1$ we get from (4)

$$\frac{\alpha'}{\alpha_0} = \left(1 - \frac{\chi}{2} \frac{\Gamma^2}{\Gamma^2 + \Omega^2}\right) \exp\left\{-\left(\frac{\Omega}{kv_0}\right)^2\right\}.$$
 (5)

In the case of small saturation, the width of the dip is equal to 2Γ and the depth is proportional to the intensity

¹⁾The exponential in the second term makes the latter more accurate when $\Omega \gg \Gamma_0$. When $\Omega \sim \Gamma_0$ it should be omitted.

of the strong field. Unlike the case of a weak standing wave, in (5) the saturation parameter χ enters only in the resonant term; there is no homogeneous part of the saturation. This is easily understood; the backward wave is weak and produces no saturation. With increasing field, the absorption coefficient α' at the center of the line decreases like $1/\sqrt{\chi}$.

In the case of strong fields² ($\chi \gg 1$)

$$\frac{\alpha'}{\alpha_0} = \frac{\Omega^2}{\Omega^2 + \tilde{\Gamma}^2} \exp\left\{-\left(\frac{\Omega}{kv_0}\right)^2\right\}.$$
 (6)

For $\Omega = 0$, the absorption coefficient of the weak wave vanishes. Indeed, as follows from (3), when $\Omega = 0$ the absorption receives contributions from the atoms for which v = 0, and the population difference tends to zero. When $\chi \gg 1$ the width of the dip in the absorption of the weak wave is equal to half the Bennett dip in the velocity distribution of the atoms.

LINE SHAPE

The strong and weak waves traveling in opposite directions will be assumed to be two longitudinal modes of a resonator of length L with wave numbers k and -k. The damping decrement of the mode will correspond to the absorption coefficient of the wave. Here and throughout we shall consider the case of weak absorption, when the change of the amplitudes of the weak and of the strong waves along the z axis can be neglected. Let E(z, t) be the electric-field component perpendicular to the z axis. The boundary conditions are chosen to be periodic:

$$E(0,t) = E(L,t).$$
 (7)

This enables us to write

$$E(z, t) = E_{+}(t)e^{ikz} + E_{-}(t)e^{-ikz},$$
(8)

and it is easy to choose a value of L such as to satisfy (7).

(The condition (7) calls for $k = 2\pi l/L$, where l is an arbitrary integer.)

A. We now find the damping coefficient of the field in the resonator. The resonant interaction of the field with the atoms leads to the appearance of polarization of the medium. Using Maxwell's equations, we write an equation for E(z, t):

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) E(z,t) = -4\pi \frac{\partial^2 P(z,t)}{\partial t^2}.$$
 (9)

The polarization of the medium, taking (8) into account, will be written in the form

$$P(z, t) = \chi_{+}E_{+}e^{-i\omega t + ikz} + \chi_{-}E_{-}e^{-i\omega t - ikz}, \qquad (10)$$

where χ_+ and χ_- are the polarizabilities of the medium for E₊ and E₋. Multiplying both halves of (9) by e^{ikz} and integrating with respect to z from 0 to L, we have

$$\frac{\partial^2 E_-(t)}{\partial t^2} + \omega^2 E_-(t) = 4\pi \omega^2 \chi_- E_- e^{-i\omega t}.$$
(11)

We seek a solution of this equation in the form

$$E_{-}(t) = E_{-}e^{-i\omega' t}.$$

Assuming χ_{-} to be a small quantity, we can always replace $e^{-i\omega t}$ in the right side of (11) by $e^{-i\omega t}$. From (11) we have

$$\omega^{\prime 2} - \omega^2 = -4\pi\omega^2 \chi_{-}.$$

Since $\omega' + \omega \approx 2\omega$, we get

$$\omega' - \omega = -2\pi\omega\chi_{-}.$$
 (12)

From (12) we get for the attenuation constant of the field (E_ $\sim e^{-\,\nu t})$

$$v = 2\pi\omega \operatorname{Im} \chi_{-}$$
.

For the power absorption coefficient per unit length of the weak wave E_{-} we obtain

$$\alpha = 4\pi k \operatorname{Im} \chi_{-}.$$
 (13)

B. Let us find the polarization of the medium

$$P = d \int_{-\infty}^{\infty} \rho_{21}(z,t,v) dv.$$
 (14)

Here $\rho_{21}(z, t, v)$ is the nondiagonal density matrix element.

In order not to clutter up the exposition, we assume the widths of the upper and lower levels to be the same, i.e., $\gamma_1 = \gamma_2 = \gamma$. We shall present later on the results for $\gamma_2 \neq \gamma_1$.

We write the equation of motion for the matrix element $\rho_{21} = \rho_{21}(z, t, v)$ of the density matrix and the population difference $\Delta n = \rho_{11} - \rho_{22}$ in the field of two opposing waves of equal frequency ω :

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial z} + \gamma\right) \Delta n = -2id[(E_+ e^{-i\omega t + ihz} + E_- e^{-i\omega t - ihz})\rho_{21} - c.c] + \gamma N(v), \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial z} + i\omega_{21} + \gamma\right)\rho_{21} = id(E_+ e^{-i\omega t + ihz} + E_- e^{-i\omega t - ihz})\Delta n;$$
(15)

Without loss of generality, we shall assume E_{\star} to be a real quantity, since this can always be done by choosing the origin for the coordinate z. The substitution

$$\rho_{21} = \rho e^{-i\omega t} \tag{16}$$

reduces (15) to the following equations:

$$\left(v \frac{\partial}{\partial z} + \gamma \right) \Delta n = -2id \left[\left(E_{+} e^{ikz} + E_{-} e^{-ikz} \right) - \mathbf{C} \cdot \mathbf{C} \cdot \right] + \gamma N(v),$$

$$\left(v \frac{\partial}{\partial z} - i\Omega + \gamma \right) \rho = id \left(E_{+} e^{ikz} + E_{-} e^{-ikz} \right) \Delta n.$$

$$(17)$$

We shall solve the system (17) by the perturbation method, i.e., we seek the solution in the form

$$\Delta n = \Delta n^{(0)} + \Delta n^{(1)} + \dots, \quad \rho = \rho^{(0)} + \rho^{(1)} + \dots, \quad (18)$$

assuming the field E_{-} to be weak.

In the zeroth approximation $\Delta n^{(0)}$ is given by (2), and $\rho^{(0)}$ is given by

$$\rho^{(0)} = \frac{dE_+ \Delta n^{(0)}}{kv - \Omega - i\gamma} e^{ikz}.$$
(19)

To find the first approximation we have from (17) the system (∂_{α})

$$\left(v \frac{\partial}{\partial z} + \gamma \right) \Delta n^{(1)} = -2iE_{+}d\left(\rho^{(1)*}e^{ihz} - \rho^{(1)}e^{-ihz}\right) - 2id\left(E_{-}e^{-ihz}\rho^{(0)*} - E_{-}*e^{ihz}\rho^{(0)}\right), \left(v \frac{\partial}{\partial z} - i\Omega + \gamma \right) \rho^{(1)} = iE_{+}de^{ihz}\Delta n^{(1)} + iE_{-}de^{-ihz}\Delta n^{(0)}.$$

²⁾ To find the line shape in this case it is possible to dispense with the condition $kv_0 \gg \Gamma \sqrt{\chi}$. The function W(1) has a sharp maximum at the point $v = \Omega/k$ and consequently the function $\Delta_n^{(0)}$ can be taken at this point outside the integral sign in (3). We note that formula (6) is written out accurate to $1/\sqrt{\chi}$

If, recognizing that Δn is real, we seek the solution in the form

$$\rho^{(1)} = aE_{-}de^{-ikz} + bE_{-}*de^{3ikz}, \quad n^{(1)} = cE_{-}de^{-2ikz} + c*E_{-}*e^{2ikz}, \quad (20)$$

then we obtain for a, b^* , and c the following system of algebraic equations:

$$(2kv + i\gamma)c = -2dE_{+}(a - b^{\bullet}) + \frac{2E_{+}d}{kv - \Omega + i\gamma}\Delta n^{(0)},$$

$$(3kv - \Omega + i\gamma)b^{\bullet} = E_{+}dc,$$

$$(kv + \Omega + i\gamma)a = -E_{+}dc - \Delta n^{(0)}.$$

Hence

$$a = -\frac{\Delta n^{(0)}}{kv + \Omega + i\gamma} \left[1 + \frac{4(E_+d)^2 f(kv)}{kv - \Omega - i\gamma} \right], \qquad (21)$$

where

$$f(x) = \frac{(3x - \Omega + i\gamma)(x + i\gamma)}{(2x + i\gamma)[(3x - \Omega + i\gamma)(x + \Omega + i\gamma) - 4(E_{+}d)^{2}]}.$$
 (22)

Using now formulas (13), (10), (14), (16), (18), (20), (21), and (2), we obtain the absorption coefficient of the wave E_:

$$\frac{\alpha}{\alpha_0} = \frac{\alpha'}{\alpha_0} + \frac{4(E_-d)^2}{\pi} \operatorname{Im} \left\{ -\int_{-\infty}^{\infty} \exp\left\{ -\frac{x^2}{(kv_0)^2} \right\} \right\}$$
$$\times \frac{(x-\Omega-i\gamma)f(x)}{[(x-\Omega)^2+\Gamma_0^2](x+\Omega+i\gamma)} dx \right\},$$
(23)

where $\Gamma_0 = \Gamma \sqrt{1 + \chi}$ is the width of the Bennett "dip."

The first term is exactly equal to the expression for the absorption coefficient (4) obtained by us on the basis of the population effects (see Sec. 1). The second term takes into account the influence of the strong field on the polarization.

Just as in the consideration of the population effects, we shall assume that $kv_0 \gg \gamma$. For different ratios of the Doppler width kv_0 to the field $E_{\star}d$, we obtain two characteristic regions of the strong field:

a) case of strong field

$$kv_0 \gg E_+d;$$
 (23a)

b) case of very strong field

$$E_+d/\gamma \gg 1.$$
 (23b)

The condition (23a) imposes an upper limit on the saturation parameter, but since $kv_0 \gg \gamma$, it follows that χ can be larger than unity. In the case of (23b) the relation between $E_{\star}d$ and kv_0 is arbitrary.

We first calculate the integral (23) subject to the condition (23a). Since the main contribution to the integral is made by a region of the order of $\Delta x \sim \text{Ed}$ in the vicinity of Ω , it follows that owing to the condition (23) we can take the exponential outside the integral sign. To this end we note that f(x) has poles in the lower half-plane. In the upper half-plane of the integral expression there is only one pole, equal to $\Omega + i\Gamma_0$. Closing the integration contour in the upper half-plane, we get

$$\frac{\alpha}{\alpha_{0}} = \frac{\alpha'}{\alpha_{0}} + 4(E_{+}d)^{2} \frac{\Gamma_{0} - \gamma}{\Gamma_{0}} \operatorname{Re} \left\{ -\frac{f(\Omega + i\Gamma_{0})}{2\Omega + i(\Gamma_{0} + \gamma)} \right\} \exp \left\{ -\left(\frac{\Omega}{kv_{0}}\right)^{2} \right\}.$$
(24)

This expression determines the absorption coefficient E_{-} under the condition (7). It differs from (4) in the second term, which leads to essential qualitative features. The most noticeable one is the deviation from (4) at $\Omega = 0$ for $E_{+}d/\gamma \gg 1$.

Formula (18) gives a finite value 3/8 for $\Omega = 0$, whereas α'/α_0 from (4) vanishes.

We now calculate the absorption coefficient in (23) in the case of a very strong field (23b). Expression (23) for α can in this case be rewritten in the form

$$\frac{\alpha}{\alpha_{\bullet}} = \frac{\alpha'}{\alpha_{\bullet}} + \frac{2(E_{+}d)^{2}}{\pi} \int_{-\infty}^{\infty} dx \frac{\exp\{-x^{2}/(kv_{\bullet})^{2}\}}{(x-\Omega)^{2}+\Gamma_{\bullet}^{2}} \cdot \times \operatorname{Im}\left\{\frac{(x-\Omega/3)(x-\Omega)}{(x+\Omega+i\gamma)\left[(x-\Omega/3)(x+\Omega)-\frac{4}{3}(E_{+}d)^{2}+i\gamma\varphi(x)\right]}\right\}, (25)$$

where

$$\varphi(x) = \frac{1}{3} \left\{ 4x + \frac{(3x - \Omega)(x + \Omega) - 4(E_+d)^2}{3x} \right\}.$$

We neglect the imaginary part in the numerator, since it is proportional to γ . The main contribution to the integral is made by regions of the order of γ in the vicinity of the points

$$x_{1,2} = -\frac{1}{3}\Omega + \frac{2}{3}\sqrt{\Omega^2 + 3(E_+d)^2}, \qquad (26)$$

which are roots of the equation

$$(x - \frac{1}{3}\Omega) (x + \Omega) - \frac{4}{3}(E_+d)^2 = 0$$

Carrying out the integration and substituting the values of x_1 and x_2 , we obtain

$$\frac{\alpha}{\alpha_{0}} = \frac{1}{4\Delta} \left[\frac{(\Delta - \Omega)^{2} (\Delta - 2\Omega)}{\frac{1}{3} (\Delta - 2\Omega)^{2} + 3(E_{+}d)^{2}} \exp\left\{-\left(\frac{2\Delta - \Omega}{3kv_{0}}\right)^{2}\right\} + \frac{(\Delta + \Omega) (\Delta + 2\Omega)}{\frac{1}{3} (\Delta + 2\Omega)^{2} + 3(E_{+}d)^{2}} \exp\left\{-\left(\frac{2\Delta + \Omega}{3kv_{0}}\right)^{2}\right\}, \quad (27)$$

where $\Delta = \sqrt{\Omega^2 + 3(E_+d)^2}$.

C. Let us generalize the results to the case of different lifetimes of the upper and lower levels. Calculations analogous to the preceding ones yield for the absorption coefficient, under the condition³⁾ $kv_0 \gg \Gamma_0$:

$$\frac{\alpha}{z_0} = \frac{\alpha'}{\alpha_0} + 4(E_+d)^2 \frac{\Gamma_0 - \Gamma}{\Gamma_0} \operatorname{Re}\left\{-\frac{f(\Omega + i\Gamma_0)}{2\Omega + i(\Gamma_0 + \Gamma)}\right\} \exp\left\{-\left(\frac{\Omega}{kv_0}\right)^2\right\},\tag{28}$$

where

$$f(x) = (3x - \Omega + i\Gamma) (x + i\Gamma) \left(2x + i\frac{\gamma_1 + \gamma_2}{2}\right)$$
$$\times \left[(3x - \Omega + i\Gamma) (x + \Omega + i\Gamma) (2x + i\gamma_1) (2x + i\gamma_2) - 4(E_+d)^2 (2x + i\Gamma) \left(2x + i\frac{\gamma_1 + \gamma_2}{2}\right) \right]^{-1},$$

and α'/α_0 is given by formula (4). Formula (28) describes a dip of complicated shape at the center of the line. An analysis of (28) shows that the additional contribution of the effects of polarization to the absorption coefficient is maximal at the line center and is determined by the parameter $\beta = \gamma/\Gamma$; at large detunings it decreases like $1/\Omega^2$.

At the center of the line we have (see Fig. 1)

$$\frac{a}{a_0}\Big|_{a=0} = \frac{1}{\sqrt{1+\chi}} + \beta\chi \frac{a-1}{a}$$

$$\times \frac{(3a+1)(2a+1)}{(3a+1)(a+1)(2a+\beta_1)(2a+\beta_2) + (2a+1)^2\chi\beta}, \quad (29)$$

where $\beta_1 = \gamma_1/\Gamma$, $\beta_2 = \gamma_2/\Gamma$, $a = \sqrt{1 + \chi}$. Comparing the first and second terms in (29), we note that allowance

³⁾We note that formula (28) is valid when $\Gamma \neq (\gamma_2 + \gamma_2)/2$, for example, in collisions leading to the collapse of the phase (see Item D of the present section).



FIG. 1. Dependence of the absorption coefficient of the weak wave on the quantity $(1 + \chi)^{-\frac{1}{2}}$ at different parameters β ; the dashed curve corresponds to $\beta \rightarrow 0$.

FIG. 2. Absorption line shape of weak field for four values of χ ; $\beta = 1$ -solid curves and $\beta \rightarrow 0$ -dashed curves; $\Gamma/kv_0 = 0.02$.

for the second term must be made when $\beta \chi \sim 1$. The following two expansions of formula (29) are useful: a) for $\chi \ll 1$

$$\frac{\alpha}{\alpha_{o}}\Big|_{\alpha=0} = \left(1 - \frac{\chi}{2} + \frac{3}{8}\chi^{2}\right) + \frac{3}{4}\beta\left[\frac{\chi^{2}}{(2+\beta_{1})(2+\beta_{2})}\right]. \quad (30)$$

The first three terms are the expansion of $1/\sqrt{1+\chi}$. The last term is due to the additional contribution made to the polarization. It is proportional to β and arises only in the second order in the saturation.

b) for $\beta \ll 1$

$$\frac{\alpha}{\alpha_{0}} = \frac{1}{\gamma + \chi} + \beta \frac{\chi^{2}}{2(1+\chi)(\gamma + \chi + 1)^{2}}.$$
 (31)

It is important to note here the following singularity. Formula (31) gives satisfactory accuracy for arbitrary values of the parameters. In the most unfavorable case $\gamma_1 = \gamma_2$ and $\chi \rightarrow \infty$, the error amounts to 30%. Recognizing that the lifetimes as a rule differ in the optical band, the use of (31) is justified in a large range of parameters.

It follows from the foregoing analysis that the contribution of the polarization effects is proportional to the parameter β . For the analysis of the line shift, in view of the complicated form of (28), we present an expansion of (28) in terms of the parameter β :

$$\frac{\alpha}{\alpha_{o}} = \frac{\alpha'}{\alpha_{o}} + \chi \frac{\beta}{2} \frac{\Gamma_{o} - \Gamma}{\Gamma_{o}} \frac{\Gamma^{2} [\Gamma_{o} (\Gamma_{o} + \Gamma)^{3} - 4\Omega^{4} - 3\Omega^{2} (\Gamma_{o} + \Gamma)^{2}]}{(\Omega^{2} + \Gamma_{o}^{2}) [4\Omega^{2} + (\Gamma_{o} + \Gamma)^{2}]^{2}} \times \exp\left\{-\left(\frac{\Omega}{kv_{o}}\right)^{2}\right\}$$
(32)

The accuracy of (32) in comparison with (31) increases with increasing Ω . In the case of a very strong field with $E_{\star}d/\gamma \gg 1$ we have in lieu of (27)

$$\frac{\alpha}{\alpha_{0}} = \frac{1}{4\psi} \left[\frac{(\psi + \nu)^{2}(\psi + 2\nu)}{\frac{1}{3}(\psi + 2\nu)^{2} + \frac{3}{4}} \exp\left\{ -\left(\frac{2\psi + \nu}{3}\frac{\Gamma_{0}}{kv_{0}}\right)^{2} \right\} + \frac{(\psi - \nu)^{2}(\psi - 2\nu)}{\frac{1}{3}(\psi - 2\nu)^{2} + \frac{3}{4}} \exp\left\{ -\left(\frac{2\psi - \nu}{3}\frac{\Gamma}{kv_{0}}\right)^{2} \right\} \right], \quad (33)$$

where $\nu = \Omega/\Gamma_0$, $\psi = \sqrt{\nu^2 + \sqrt[3]{4}\beta}$. For $\nu = 0$ we get

$$\frac{\alpha}{\alpha_0}\Big|_{\mathfrak{g}=\mathfrak{o}} = \frac{3}{2} \frac{\beta}{3+\beta} \exp\left\{-\frac{1}{3} \beta \left(\frac{\Gamma_0}{kv_0}\right)^2\right\}.$$
 (34)

For large detunings $\nu^2 \gg \frac{3}{4}\beta$ we obtain from (33) $\frac{\alpha}{\alpha_e} = \left\{ 1 + \frac{1}{4} \frac{\beta}{\nu^2} \left[\frac{1}{2} - \frac{\nu^2}{\nu^2 + \frac{1}{4}} \right] \right\}$

$$\frac{v^2}{v^2+\frac{1}{4}}\exp\left\{-\left(\frac{\Omega}{kv_0}\right)^2\left(1+\frac{1}{2}\frac{\beta}{v^2}\right)\right\}.$$
(35)

From Fig. 2 and from expansion (35) we see that the additional contribution of the polarization effects has a different sign, depending on the detuning frequency. At $\Omega \approx 0$ it is positive, and at $\Omega \gg \Gamma_0$ it is negative.

D. Let us consider the influence of collisions. Formula (28) can be generalized to the case of collisions by redefining the relaxation constants γ_1 and γ_2 and Γ , and consequently also the saturation parameter χ :

$$\gamma_1 \rightarrow \gamma_1 + \nu_1, \quad \gamma_2 \rightarrow \gamma_2 + \nu_2, \quad \Gamma \rightarrow \frac{1}{2}(\gamma_1 + \gamma_2) + \tilde{\nu},$$

where ν_1 and ν_2 are respectively the frequencies of the quenching collisions at the levels 1 and 2, and $\tilde{\nu}$ is the collision frequency that leads to a collapse of the phase. Depending on the collision model, the influence of the latter may turn out to be qualitatively different. We shall demonstrate this separately using as an example quenching collisions and collisions that collapse the phase. We shall analyze their influence only on the parameter β , which indeed determines the features of the absorption of a weak wave in the presence of a strong one.

Collisions that lead to collapse of the phase increase the line width without changing the lifetime at the levels. For the parameter β we have in this case

$$\beta_{i} = \beta_{0} \frac{\gamma_{i} + \gamma_{2}}{\gamma_{i} + \gamma_{2} + 2\tilde{\nu}},$$

where β_0 is the parameter in the absence of collisions. It is easy to see that such collisions decrease the contribution of the effect of the strong field to the polarization. When $\tilde{\nu} \to \infty$ the parameter $\beta_1 \to 0$ and the main effect is the effect of the population.

In the case of quenching collisions the picture can be different. Here

$$\beta_{2} = \frac{4(\gamma_{1} + \nu_{1})(\gamma_{2} + \nu_{2})}{(\gamma_{1} + \gamma_{2} + \nu_{1} + \nu_{2})^{2}}.$$

At equal collision frequencies on the upper and lower levels, the parameter β_2 increases, corresponding to an increase of the contribution of the polarization to the absorption coefficient of the weak wave. When ν_1 and ν_2 differ greatly, the parameter β_2 decreases. We emphasize once more that in each of the considered cases the collisions influence also the saturation parameter χ .

E. Let us consider the case of opposing waves with different frequencies. We shall show that the results can be generalized to this case of interaction of a strong and weak wave. The system of equations (15) now takes the form

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial z} + \gamma\right) \Delta n = -2id[(E_+ e^{-i\omega_+ t + ik_+ z} + E_- e^{-i\omega_- t - ik_- z})\rho_{21} - C_+ C_-] + \gamma N(v),$$

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial z} + i\omega_{21} + \gamma\right) \rho_{21} = id[E_+ e^{-i\omega_+ t + ik_+ z} + E_- e^{-i\omega_- t - ik_- z}]\Delta n. (36)$$

Here $2E_{+}$ and ω_{+} are the amplitude and frequency of the strong-field waves, and $2E_{-}$ and ω_{-} are those for the weak field; $k_{+} = \omega_{+}/c$, $k_{-} = \omega_{-}/c$. The absorption coefficient of the weak wave is best sought in a coordinate system moving with velocity $V = -\Delta/2k$, where $\Delta = \omega_{-} - \omega_{+}$ is the detuning of the frequency of the weak wave relative to the strong one. The new coordinate z' is connected with the old one by the usual relation

$$z \doteq z' + Vt. \tag{37}$$

On going over to the new coordinate system we note that in the optical region $\Delta^2/ck\gamma \ll 1$, $V\Delta/c\gamma \ll 1$. This makes it possible to discard in the exponential the terms of the order of $t\Delta^2/ck$ and $z'\Delta/c$. Equations (36) in the new coordinate system coincide with (15), and instead of a Maxwellian distribution function we now have

$$f(v) = \frac{1}{\sqrt{\pi} v_0} \exp\left(\frac{v - \Delta/2k}{v_0}\right)^2$$

Therefore to find the absorption coefficient of the weak wave we can use expressions (3) and (23), and Ω should be replaced by $\Omega_+ + \Delta/2$, where $\Omega_+ = \omega_+ - \omega_{21}$ is the frequency difference between the strong wave and the center of the line. We have

$$\frac{a}{a_{0}} = \frac{a_{1}}{a_{0}} + \frac{4(E_{+}d)^{2}}{\pi} \operatorname{Im} \left\{ -\int_{-\infty}^{\infty} \exp \left\{ -\left(\frac{x - \Delta/2}{kv_{0}}\right)^{2} \right\} \\ \times \frac{(x - \Omega_{+} - \Delta/2 - i\gamma)f(x)}{\left[(x - \Omega_{+} - \Delta/2)^{2} + \Gamma_{0}^{2}\right][x + \Omega_{+} + \Delta/2 + i\gamma]} dx, \quad (38)$$

where f(x) is given by (28) with the substitution $\Omega \to \Omega_+ + \Delta/2$, and

$$\frac{a_{1}}{a_{0}} = \frac{1}{\pi} \int_{-\infty}^{\infty} \exp\left\{-\left(\frac{x-\Delta/2}{kv_{0}}\right)^{2}\right\} \left[1 - \frac{\Gamma^{2}\chi}{(x-\Omega_{+}-\Delta/2)^{2}+\Gamma^{2}(1+x)}\right] \times \frac{\Gamma^{2}}{(x+\Omega_{+}+\Delta/2)^{2}+\Gamma^{2}} dx.$$
(39)

Just as in Sec. 2, let us analyze expression (39) first for $kv_0 \gg \Gamma_0$. Comparing (3) and (4) with (39), we note that the second term makes the main contribution when $\Omega_+ + \Delta/2 \sim \Gamma$ in the vicinity of $\Omega_+ + \Delta/2 = 0$. This is physically understandable. The wave E₊ interacts resonantly with atoms whose velocities are equal to kv_1 = Ω_+ , and the wave E₋ with atoms for which $kv_2 = -\Omega_-$ - Δ . The condition $\Omega_+ + \Delta/2 = 0$ defines those atoms that interact simultaneously with both waves. This corresponds to symmetrical location of the frequencies of the strong and weak fields relative to the center of the line. On the other hand, if $\Omega_+ + \Delta/2 \gg \Gamma_0$, then the second term is small and the absorption coefficient of the weak wave is equal to the unsaturated value. This makes it possible to take the exponential at the point x = 0 outside the integral sign. We then have for α_1

$$\frac{a_{i}}{a_{o}} = \exp\left\{-\left(\frac{\Omega_{+}+\Delta}{kv_{o}}\right)^{2}\right\} - \frac{b\tilde{\Gamma}^{2}}{\left(\Omega_{+}+\Delta/2\right)^{2}+\tilde{\Gamma}^{2}}\exp\left\{-\left(\frac{\Omega_{+}}{kv_{o}}\right)^{2}\right\}, (40)$$

where Γ and b are given in (4).

We can calculate analogously the second term in (38):

$$\frac{\alpha}{\alpha_{0}} = \frac{\alpha_{1}}{\alpha_{0}} + 4(E_{+}d)^{2} \frac{\Gamma_{0} - \gamma}{\Gamma_{0}}$$

$$\cdot \operatorname{Re}\left\{-\frac{f(\Omega_{+} + \Delta/2 + i\Gamma_{0})}{2(\Omega_{+} + \Delta/2) + i(\Gamma_{0} + \gamma)}\right\} \exp\left\{-\left(\frac{\Omega_{+}}{kv_{0}}\right)^{2}\right\}, \quad (41)$$

where f(x) was defined in (28).

For a very strong field $E_{+}d/\gamma \gg 1$ we obtain

$$\frac{a}{a_{0}} = \frac{1}{4\psi} \left[\frac{(\psi + v)(\psi + 2v)}{\frac{1}{3}(\psi + 2v)^{2} + \frac{3}{4}} \exp\left\{ -\left(\frac{2\psi + v}{3} + \frac{\Delta}{2\Gamma_{0}}\right) \left(\frac{\Gamma_{0}}{kv_{0}}\right)^{2} \right\} + \frac{(\psi - v)^{2}(\psi - 2v)}{\frac{1}{3}(\psi - 2v)^{2} + \frac{3}{4}} \exp\left\{ -\left(\frac{2\psi - v}{3} - \frac{\Delta}{2\Gamma_{0}}\right) \left(\frac{\Gamma_{0}}{kv_{0}}\right)^{2} \right\} \right], \quad (42)$$

where $\nu = (\Omega_+ + \Delta/2)/\Gamma_0$, $\psi = \sqrt{\nu^2 + \frac{3}{4}\beta}$.

3. EFFECTS OF SPLITTING

Let us now explain the physical meaning of the results. In Sec. 2 it was shown that the first term in (23) is connected with population effects. The second term, as already mentioned, is connected with the influence of the strong field on the polarization of the medium. The probability of finding the atom at a level oscillates with a low frequency that depends on the value of the field and on the detuning (see, for example^[11], Sec. 40). This leads to a change in the dipole moment of the atom and, as a consequence, to a change of the absorption line shape of the second signal^[12]. Sometimes phenomena of this type are conveniently interpreted as splitting of the levels in a strong rapidly oscillating field^[13]. The use of the model of the splitting of the levels makes it possible to reveal easily the resonance conditions.

Assume that in the absence of the strong field the energies of the upper and the lower levels are respectively E_m and E_n . If the moving atom is acted upon by a strong monochromatic field whose frequency deviates from the transition frequency ω_{mn} by an amount Ω , then the level E_m splits into two, with

$$E_m^{(1)} = E_m + \epsilon/2 + \sqrt{(\epsilon/2)^2 + (E_+d)^2},$$

$$E_m^{(2)} = E_m + \epsilon/2 - \sqrt{(\epsilon/2)^2 + (E_+d)^2}.$$

The level E_n also splits:

$$E_n^{(1)} = E_n - \varepsilon/2 - \sqrt{(\varepsilon/2)^2 + (E_+d)^2},$$

$$E_n^{(2)} = E_n - \varepsilon/2 + \sqrt{(\varepsilon/2)^2 + (E_+d)^2},$$

where $\epsilon = \Omega - kv$. The indicated splittings causes the moving atom to have not one but three resonant frequencies in the strong field:

$$\omega_{1,2} = \omega_{mn} + \varepsilon \pm \sqrt{\varepsilon^2 + 4(E_+d)^2}, \quad \omega_3 = \omega_{mn} + \varepsilon$$

The weak opposing wave will apply to such an atom a field of frequency $\omega' = \omega_{mn} + \Omega + kv$. The resonance conditions determine the velocities of the atoms that effectively absorb the weak wave. From the conditions $\omega_{1,2} = \omega'$, $\omega_3 = \omega'$ we obtain

$$(kv)_{1,2} = -\frac{1}{3}\Omega \pm \frac{2}{3}\sqrt{\Omega^2 + 3(E_+d)^2}, \quad (kv)_3 = \Omega.$$
(43)

As already seen in the preceding section, regions of the order of Γ (see (26)) near the same velocities made the main contribution in the integration over the velocities in (25). This allows us to conclude that the atoms whose velocities satisfy the condition (43) are resonant for the weak opposing wave. For $\Omega = 0$ these velocities are

$$(kv)_{1,2} = \pm \frac{2}{\sqrt{3}} E_+ d, \quad (kv)_3 = 0.$$

In the absence of an external field, with allowance for the Doppler shift, the resonant frequency of these atoms for the weak field was $\omega = \omega_0 \pm (kv)_{1,2}$. Since such atoms now turn out to be at resonance, the effect of the field can be interpreted as compensation of the Doppler shift by the level splitting.

It is now easy to explain why the absorption coefficient tends to the constant value (24) with increasing field, and not to zero, as would follow from a consideration of the population effects (4). When $\Omega = 0$ the main contribution is made by atoms whose velocities are equal to $\pm 2E_{\pm}d/\sqrt{3}$. It is obvious that the contribution of



FIG. 3. Distribution of the population difference with respect to velocity for two values of the strong field, E_+ and E'_+ , at $\Omega = 0$. The shaded regions correspond to the number of atoms that interact resonantly with the weak wave.

these atoms to the absorption will be proportional to their number. Qualitatively, the value of α/α_0 can be estimated as the ratio of the density of the atoms in the regions $\pm 2E_{\star}d/\sqrt{3}$ in the presence of a strong field to the density of the atoms with v = 0 in its absence. For a very strong field, the velocity distribution of the atoms is given by

$$\Delta n(kv) = \frac{(kv)^2}{(kv)^2 + \Gamma_0^2} \exp\left(-\frac{v^2}{v_0^2}\right).$$
 (44)

We then obtain for the absorption coefficient

$$\frac{\alpha}{\alpha_0}\Big|_{\alpha_0} = 2\left[\left(\frac{E_+d}{\Gamma_0}\right)^2 + \frac{3}{4}\right]^{-1} \left(\frac{E_+d}{\Gamma_0}\right)^2 \exp\left\{-\frac{1}{3}\left(\frac{4E_+d}{kv_0}\right)\right\}_{j}$$

Since in a strong field $\Gamma_0^2 = 4(E_+d)^2/\beta$, we get for α/α_0

$$\frac{\alpha}{\alpha_{o}}\Big|_{\alpha=0} = 2\beta \frac{1}{3+\beta} \exp\left[-\frac{1}{3}\beta \left(\frac{\Gamma_{o}}{kv_{o}}\right)^{2}\right]$$
(45)

(see the exact formula (34)!).

Figure 3 illustrates the foregoing and explains the meaning of the result. The velocity of the atoms that interact resonantly with the weak wave increases in proportion to the field. The width of the dip in the velocity distribution of the atoms, which is equal to $2\sqrt{\beta} E_{\star} d$, also increases in proportion to the field, in such a way that the number of interacting atoms remains unchanged. At equal relaxation constants of the upper and lower levels, $\beta = 1$, the splitting is comparable with the magnitude of the field, and the splitting effects are most appreciable. With decreasing β (different relaxation constants of the levels) the number of resonant atoms decreases, and when $\beta \ll 1$ the contribution of the splitting effect turns out to be small.

Thus, the parameter $\beta = \gamma/\Gamma$ can be ascribed a fully defined meaning, namely, it determines the additional contribution of the coherent effects to the absorption when simultaneous account is taken of the level splitting and of the change of the velocity distribution of the atoms, arising in a strong monochromatic field. When the splitting dE₊ is much smaller than the width of the Bennett dip Γ_0 , i.e., $\beta \ll 1$, we arrive at the important conclusion that the saturation effects in the interaction of two opposing waves can be described within the framework of the populations. In other words, a gas medium with essentially different level relaxation constants is similar to a medium with transverse and longitudinal relaxation constants Γ and γ , respectively.

The splitting effects can be disregarded in the weakfield approximation, $\chi \ll 1$. Here the splitting is always much smaller than the width of the dip Γ_0 , and it does not appear in first order in the saturation (30). These effects become significant when $\chi\beta \sim 1$, as is clearly seen from (29).

4. APPLICATIONS OF THE THEORY

Let us consider some applications of the theory for the solution of problems in the stability of generation of gas lasers and for spectroscopic investigations.

A. The theory can be applied directly for the analysis of the stability of symmetrical modes in an ordinary laser with a Fabry-Perot resonator, and also the stability of unidirectional generation in a laser with a ring resonator for arbitrary excesses of the gain over threshold. Within the framework of the theory considered here, the stability problems are easy to solve, namely, the gain of the weak opposing wave is always larger than the saturated gain of the strong wave. Generation at one mode with symmetrically disposed modes in an ordinary laser and the unidirectional generation in a laser with ring resonator turn out to be unstable.

An interesting application of the results of the theory is a consideration of stability problems in lasers with nonlinear absorption. The use of the distinguishing features of the saturation in these lasers has made it possible to realize a single-frequency generation regime with a large gain-to-threshold ratio^[14,15]. To explain the mechanism whereby stable generation is obtained at one mode, a simple model was used $in^{[14,16]}$ for the formation of dips in the velocity distribution of the atoms in absorbing and amplifying media. An examination of this question from the population point of view describes well the regions where the interaction of two waves turns out to be weak, and explains the condition for the selection of remote modes. However, such a consideration is insufficient when the oscillation modes are symmetrical with respect to the line center. The difference between the amplification and absorption is equal to the loss in the resonator for both the strong and the weak fields, and the question of generation stability at one mode remains open. $In^{[15]}$ they considered the stability in first order in the saturation. However, as seen from (30), the main features in the gain and absorption arise when terms of order χ^2 are considered, and these were not considered in^[15]. The results of Item E of Sec. 2 enable us to understand and to analyze this question.

Let $\Delta \gg \Gamma_0$. The field of the standing wave at the generation frequency can be represented as a sum of two traveling waves interacting with different atoms. We represent the field of the weak wave at the mirror frequency also in the form of two traveling waves. We note that waves traveling in opposite directions with different frequencies interact with the same atoms. Thus, the weak and strong interacting traveling waves are opposed, making it possible for us to use the results of Sec. 2. It is easy to see that the additional term in (41), which is connected with splitting effects, is equal to the difference between the gain (absorption) of the weak signal and the saturated gain (absorption) of the strong one. Therefore the ratio of these additional terms in amplification and absorption of the weak signal solves in essence the problem of stability of one mode:

if the additional contribution of the effects of splitting to absorption of the weak signal is larger than that to the amplification, then the generation regime with one mode is stable. Taking into account (29) and (41), and the remark made in Item E of Sec. 2 in connection with allowance for the collisions, this condition can be written for symmetrically placed modes in lasers with ring resonators.

$$\alpha_0{}^a\beta_{aga} > \alpha_0{}^g\phi_g\beta_{7}, \qquad (46)$$

where

$$\varphi = \chi \frac{a-1}{a} (3a+1) \left(2a + \frac{\gamma_1 + \gamma_2}{2\Gamma}\right)$$

$$\times \left[(3a+1) (a+1) \left(2a + \frac{\gamma_1}{\Gamma}\right) \right]$$

$$\left(2a + \frac{\gamma_2}{\Gamma}\right) + (2a+1) \left(2a + \frac{\gamma_1 + \gamma_2}{2\Gamma}\right) \beta\chi$$

the indices a and g correspond to absorbing and amplifying media.

For large saturation parameters in both media, the condition assumes the simple form

$$\alpha^{a}\beta^{a}(3+\beta^{a})^{-i} > \alpha^{g}\beta^{g}(3+\beta^{g})^{-i}.$$

Thus, stability of single-mode generation is determined at large saturation parameters only by the parameters β^a and β^g . Condition (46) also determines the stability of unidirectional generation on symmetrically placed modes in lasers with ring resonators.

B. The use of the present results for spectroscopic investigations is interesting from the point of view of measuring line widths and the relaxation constants of individual levels. Spectroscopic investigations can be carried out in accordance with two schemes: using two generators with different frequencies and with the aid of one scanned generator, where the weak opposing wave is obtained by reflection from a mirror. In both cases, the line shapes are similar, but they differ in frequency scale by a factor of two (see (41) and (42)). The second scheme is preferable and we shall henceforth have only this scheme in mind. At small saturation parameters, the width of the dip at the center of the line is equal to the homogeneous line width. At large saturation dimensions it is necessary to take into account the effect of broadening by the field. Such a scheme is interesting in the investigation of processes that lead to homogeneous saturation: the decrease of the absorption of the weak signal at $\Omega \gg \Gamma_0$ indicates directly the presence of a homogeneous part in the saturation.



FIG. 4. Experimental setup for the measurement of the absorption coefficient of a weak opposing wave: AC-absorbing cell, F-decoupling filter, SM-semitransparent mirror, M_1 , M_2 -mirrors, D-diaphragm, PC₁, PC₂-photoreceivers, Σ -summing device.

The new qualitative possibilities of the method are connected with singularities of the absorption in strong fields. By comparing the absorption of the strong and weak signals we can determine the parameter β and consequently the relaxation constants of the individual levels. In the presence of collisions, the behavior of the parameter β indicates the collision mechanism (see Sec. 2, Item D). Thus, the observed regularities in the behavior of the absorption of the weak signal make this method promising.

We shall show that the experiment can be organized in such a way that the results of the theory can be compared directly with the results of the experiment. The experimental setup is shown in Fig. 4. The laser radiation passes through a semitransparent mirror and an absorbing cell.

After reflection from a mirror with a small reflection coefficient, the radiation again passes through the absorbing medium and goes from the semitransparent mirror to photoreceiver PC_2 . The saturation parameter is measured with the aid of photoreceiver PC_1 . The setup was adjusted in such a way that the currents \Box_1 both photocells coincided in the absence of absorption. At small absorption, the difference between the photocell currents I_1 and I_2 then corresponds to absorption of the weak wave, and the ratio $(I_1 - I_2)/I_1$ is the absorption coefficient of the weak wave. The diameter of the diaphragm is much smaller than the beam diameter, making it possible to exclude the influence of spatial inhomogeneity of the field over the beam cross section.

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