THE ELECTROSTATIC POTENTIAL OF THE EARTH AND EXPERIMENTS ON VERIFICATION OF COULOMB'S LAW

A. D. DOLGOV and V. I. ZAKHAROV

Institute of Theoretical and Experimental Physics

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The role of the electrostatic potential of the Earth in experiments on verification of Coulomb's law is discussed. It is shown that the effect of the Earth's potential is extremely important in the case where Coulomb's law is violated as the result of a finite rest mass for the photon. In this case, in verification of Coulomb's law in a Cavendish experiment there is no need of charging the external sphere; on the contrary it should be grounded. If the violation of Coulomb's law reduces to a change in the exponent of the distance, the effect of the Earth's field in these experiments is unimportant.

IN this note we discuss the role of the electrostatic field of the Earth in experiments on verification of Coulomb's law. In the experiment first performed by Cavendish in 1773 and later repeated with improved accuracy,[1,2] the potential difference was measured between two concentric conducting spheres. The value of $\delta\varphi$ is different from zero only in the case where Coulomb's law is not exact. Two modifications of this law have been suggested:

$$\varphi(r) = Qe^{-\mu r}/r \text{ (de Broglie [3])}, \tag{1}$$

$$\varphi(r) = Q/r^{1+\varepsilon} \text{ (Maxwell [4])}, \tag{2}$$

where $\varphi(r)$ is the potential due to a point charge Q.

The first expression can be obtained systematically if we assign to the photon a rest mass μ, and the second can best be discussed as a phenomenological description of a possible change of Coulomb's law.

We will show that in verifying a change of Coulomb's law to the form given by Eq. (1) it is necessary to take into account the electrostatic potential of the Earth. For the second modification, the effect of the Earth's field is unimportant. The reason for this lies in the fact that Eq. (1) changes Coulomb's law mainly at large distances, and Eq. (2) at small distances. Therefore inclusion of the effect of remote charges (the charge of the Earth) is important only in the first case.

We note that the limitation imposed on the photon mass which follows from existing experiments on measurement of $\delta\varphi$ is weaker than the limit obtained on the basis of data on the Earth's magnetic field[5,6]: $\mu < (3\times 10^4\text{ km})^{-1}$. However, taking account of the electrostatic potential of the Earth permits the experimental arrangement for measurement of $\delta\varphi$ to be substantially simplified and, possibly, better accuracy to be achieved for this reason.

2. In this section we will discuss in detail case (1). If the photon has the mass μ, then it is possible to show that the static potential φ satisfies the equation

$$(\Delta - \mu^2)\varphi = -4\pi\rho, \tag{3}$$

where ρ is the electric charge density.

An important difference between Eq. (3) and the usual equation with $\mu = 0$ is that it does not permit the calibrational transformation $\varphi \to \varphi + \text{const}$ which allows arbitrary choice of a reference potential. For this reason the potential of a given point of space becomes observable, and the difference of potential between two closed conducting shells, of which the internal shell is not charged, will be proportional to the absolute value of the potential of the outer shell.

It is known (see, for example, ref. 7) that the potential difference between the surface of the Earth and the ionosphere is 4×10^5 V (in electrical storms, local increase of the potential difference to 10^8 V is possible). If we assume that the potential at great distances is zero, then the absolute value of the Earth's potential φ_E is 4×10^5 V (in electrical storms 10^8 V) and consequently the potential of the apparatus located on the Earth is also close to this value.

For the potential difference between two spheres of which the inner is uncharged and the outer is grounded, we can obtain the following expression

$$\delta\varphi = {}^1/_6\varphi_E\mu^2(R_1{}^2 - R_2{}^2) + O(\mu^4). \tag{4}$$

In the experiments on verification of Coulomb's law, the outer sphere was charged to a rather high potential relative to the Earth--to 6×10^3 V in the work of Plimpton and Lawton[1] and to 10^5 V in the work of Bartlett and Phillips.[2] From what we have said above it follows that in checking modification (1) of Coulomb's law there is no need to charge the outer sphere, since in this case its potential changes by no more than 25%; in addition, use of such a high voltage requires rather complicated apparatus.

However, in order to use the large value of the Earth's potential it is necessary to see to it that the charge of the inner sphere is actually zero as was assumed in derivation of Eq. (4). We note that the usual procedure for discharging the inner sphere is not suitable in this case. Usually, in fact, in order to remove accidental charges from the sphere, it is connected with a wire to the outer sphere and in this way the two spheres acquire the same potential. In this case the inner sphere retains a charge $Q = ({}^1/_6)\mu^2 R_1 R_2(R_1 + R_2)\varphi_E$, which compensates the potential difference of Eq. (4). Therefore the discharging procedure must be changed.

The following procedure can be suggested, for example. Introduce an auxiliary sphere, concentric with the first two and with a radius R ($R_1 > R \geq R_2$). At the beginning all the spheres must be temporarily connected together, in order to remove background charges. For this reason the potentials of all these spheres will be equal; this equality will be preserved even after the connecting wire is removed. If we now remove the intermediate sphere, then a potential difference arises between the inner and outer spheres:

$$\delta\varphi' = \frac{1}{6}\varphi_E\mu^2(R_1 - R)(R_1 - R_2)\frac{R_1 + R_2 + R}{R_1} + O(\mu^4). \tag{5}$$

The value of $\delta\varphi'$ is proportional to the absolute value of the Earth's potential.

Allowance for the fact that in the measurement it is not necessary to charge the outer sphere will apparently permit simplification of the experimental apparatus and improvement of the experimental limit to the value of the photon mass, in particular, as the result of increasing R_1.

3. In this section we will discuss the effect of the electrostatic potential of the Earth on the potential difference between the spheres in the case in which the interaction of two charges is described by Eq. (2).

The potential of a system of charges distributed with density $\rho(\mathbf{r})$ is

$$\varphi(\mathbf{r}) = \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^{1+\varepsilon}}. \tag{6}$$

If the charge distribution is assumed to be given, then $\varphi(\mathbf{r})$ satisfies Poisson's equation:

$$\Delta\varphi(\mathbf{r}) = \varepsilon(1 + \varepsilon)\int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^{3+\varepsilon}}. \tag{7}$$

In contrast to the usual case, this equation determines the potential only in the region of space where there are no charges ($\rho(\mathbf{r}) = 0$), since in the opposite case the integral in the right-hand part of Eq. (7) diverges for $\epsilon > 0$.

Since the value of ϵ is extremely small (according to Bartlett and Phillips[2], $\epsilon \sim 10^{-12}$), in the right-hand side of Eq. (7) we can limit ourselves to terms of first order in ϵ. In this case instead of $\rho(\mathbf{r}')$ we can take the distribution $\rho_0(\mathbf{r}')$ which exists in classical electrostatics and is assumed known. Solution of Eq. (7) in this case will determine the correction to φ of first order in ϵ.

Since the distribution of charges for $\epsilon = 0$ is determined by the field intensity and not by the potential, it is clear that the quantity $\delta\varphi$ in case (2) does not depend on the absolute potential of the Earth but depends only on the field intensity near the surface of the Earth. Therefore the corresponding effects are small.

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