ELECTRIC CONDUCTIVITY TENSOR OF POLARONS IN A MAGNETIC FIELD

A. A. KLYUKANOV and E. P. POKATILOV

Kishinev State University

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A method is developed for calculating the components of the electric conductivity σ_{ik} of an electron gas in a spatially homogeneous electric field of frequency ν, with allowance for the polaron effect, when the principal role in the scattering is played by the interaction of the carriers with the thermal vibrations of the lattice. This approach is a generalization of the Feynman method to the case of arbitrary intensities of the constant magnetic fields H. The influence phase Φ, which describes the potential (real part) and dissipative (imaginary part) properties of the phonon subsystem, is simulated in the zeroth approximation by functionals that are quadratic in the velocity, with complex tensor coefficients that take into account the anisotropy of the problem and are determined from the condition that the first corrections to the admittance vanish. By way of examples we present calculations of the longitudinal and transverse magnetoresistance, and also of the polaron masses. We consider in detail cyclotron resonance of piezoelectric polarons in CdS. It is shown that the absorption depends on the orientation of the magnetic and electric fields relative to the principal axis C_6 of the crystal. It is established that the anisotropy that depends on the direction of the field **H** is stronger than that dependent on the direction of the electric field. The anisotropy of the former type was observed in experiment, and agreement was obtained with the experimental data.

ELECTRON-PHONON interaction in semiconductors leads not only to scattering of the carriers but also to renormalization of their energy and mass (the polaron effect). The mobility of optical polarons of large radius (in the continual approximation) was calculated by a number of authors [1-3] under different assumptions concerning the value of the electron-phonon-coupling parameter. A consistent quantum theory of the conductivity of the polaron was developed by Feynman.[3] The component of the electric conductivity tensor $\sigma_{zz}(\nu)$ of the electron-phonon system, placed in a spatially-homogeneous electric field of frequency ν, was expressed in terms of the response function (admittance), for the calculation of which a special technique of continual integration was used. The polaron effect was imitated by a fictitious particle coupled elastically to the electron. In such an approach, however, it is possible to take into account only weak magnetic fields, since the functional of the action of the polaron in the magnetic field cannot be reduced to an integrable form.

In this article we present a method for calculating the electric-conductivity tensor σ_{ik} of polarons in a homogeneous magnetic field of arbitrary intensity. The polaron effect is taken into account in the weak-coupling approximation, which is realized in most cases of practical importance.

1. ELECTRIC CONDUCTIVITY TENSOR IN THE REPRESENTATION OF CONTINUAL INTEGRATION

The components of the tensor $\sigma_{ik}(\nu)$ for an anisotropic medium in a weak spatially-homogeneous electric field of frequency ν are expressed in terms of spectral representations of the response function $G_{ik}(\nu)$ (admittance):

$$\sigma_{ik}(\nu) = e^2\nu G_{ik}(\nu), \tag{1}$$

$$G_{ik}(\nu) = \int_{-\infty}^{\infty} e^{-i\nu t} G_{ik}(t)\,dt, \quad G_{ik}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\nu t} G_{ik}(\nu)\,d\nu. \tag{2}$$

According to the Feynman method,[3] the components $G_{ik}(\nu)$ can be calculated with the aid of a specially chosen function $g(\nu)$:

$$G_{ik}(\nu) = \frac{\hbar}{2e^2}\frac{\partial^2}{\partial\zeta_i\,\partial\varepsilon_k} g(\nu)|_{\zeta=\varepsilon=0}. \tag{3}$$

Without stopping to discuss the method of choosing the trace $g(t)$ and representing it in the form of a continual integral with respect to the electronic variables only, we present the general formulas that will be needed in what follows:

$$g(\tau-\sigma) = \mathrm{Sp}\int \exp\left[\frac{i}{\hbar}F(\mathbf{r},\mathbf{r}')\right] D\mathbf{r}\,D\mathbf{r}'\,d\mathbf{r}_{t''}\,d\mathbf{r}_{t''}'. \tag{4}$$

Here $D\mathbf{r}$ is the symbol of continual integration over the electron trajectories;

$$F(\mathbf{r},\mathbf{r}') = S_e(\mathbf{r}) - S_e'(\mathbf{r}') + \Phi(\mathbf{r},\mathbf{r}'), \quad S_e(\mathbf{r}) = \int_{t'}^{t''} L_e(\mathbf{r},\dot{\mathbf{r}},t)\,dt \tag{5}$$

(S' is expressed analogously in terms of L'), S_e is the action of the electron, described by the Lagrangian

$$L_e(\mathbf{r},\dot{\mathbf{r}},t) = \sum_{i=1}^{3}\frac{1}{2}m_{ii}^{(0)}\dot{r}_i^2 - \frac{e}{c}\dot{r}_iA_i + er_iE_i, \quad i=1,2,3=x,y,z, \tag{6}$$

where $m_{ii}^{(0)}$ are the diagonal components of the mass tensor of the band electron, e is the absolute charge of the electron, $A = (0, r_1H, 0)$ is the vector potential of a constant and homogeneous magnetic field. The electric fields **E** in L_e and **E**′ in L_e' are chosen with field coefficients ϵ and ζ:

$$\mathbf{E}(s) = \varepsilon\delta(\sigma-s) + \zeta\delta(\tau-s), \qquad \mathbf{E}'(s) = \varepsilon\delta(\sigma-s) - \zeta\delta(\tau-s). \tag{7}$$

The phonon subsystem makes a contribution in the form of the influence phase $\Phi(\mathbf{r}, \mathbf{r}')$, which is a functional of the electron trajectory; for an aggregate of phonon harmonic oscillators interacting with the electron, in an approximation linear in the oscillator coordinates, the phase can be calculated exactly:[3, 4]

$$\Phi(\mathbf{r},\mathbf{r}')=\sum_{\varkappa j}\Phi_{\varkappa j}(\mathbf{r},\mathbf{r}') \tag{8}$$

(κ is the wave vector and j is the number of the branch of the lattice vibrations),

$$\exp\left[\frac{i}{\hbar}\Phi_{\varkappa j}(\mathbf{r},\mathbf{r}')\right]=\operatorname{Sp}\int\exp\left[\frac{i}{\hbar}\int\limits_{t'}^{t''}(L_{\varkappa j}-L_{\varkappa j}')\,dt\right]$$
$$\cdot\rho(q_{\varkappa jt'},\,q'_{\varkappa jt'})\,Dq_{\varkappa j}\,Dq_{\varkappa j}'\,dq_{\varkappa jt'}\,dq'_{\varkappa jt'}. \tag{9}$$

In (9), $\rho(q, q')$ is the density matrix of the phonon subsystem at the initial instant of time $t' \to \infty$, when the lattice oscillators can be regarded as not interacting with the electron, and $L_{\kappa j}$ is the Lagrangian of the phonon oscillator, on which an electron with a force $\gamma_{\kappa j}$ acts:

$$\tag{10}$$

The calculation of the integrals in (9) leads to the following result:[3]

$$\Phi_{\varkappa j}=\frac{1}{2}\iint\limits_{t'}^{t''}dt\,ds\,[\gamma_{\varkappa j}(t)-\gamma_{\varkappa j}'(t)]\{[\gamma_{\varkappa j}(s)+\gamma_{\varkappa j}'(s)]\,J(\omega_{\varkappa j},t-s)$$
$$+i\,[\gamma_{\varkappa j}(s)-\gamma_{\varkappa j}'(s)]\,A(\omega_{\varkappa j},t-s)\}, \tag{11}$$

$$\gamma_{\varkappa j}=v_{\varkappa j}\left(\frac{2}{L^3}\right)^{1/2}\begin{cases}\sin\varkappa\mathbf{r}, & \varkappa_x\geqslant 0\\ \cos\varkappa\mathbf{r}, & \varkappa_x<0\end{cases}. \tag{12}$$

The form of $v_{\kappa j}$ for different interaction mechanisms will be specified more concretely later on;

$$J(\omega,t)=\begin{cases}\omega^{-1}\sin\omega t, & t>0\\ 0, & t<0\end{cases}, \tag{13}$$

$$A(\omega,t)=\frac{1}{2\omega}\operatorname{cth}\left(\frac{\lambda\hbar\omega}{2}\right)\cos\omega t,\quad \lambda=\frac{1}{kT}. \tag{14}$$

Substituting (11) in (8), we obtain, with account taken of (12)

$$\Phi(\mathbf{r},\mathbf{r}')=\frac{1}{2}\sum_j\int\frac{d\varkappa}{(2\pi)^3}\,|v_{\varkappa j}|^2\iint\limits_{-\infty}^{\infty}dt\,ds\{[\cos(\varkappa,\mathbf{r}_t-\mathbf{r}_s)$$
$$-\cos(\varkappa,\mathbf{r}_t'-\mathbf{r}_s)]\,f(\omega_{\varkappa j},t-s)$$
$$+[\cos(\varkappa,\mathbf{r}_t-\mathbf{r}_s')-\cos(\varkappa,\mathbf{r}_t'-\mathbf{r}_s')]\,f^*(\omega_{\varkappa j},t-s)\}, \tag{15}$$
$$f(\omega,t)=J(\omega,t)+iA(\omega,t).$$

Formulas (1) and (3) together with (4), (5), and (15) determine the exact values of the components of the tensor σ_{ik}.

2. METHOD OF APPROXIMATE CALCULATION

It is impossible to carry out continual integration in (4) with the functional (15). Feynman's approximation[3] consists of replacing the phonon system with a fictitious particle elastically coupled to the electron. After eliminating its coordinates, one obtains a trial functional that is quadratic in the electron variables. Continual integrals with functionals of this type can be evaluated,[3] but when the magnetic field is taken into account this method turns out to be suitable only in the case of small H, when (4) can be expanded in powers of H.

We call attention to two circumstances that explain the choice of the approximation suitable for all H and describing the polaron effect: integration in (4) with the Lagrangian (6), but without $\Phi(\mathbf{r}, \mathbf{r}')$, can be carried out exactly; allowance for the influence phase leads to a renormalization and dissipation of the electron energy.

The polaron effect can be described by replacement of the mass. Further, we note that the dissipation function[5] leads formally to the appearance in the action of a term in the form

$$\frac{i\gamma_{ik}}{\nu}\int\limits_{t'}^{t''}\dot{r}_i\dot{r}_k\,dt.$$

Consequently, both effects can be taken into account by making the following substitution in the action (5):

$$m_{ik}^{(0)}\delta_{ik}\to M_{ik}(1-i\Delta_{ik})\equiv m_{ik}, \tag{16}$$

$$S_0(\mathbf{r})=\int\limits_{t'}^{t''}\left(\sum_{i,k}\frac{1}{2}m_{ik}\dot{r}_i\dot{r}_k-\frac{e}{c}Hr_1\dot{r}_2+er_iE_i\right)dt; \tag{17}$$

here M_{ik} and Δ_{ik} are the sought parameters of the trial action S_0.

We write $g(\tau-\sigma)$ in the form

$$g(\tau-\sigma)=\operatorname{Sp}\int D\mathbf{r}\,D\mathbf{r}'\,d\mathbf{r}_{t'}\,d\mathbf{r}_{t'}'\exp\left[\frac{i}{\hbar}F_0(\mathbf{r},\mathbf{r}')\right]\left[1\right.$$
$$\left.+\frac{i}{\hbar}(F-F_0)+\frac{1}{2}\left(\frac{i}{\hbar}\right)^2(F-F_0)^2+\ldots\right] \tag{18}$$
$$=g_0(\tau-\sigma)\left[1+\frac{i}{\hbar}\langle F-F_0\rangle_0+\frac{1}{2}\left(\frac{i}{\hbar}\right)^2\langle(F-F_0)^2\rangle_0+\ldots\right]$$
$$=g_0(\tau-\sigma)+g_1(\tau-\sigma)+g_2(\tau-\sigma)+\ldots$$

In (18), $F_0 = S_0 - S_0'$, and $\langle\ldots\rangle$ is the continual mean value:

$$\langle T(\mathbf{r},\mathbf{r}')\rangle_0=\frac{1}{g_0}\operatorname{Sp}\int T(\mathbf{r},\mathbf{r}')\exp\left[\frac{i}{\hbar}F_0(\mathbf{r},\mathbf{r}')\right]D\mathbf{r}\,D\mathbf{r}'\,d\mathbf{r}_{t'}\,d\mathbf{r}_{t'}'. \tag{19}$$

In normal coordinates that diagonalize the kinetic energy in (16), g_0 and all the mean values are calculated exactly.

The parameters of the trial action are obtained from the condition that the corrections to

$$G_{ik}^{(0)}=\frac{\hbar}{2e^2}\frac{\partial^2}{\partial\zeta_i\,\partial\varepsilon_k}g_0\Big|_0$$

vanish. We shall find them in the first approximation in the constants of the electron-phonon coupling $|v_{\kappa j}|^2$ from the equations

$$G_{ik}^{(1)}(\nu)=\frac{\hbar}{2e^2}\frac{\partial^2}{\partial\zeta_i\,\partial\varepsilon_k}g_1(\nu)|_{\zeta=\varepsilon=0}=0. \tag{20}$$

In the calculation in g_1, we can replace S_0 by S_e.

Let us calculate the trace g_0 with the trial actions S_0 and S_0', in which we neglect m_{ik} with $i \neq k$, i.e., we take into account the non-diagonality of σ_{ik}, which is connected only with the magnetic field (the Hall conductivity). This non-principal simplification is justified by the fact that usually the mixed components σ_{ik} are small when H = 0.

The result of the integration in g_0 with respect to $\mathbf{r}_1$ and $\mathbf{r}_3$ can be written down immediately, by using formulas (9) and (11), as for oscillators with zero frequencies perturbed by the forces

$$\gamma_1(t)=\frac{e}{\sqrt{m_{11}}}E_1(t)-\tilde{\omega}_c\dot{r}_2(t),\quad \gamma_1'(t)=\frac{e}{\sqrt{m_{11}}}E_1'(t)-\tilde{\omega}_c\dot{r}_2'(t),$$

$$\gamma_3(t)=\frac{e}{\sqrt{m_{33}}}E_3(t),\quad \gamma_3'(t)=\frac{e}{\sqrt{m_{33}}}E_3'(t),\quad \tilde{\omega}_c^2=\frac{e^2H^2}{c^2m_{11}m_{22}}. \tag{21}$$

The functional obtained in this case, as seen from (21), contains the terms $\dot{r}_2(t)\dot{r}_2(s)$, and after integration by parts it reduces to a quadratic form with frequency $\tilde{\omega}_c$ and forces

$$\gamma_2(t)=\frac{e}{\sqrt{m_{22}}}E_2(t)+\frac{e\tilde{\omega}_c}{\sqrt{m_{11}}}\int_{-\infty}^{t}dt'E_1(t'),$$

$$\gamma_2'(t)=\frac{e}{\sqrt{m_{22}}}E_2'(t)+\frac{e\tilde{\omega}_c}{\sqrt{m_{11}}}\int_{-\infty}^{t}dt'E_1'(t'). \qquad (22)$$

After integration with respect to r_2 and certain transformations, we obtain ultimately

$$g_0(\tau-\sigma)=g_{12}^{(0)}(\tau-\sigma)\prod_{i=1}^{3}g_{ii}^{(0)}(\tau-\sigma), \qquad (23)$$

where

$$g_{ii}^{(0)}(\tau-\sigma)=\exp\left\{\frac{2ie^2}{\hbar m_{ii}}[\zeta_i\varepsilon_iJ(\omega_{ii},\tau-\sigma)+i\zeta_i^2A(\omega_{ii},0)]\right\},$$

$$g_{12}^{(0)}(\tau-\sigma)=\exp\left\{\frac{2ie^2}{\hbar\sqrt{m_{11}m_{22}}}[\varepsilon_1\zeta_2-\varepsilon_2\zeta_1]J_1(\tilde{\omega}_c,\tau-\sigma)\right\}; \qquad (24)$$

$$\omega_{ii}=\tilde{\omega}_c,\quad i=1,2,\quad \omega_{33}=0,$$

$$J_1(\omega,t)=\begin{cases}\omega^{-1}(1-\cos\omega t), & t>0\\ 0, & t<0\end{cases}. \qquad (25)$$

The function J_1 appears in connection with allowance for the magnetic field. In the limit as $H_1\to 0$ we get $J_1=0$ and $g^{(0)}=1$.

Substituting (23) in (3), we obtain

$$G_{ii}^{(0)}(\nu)=\frac{i}{m_{ii}}J(\omega_{ii},\nu),$$

$$G_{12}^{(0)}(\nu)=-G_{21}^{(0)}(\nu)=-\frac{i}{\sqrt{m_{11}m_{22}}}J_1(\tilde{\omega}_c,\nu),$$

$$J_1(\omega,\nu)=-\frac{i\omega}{\nu}J(\omega,\nu),\qquad J(\omega,\nu)=\frac{1}{\omega^2-\nu^2}. \qquad (26)$$

Now, in accordance with the definition (1) we can readily obtain formulas for the components of the tensor σ_{ik}:

$$\sigma_{ii}(\nu)=\frac{i\nu e^2}{M_{ii}}\left[\frac{m^2\omega_c^2}{M_{ii}M_{ll}(1-i\Delta_{ll})}-\nu^2(1-i\Delta_{ii})\right]^{-1},\quad l\neq i,\quad i,l=1,2; \qquad (27)$$

$$\sigma_{33}(\nu)=-\frac{ie^2}{M_{33}\nu}\frac{1}{1-i\Delta_{33}}; \qquad (28)$$

$$\sigma_{12}(\nu)=-\sigma_{21}(\nu)=-\frac{e^2m\omega_c}{M_{11}M_{22}}\left[\frac{m^2}{M_{11}M_{22}}\omega_c^2-\nu^2(1-i\Delta_{11})(1-i\Delta_{22})\right]^{-1};$$

$$\omega_c=eH/mc.$$

In the final formulas (27)–(29) we assume, to simplify the notation, that the band mass is isotropic: $m_{11}^{(0)}=m_{22}^{(0)}=m_{33}^{(0)}=m$.

We present the real parts of σ_{ik} in the limit $\nu=0$; these describe galvanomagnetic phenomena in a constant field:

$$\mathrm{Re}\,\sigma_{ii}=\frac{e^2M_{ll}}{\tau_{ll}m^2\omega_c^2}\left[1+\frac{1}{\tau_{11}\tau_{22}\Omega^2}\right]^{-1},\quad i,l=1,2,\quad i\neq l; \qquad (30)$$

$$\mathrm{Re}\,\sigma_{33}=\frac{e^2\tau_{33}}{M_{33}},\qquad \frac{1}{\tau_{ii}}=\nu\Delta_{ii}, \qquad (31)$$

$$\mathrm{Re}\,\sigma_{12}=-\mathrm{Re}\,\sigma_{21}=-\frac{e^2}{m\omega_c}\left[1+\frac{1}{\Omega^2\tau_{11}\tau_{22}}\right]^{-1}; \qquad (32)$$

here $\Omega^2=\omega_c^2m^2/M_{11}M_{22}$.

We obtain also the real part of $\sigma_{ii}(\nu)$ when $\nu\sim\omega_c$ and the cyclotron-resonance conditions $\tau\nu>1$ are satisfied:

$$\mathrm{Re}\,\sigma_{ii}=\frac{e^2\tau_{ii}}{M_{ii}}\frac{\Omega^2\tau_{11}\tau_{22}+\nu^2\tau_{ll}^2}{(\Omega^2-\nu^2)^2\tau_{11}^2\tau_{22}^2+(1+\Omega^2\tau_{ii}/\nu^2\tau_{ll})(\Omega^2\tau_{11}\tau_{22}+\nu^2\tau_{ll}^2)} \qquad (33)$$

Formulas (30)–(33) and the corresponding classical expressions are outwardly similar. The essential difference lies in the fact that the ordinary masses and relaxation times are replaced in (30)–(33) by the polaron masses M_{ii} and by the functions τ_{ii} that depend on the magnetic field.

3. CALCULATION OF THE PARAMETERS OF THE TRIAL ACTION

To obtain Eqs. (20) in explicit form it is necessary to calculate the quantity $g_1\equiv i\hbar^{-1}g_0\langle F-F_0\rangle_0$. It is easily seen that all the mean values in $\langle F-F_0\rangle$ can be expressed in terms of generating functions of the type

$$\Psi_{\varkappa}^{(1)}(\xi,\eta)=\langle\exp[i(\varkappa,\xi\mathbf{r}(t)-\eta\mathbf{r}(s))]\rangle_0. \qquad (34)$$

(The remaining necessary functions $\Psi^{(2)}$, $\Psi^{(3)}$, and $\Psi^{(4)}$ are obtained from (34) by replacing $\mathbf{r}(t)$ and $\mathbf{r}(s)$ by other pairs from the set $\mathbf{r}(t)$, $\mathbf{r}(s)$, $\mathbf{r}'(t)$, and $\mathbf{r}'(s)$.) We then carry out, for example, the transformation

$$\left\langle\int_{-\infty}^{\infty}\dot{r}_1^2(t)dt\right\rangle_0=-\iint_{-\infty}^{\infty}dt\,ds\,\delta(t-s)\frac{\partial^2}{\partial s^2}\langle r_1(t)r_1(s)\rangle_0. \qquad (35)$$

In (35) it is first necessary to differentiate with respect to s, then integrate with allowance for the delta function. The quadratic form (35) is connected with $\Psi_K^{(1)}$:

$$\langle r_1(t)r_1(s)\rangle_0=\frac{1}{\varkappa_1^2}\frac{\partial^2}{\partial\xi\,\partial\eta}\Psi_{\varkappa_1}^{(1)}(\xi,\eta)\big|_{\xi=\eta=0}. \qquad (36)$$

The calculation of $\Psi^{(n)}$ is analogous to that of g_0 and leads, in particular, to the result

$$\Psi_{\varkappa}^{(1)}(\xi,\eta)=\Psi_{12}^{(1)}(\xi,\eta)\prod_{i=1}^{3}\Psi_{ii}^{(1)}(\xi,\eta), \qquad (37)$$

where

$$\ln\Psi_{ii}^{(1)}(\xi,\eta)=\frac{ie\varkappa_i}{2\pi m}\Big\{\varepsilon_i\int_{-\infty}^{\infty}d\nu\,e^{-i\nu\sigma}(\xi e^{i\nu t}-\eta e^{i\nu s})J(\omega_{ii},\nu)$$
$$+\zeta_i\int_{-\infty}^{\infty}d\nu\,e^{i\nu\tau}(\xi e^{-i\nu t}-\eta e^{-i\nu s})[J(\omega_{ii},\nu)+2iA(\omega_{ii},\nu)]\Big\}$$
$$+\frac{i\hbar\varkappa_i^2}{4\pi m}\int_{-\infty}^{\infty}d\nu[\xi^2+\eta^2-2\xi\eta\cos\nu(t-s)][J(\omega_{ii},\nu)+iA(\omega_{ii},\nu)]; \qquad (38)$$

$$\ln\Psi_{12}^{(1)}(\xi,\eta)=\frac{ie}{2\pi m}\int_{-\infty}^{\infty}d\nu\{-e^{i\nu\tau}(\zeta_1\varkappa_2$$
$$-\zeta_2\varkappa_1)(\xi e^{-i\nu t}-\eta e^{-i\nu s})[J_1(\omega_c,\nu)+2iA_1(\omega_c,\nu)] \qquad (39)$$
$$+e^{-i\nu\sigma}(\varepsilon_1\varkappa_2-\varepsilon_2\varkappa_1)(\xi e^{i\nu t}-\eta e^{i\nu s})J_1(\omega_c,\nu)\};$$

$$A(\omega,\nu)=\frac{\pi}{2\omega}[\delta(\omega-\nu)+\delta(\omega+\nu)]\coth\frac{\lambda\hbar\omega}{2};$$

$$A_1(\omega,\nu)=\frac{\pi}{2i\omega}[\delta(\omega-\nu)-\delta(\omega+\nu)]\coth\frac{\lambda\hbar\omega}{2}, \qquad (40)$$

$$\omega_{ii}=\omega_c,\quad i=1,2,\quad \omega_{33}=0.$$

Further, the contribution of the first term of formula (15) to g_1, denoted $G_{ik}^{(1)1}(\tau-\sigma)$, is given by

$$G_{ik}^{(1)1}(\tau-\sigma)=\frac{i}{2e^2}\frac{\partial^2}{\partial\zeta_i\,\partial\varepsilon_k}g_0(\tau-\sigma)\frac{1}{4}\sum_j\int\frac{d\varkappa}{(2\pi)^3}|v_{\varkappa j}|^2$$
$$\times\iint_{-\infty}^{\infty}dt\,ds\,f(\omega_{\varkappa j},t-s)[\Psi_{\varkappa}^{(1)}(1,1)+\Psi_{\varkappa}^{(1)}(-1,-1)]|_{\varepsilon=\zeta=0}. \qquad (41)$$

From the definition (18) of g_0 it follows that when $\zeta = \epsilon = 0$ the trace is $g_0 = 1$ and $\partial g_0/\partial\epsilon_i = \partial g_0/\partial\zeta_i = 0$. Since g for $\zeta = \epsilon = 0$ is the trace of the equilibrium density matrix, it follows that $g(\epsilon = \zeta = 0) = g_0(\epsilon = \zeta = 0) = 1$ and $g_1(\epsilon = \zeta = 0) = 0$. Therefore in formula (41) it is necessary to differentiate only $\Psi^{(1)}$. Substituting (37) in (41), carrying out the necessary transformations, and changing over to the spectral representation, we obtain (we write out only $G_{11}^{(1)1}(\nu)$)

$$
\begin{aligned}
G_{11}^{(1)1}(\nu) = {} & \frac{i}{2m^2}\sum_j\int\frac{d\varkappa}{(2\pi)^3}|v_{\varkappa j}|^2\Big\{\varkappa_1^2 J(\omega_c,\nu)\left[J(\omega_c,\nu)\right. \\
& \left.+2iA(\omega_c,\nu)\right]-\varkappa_2^2 J_1(\omega_c,\nu)\left[J_1(\omega_c,\nu)+2iA_1(\omega_c,\nu)\right]\Big\} \\
& \times\int_{-\infty}^{\infty}dt\, f(\omega_{\varkappa j},t)(\cos\nu t-1)\exp\Big(\frac{i\hbar}{2\pi m}\int_{-\infty}^{\infty}d\mu(1-\cos\mu t) \\
& \times\{\varkappa_3^2[J(0,\nu)+iA(0,\nu)]+\varkappa_\perp^2[J(\omega_c,\nu)+iA(\omega_c,\nu)]\}\Big)
\end{aligned}
\tag{42}
$$

We calculate analogously the remaining terms in $G_{11}^{(1)}(\nu)$ (there are ten of them, four from the averaging of the phase and six from the kinetic energy).

Using the exclusive form of $G_{ik}^{(1)}(\nu)$, we get from (20)

$$
\begin{aligned}
\frac{m-M_{ii}}{m}+\frac{i}{\nu\tau_{ii}} = {} & \sum_j\int\frac{d\varkappa}{(2\pi)^3}\frac{\varkappa_i^2}{m\nu^2}\frac{|v_{\varkappa j}|^2}{\omega_{\varkappa j}\,\mathrm{sh}(\lambda\hbar\omega_{\varkappa j}/2)} \\
& \times\int_0^\infty dt(1-e^{-i\nu t})\,\mathrm{Im}\cos\omega_{\varkappa j}\Big(t-\frac{i\lambda\hbar}{2}\Big)\exp\Big[-\frac{\hbar\varkappa^2}{2m\omega_c}D(t)\Big] \\
\equiv {} & \int_0^\infty dt(1-e^{-i\nu t})\,\mathrm{Im}\,S_{ii}(t), \\
D(t) = {} & \frac{\varkappa_\perp^2}{\varkappa^2}\Big[\mathrm{cth}\frac{\lambda\hbar\omega_c}{2}-\frac{\cos\omega_c(t-i\lambda\hbar/2)}{\mathrm{sh}(\lambda\hbar\omega_c/2)}\Big] \\
& +\frac{\varkappa_3^2}{\varkappa^2}\Big[\frac{\lambda\hbar\omega_c}{4}+\frac{(\omega_c t-i\lambda\hbar\omega_c/2)^2}{\lambda\hbar\omega_c}\Big].
\end{aligned}
\tag{43}
$$

Separating in (43) the real and imaginary parts, we obtain the components of the effective-mass tensor and of the relaxation time:

$$
\frac{M_{ii}}{m} = 1-\mathrm{Im}\int_0^\infty dt(1-\cos\nu t)S_{ii}(t), \tag{44}
$$

$$
\frac{1}{\tau_{ii}} = \nu\,\mathrm{Im}\int_0^\infty dt\sin\nu t\, S_{ii}(t). \tag{45}
$$

The function $S_{ii}(t)$ has no singularities in the complex t plane in the region $0 \le \mathrm{Re}\, t \le \infty$ and $0 \le \mathrm{Im}\, t \le \lambda\hbar$. We therefore replace the integrals in (44) and (45) along the real axis by integrals from 0 to $i\lambda\hbar/2$ along the imaginary axis and from $i\lambda\hbar/2$ to $i\lambda\hbar/2 + \infty$ along a line parallel to the real axis. The section of the contour from $i\lambda\hbar/2 + \infty$ to ∞ makes no contribution. The integral in τ_{ii}^{-1} over the section $0 - i\lambda\hbar/2$ is equal to zero, since $S_{ii}(it)$ and $i\,dt \sin i\nu t$ are real functions.

Thus, the final expressions become

$$
\begin{aligned}
\frac{M_{ii}}{m} = {} & 1-\mathrm{sh}\frac{\lambda\hbar\nu}{2}\int_0^\infty dt\sin\nu t\, S_{ii}\Big(t+\frac{i\lambda\hbar}{2}\Big) \\
& +\int_0^{\lambda\hbar/2}dt(\mathrm{ch}\,\nu t-1)S_{ii}(it),
\end{aligned}
\tag{46}
$$

$$
\frac{1}{\tau_{ii}} = \nu\,\mathrm{sh}\frac{\lambda\hbar\nu}{2}\int_0^\infty dt\cos\nu t\, S_{ii}\Big(t+\frac{i\lambda\hbar}{2}\Big). \tag{47}
$$

As $\lambda \to \infty$ $(T \to 0)$ and $\nu = 0$, the first integral in (46) vanishes, but at finite values of λ this integral is the principal one. Therefore, with increasing temperature the polaron correction changes from positive to negative.

Formula (47) gives the relaxation time with allowance for the inelasticity. In the elastic approximation it is necessary to set $\cos\omega_{\kappa j}t$ equal to unity in $S_{ii}(t + i\lambda\hbar/2)$.

4. LONGITUDINAL AND TRANSVERSE MAGNETORESISTANCE

The longitudinal magnetoresistance is given by the formula

$$
\rho_{33} = \frac{1}{\sigma_{33}} = \frac{M_{33}}{e^2\tau_{33}},
$$

from which it is seen that the different scattering mechanisms are additive. Let us consider some particular cases.

For a deformation interaction of the carriers with acoustic oscillations

$$
|v_{\varkappa j}|^2 = a_0^2\varkappa^2/\rho \tag{48}
$$

at temperatures that are high relative to the active phonon $\lambda\hbar\omega_{\kappa j}/2 < 1$ and in weak fields $\lambda\hbar\omega_c/2 < 1$ we have

$$
\rho_{33}(H) = \frac{M_{33}}{m}\rho_{33}(0)\Big[1+\frac{1}{8}\Big(\frac{\lambda\hbar\omega_c}{2}\Big)^2\Big], \tag{49}
$$

$$
\frac{M_{33}}{m} = 1-\frac{1}{12\sqrt{2\pi}}\frac{a_0^2 m^{3/2}}{\hbar^3\lambda^{1/2}\rho w^2}\Big(\frac{\lambda\hbar\omega_c}{2}\Big)^2. \tag{50}
$$

Here a_0 is the constant of the deformation potential, ρ is the density of the medium, and w is the longitudinal velocity of sound. The value of $\rho_{33}(0)$ coincides with that obtained by Davydov and Shmushkevich.[6] The correction to the mass (50) at H = 0 vanishes, in agreement with the result of Krivoglaz and Pekar.[7]

There is no difficulty in calculating ρ_{33} for the cases of piezoelectric interaction with acoustic oscillations at $\lambda\hbar\omega_{\kappa j}/2 < 1$ and $\lambda\hbar\omega_c/2 < 1$ and with longitudinal optical oscillations at $\lambda\hbar\omega/2 > 1$ and $\lambda\hbar\omega_c/2 < 1$, where ω is the limiting frequency of the optical oscillations. The expression for $\rho_{33}(0)$ for the piezo-interaction coincides with the result obtained in [8]; for the interaction with the longitudinal optical oscillations the results agree with those of [3].

In the case of strong fields $\lambda\hbar\omega_c/2 > 1$ and high temperatures $\lambda\hbar\omega_{\kappa j}/2 < 1$, for piezo-interaction of electrons with lattice vibrations described by the constants

$$
|v_{\varkappa j}|^2 = \frac{16\pi^2 e^2}{\varepsilon_0^2\rho}(B_n e_n^j)^2, \quad B_n e_n^j = \frac{\beta_{i,kn}\varkappa_i\varkappa_k e_n^j}{\varkappa^2}, \tag{51}
$$

where $\beta_{i,kn}$ are the piezo-moduli, we have

$$
\rho_{33} = 2\sqrt{\frac{2}{\pi}}\frac{M_{33}m^{1/2}}{\hbar^2\lambda^{1/2}}\mathscr{K}^2\ln\frac{\lambda\hbar\omega_c}{4}, \tag{52}
$$

$$
\mathscr{K}^2 = \sum_j\int\frac{(B_n e_n^j)^2}{\varepsilon_0^2\rho w_j^2}\sin\vartheta\, d\vartheta\, d\varphi,
$$

$$
\frac{M_{33}}{m} = 1-\frac{\pi^{3/2}}{16}\frac{m^{1/2}e^2\lambda^{1/2}}{\hbar}\mathscr{K}^2\Big(\frac{\lambda\hbar\omega_c}{2}\Big)^{1/2}. \tag{53}
$$

The main relation $\rho_{33} \propto T^{1/2}$ coincides with that given by Adams and Holstein.[9] We note that the dependence of the polaron mass on H may change ρ_{33} in CdS (by about 30% at $T \sim 1°K$) and can lead to a decrease of ρ_{33} with increasing field.

For the optical interaction

$$|v_{\varkappa j}|^2 = \frac{4\pi c e^2 \omega}{\varkappa^2} = \alpha \frac{4\pi \cdot 2^{1/2} \omega^{5/2} \hbar^{3/2}}{m^{1/2} \varkappa^2} \tag{54}$$

(where α is the Frölich interaction constant) the case of importance is that of low temperatures $\lambda\hbar\omega/2 > 1$. Interest attaches to strong fields $\omega_c \sim \omega$, in which oscillations of the longitudinal magnetoresistance, which were investigated by Gurevich and Firsov,[11] were observed.[10] The non-oscillating part of ρ_{33} is given by

$$\rho_{33} = \sqrt{2\pi} \frac{M_{33}\omega}{e^2} \alpha \lambda \hbar \omega_c e^{-\lambda\hbar\omega}. \tag{55}$$

The linear increase of ρ_{33} with H (a similar dependence is obtained for strong fields in the deformation interaction, see (64) with $\nu \to 0$) is in agreement with the experimental results.[10] The value of M_{33} is given by formula (3) of [12]. The oscillating part of $\Delta\rho_{33}$ is small and is given approximately by

$$\Delta\rho_{33} = 4 \frac{M_{33}\omega}{e^2} \alpha \left(\frac{\lambda\hbar\omega}{2}\right)^{1/2} e^{-\lambda\hbar\omega} \cdot \frac{\lambda\hbar|\omega - \omega_c|}{2} K_1 \left(\frac{\lambda\hbar|\omega - \omega_c|}{2}\right); \tag{56}$$

Here $K_n(z)$ is the Macdonald function. Formula (56) gives a maximum at $\omega_c = \omega$, in agreement with the conclusions of Gurevich and Firsov[11] for the case when the scattering by the optical oscillations plays the principal role.

The transverse magnetoresistance $\rho_\perp = \sigma_{11}/(\sigma_{11}^2 + \sigma_{12}^2)$ in the case of strong fields $\omega_c\tau > 1$ is determined in the isotropic approximation as follows:

$$\rho_\perp = \sigma_{11}/\sigma_{12}^2 = M_{11}/e^2\tau_{11}. \tag{57}$$

Considering different interaction mechanisms, we can obtain from (57) the results of Adams and Holstein[9] and of Gurevich and Firsov,[13] refined by allowance for the polaron mass M_{11}, which depends on the temperature and on the field. We shall discuss in greater detail the case of the interaction with optical oscillations, wherein $\rho_\perp$ experiences resonant oscillations. As a result of neglecting the dispersion of the optical phonons and other mechanisms that limit the heights of the oscillations, $\rho_\perp(H)$ diverges logarithmically at the points $\omega_c = \omega/N$. In the case $\lambda\hbar\omega_c/2 > 1$ we have

$$\rho_\perp = \frac{1}{2\sqrt{\pi}} \frac{M_{11}\omega_c}{e^2} \frac{\alpha}{\operatorname{sh}(\lambda\hbar\omega/2)} (\lambda\hbar\omega)^{3/2} \left\{ K_0\left(\frac{\lambda\hbar\omega}{2}\right) + \frac{1}{2\operatorname{ch}(\lambda\hbar\omega_c/2)} \times K_0\left(\frac{\lambda\hbar|\omega - \omega_c|}{2}\right) + \frac{1}{4\operatorname{ch}^2(\lambda\hbar\omega_c/2)} K_0\left(\frac{\lambda\hbar|\omega - 2\omega_c|}{2}\right) + \ldots \right\}. \tag{58}$$

For $\lambda\hbar\omega/2 < 1$ (high temperatures) we have $\rho_\perp \propto HT^{1/2}$, just as in [9]. If $\omega_c > \omega$ and $\lambda\hbar\omega/2 > 1$, then $\rho_\perp \propto \lambda\hbar\omega_c \exp(-\lambda\hbar\omega)$, which agrees with formula (38) of Gurevich and Firsov.[13]

It is interesting to note that M_{11} also has resonant peaks at the points $\omega_c = \omega/N$, but of delta-like character. Taking into account the cutoff due to the broadening of the levels,[9] we obtain

$$\frac{M_{11}}{m} = 1 - \frac{\alpha}{\sqrt{\pi}} \frac{\lambda\hbar\omega_c(\lambda\hbar\omega)^{1/2}}{e^{\lambda\hbar\omega/2}} \cdot \sum_{N=0}^{\infty} \frac{1}{e^{N\lambda\hbar\omega_c/2}} \frac{\tau}{\lambda\hbar[1 + (\omega - N\omega_c)^2\tau^2]}. \tag{59}$$

5. CYCLOTRON RESONANCE OF POLARONS

The cyclotron resonance is described by formula (33). We consider the piezoelectric interaction, since there is no complete agreement between the theoretical papers[14-17, 20] and the experiment of Baer and Dexter.[18] The decrease of the effective mass $m^* = eH_{max}/c\nu$, determined by measuring the shift of the maximum of the band in CdS, was first observed by Sawamoto.[19] Later Baer and Dexter observed an anisotropy, connected with the orientation of the magnetic field, in this effect. It turned out that at $H \parallel C_6$ (C_6 is the hexagonal axis in CdS), the shift of the maximum is smaller than at $H \perp C_6$. Mahan and Hopfield[14] attributed this shift to the piezopolaron effect, but in calculating M they neglected the influence of the magnetic field and the anisotropy. Larsen[15] calculated M_{11} in the approximation $T = 0$ and obtained $M_{11} > m$. Saitoh and Kawabata[16] used the Kubo formula and employed the Mori method to calculate the correlators. But $\sigma_\perp$ as obtained by them contains sums over the quantum numbers of the electronic states; since these sums are difficult to calculate, Saitoh and Kawabata confined themselves to the zeroth Landau band, a procedure that is valid only in the case of the strong inequality $\lambda\hbar\omega_c \gg 1$, and obtained $M_{11} \approx m$. Miyake[17] used the temperature Green's functions method, but put $k_z = 0$ in the calculation and went beyond the accuracy of the method, taking into account the corrections in the energy denominators.

The present authors[20] obtained the correct order of magnitude of the shift of the maximum of the cyclotron resonance (CR) band, but could not obtain the weaker anisotropy effect. Furthermore, for a comparison with [18] they used the experimental values of τ. We note that unlike the galvanomagnetic phenomena, in which the polaron change of mass is weakly pronounced, experiments on CR are of particular interest for polaron theory, since M and τ are measured independently, respectively from the shift of the maximum and the half-width of the CR line.

In the discussed experiments of Baer and Dexter[18] $m = 0.2m_e$, $\nu = 4.26 \times 10^{11}\ \text{sec}^{-1}$, $T = 1.3°K$, and $\lambda\hbar\nu/2 = 1.25$, corresponding to intermediate magnetic fields. Two calculation methods are possible, starting from the strong-field approximation, when $S_{ii}(t + i\lambda\hbar/2)$ is expanded in powers of $\cos\omega_c t/\cosh(\lambda\hbar\omega_c/2)$, and of relatively weak fields, when the expansion of the quantities $\cosh(\lambda\hbar\omega_c/2) - \cos\omega_c t$ and $\cosh(\lambda\hbar\omega_c/2) - \cosh\omega_c(t - \lambda\hbar/2)$ in $S_{ii}(it)$ is carried out with respect to ω_c. The first approach calls for summation of many terms of the expansion, which is equivalent to inclusion of many Landau bands. In the second approach it was sufficient to retain two terms of the series, thereby ensuring an accuracy of $\sim 1\%$.

The energy of the effective phonon is $\lambda\hbar\omega l^{-1} \sim 0, 1$, $l^2 = c\hbar/eH$ and it can be assumed that the phonon temperatures are high. In the elastic approximation we obtain

$$\frac{m-M_{ii}}{m}+\frac{i}{\nu\tau_{ii}}=16\sqrt{2\pi}\,\frac{e^2m^{1/2}}{\nu^2\hbar^3\lambda^{3/2}\varepsilon_0{}^2\rho}\left(\frac{2\,\mathrm{sh}\,(\lambda\hbar\omega_c/2)}{\lambda\hbar\omega_c}\right)^{3/2}$$
$$\times\left\{\frac{1}{w_1{}^2}\left[\beta_{3,33}^2 I_{ii}^{(1)}(x)+2\beta_{3,33}(\beta_{3,11}+2\beta_{1,13})I_{ii}^{(2)}(x)+(\beta_{3,11}+2\beta_{1,13})^2 I_{ii}^{(3)}(x)\right]\right.$$
$$+\frac{1}{w_3{}^2}\left[\beta_{1,13}^2 I_{ii}^{(4)}(x)+2\beta_{1,13}(\beta_{3,33}-\beta_{3,11}-\beta_{1,13})I_{ii}^{(3)}(x)\right.$$
$$\left.\left.+(\beta_{3,33}-\beta_{3,11}-\beta_{1,13})^2 I_{ii}^{(2)}(x)\right]\right\},$$
$$l=1,\ 2=x,\ y;\qquad x=1-2\,\mathrm{sh}\,(\lambda\hbar\omega_c/2)/\lambda\hbar\omega_c. \tag{60}$$

In the case $C_6 \parallel \mathbf{H} \parallel z$ we have $1-M_{11}/m = 1-M_{22}/m$ and $1/\tau_{11} = 1/\tau_{22}$, since $I_{11}^{(n)} = I_{22}^{(n)} = I^{(n)}$, with

$$I^{(1)}(x)={}^2/_{63}f_1(\nu)F_{1,3}(x)-{}^4/_{231}f_2(\nu)F_{2,3}(x),$$
$$I^{(2)}(x)={}^8/_{315}f_1(\nu)F_{1,2}(x)-{}^8/_{385}f_2(\nu)F_{2,2}(x),$$
$$I^{(3)}(x)={}^{16}/_{315}f_1(\nu)F_{1,1}(x)-{}^{64}/_{231}f_2(\nu)F_{2,1}(x),$$
$$I^{(4)}(x)={}^{128}/_{315}f_1(\nu)F_{1,0}(x)-{}^{128}/_{231}f_2(\nu)F_{2,0}(x), \tag{61}$$

where

$$F_{1,m}(x)={}_2F_1\left(\frac{2m+1}{2},\frac{3}{2};\frac{11}{2};x\right),$$
$$F_{2,m}(x)=\frac{1}{12}\left(\frac{\lambda\hbar\omega_c}{2}\right)^2{}_2F_1\left(\frac{2m+1}{2},\frac{5}{2};\frac{13}{2};x\right),$$

${}_2F_1$ is the hypergeometric function.

The real and imaginary parts of the function f are equal to

$$\mathrm{Re}\,f_1(y)={}^1/_2\pi y\,\mathrm{sh}\,y[L_{-1}(y)-I_1(y)]-(1-{}^1/_4\pi)y^2,$$
$$\mathrm{Re}\,f_2(y)={}^1/_2\pi\,\mathrm{sh}\,y(2y(L_{-1}(y)-I_1(y))-(I_0(y)-L_0(y))]$$
$$-({}^2/_3-{}^1/_8\pi)y^2, \tag{62}$$
$$\mathrm{Im}\,f_1(y)=y\,\mathrm{sh}\,yK_1(y),$$
$$\mathrm{Im}\,f_2(y)=\mathrm{sh}\,y[2yK_1(y)-K_0(y)],\qquad y=\lambda\hbar\nu/2,$$

$L_n(y)$ are Struve functions and $I_n(y)$ are Bessel functions of imaginary argument. The second terms with f_2 in (61) determine the corrections for H, which under the conditions of the Baer and Dexter experiment turn out to be of the order of 1%.

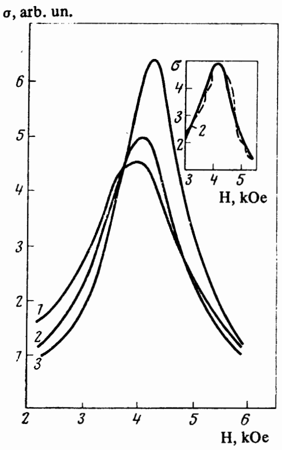
At the experimental configuration $C_6 \parallel y \perp \mathbf{H}$ we have $1/\tau_{11} < 1/\tau_{22}$ and $1-M_{11}/m < 1-M_{22}/m$. The functions I take the form

$$I_{11}^{(1)}={}^5/_{64}I^{(4)},\ I_{11}^{(2)}={}^1/_8I^{(3)}+{}^3/_{64}I^{(4)},\ I_{11}^{(3)}={}^1/_4I^{(2)}+{}^1/_4I^{(3)}+{}^5/_{64}I^{(4)},$$
$$I_{11}^{(4)}=I^{(1)}+{}^9/_4I^{(2)}+{}^{15}/_8I^{(3)}+{}^{35}/_{64}I^{(4)},$$
$$I_{22}^{(1)}={}^{35}/_{64}I^{(4)},\ I_{22}^{(2)}={}^5/_8I^{(3)}+{}^5/_{64}I^{(4)},\ I_{22}^{(3)}={}^3/_4I^{(2)}+{}^1/_4I^{(3)}+{}^3/_{64}I^{(4)},$$
$$I_{22}^{(4)}=I^{(1)}+{}^3/_4I^{(2)}+{}^3/_8I^{(3)}+{}^5/_{64}I^{(4)}. \tag{63}$$

Substituting (61) and (63) in (60), we obtain $1-M_{ii}/m$ and $1/\tau_{ii}$ which are needed for the calculation of σ_{ii} from (33). Owing to the indicated relations for $1-M_{ii}/m$ and $1/\tau_{ii}$, the values of σ_{ii}, generally speaking, are not equal to each other, and this reflects the anisotropy of the absorption.

Substituting the numerical values of the parameters of CdS (taken from the work of Pokatilov[21]), we have plotted (see the figure) $\sigma_{22}(H)$ for the case $C_6 \parallel y \perp \mathbf{H}$, $\mathbf{E} \parallel C_6$ (1) and $\sigma_{11}(H)$ for the cases $C_6 \parallel y \perp \mathbf{H}$, $\mathbf{E} \parallel x$ (2) and $C_6 \parallel \mathbf{H}$, $\mathbf{E} \parallel x \perp C_6$ (3). The position of the maximum and the form of the absorption band in case (2) agree well with experiment[18] (dashed curve in the insert). For case (3), the maximum position indicated in [18] agrees with curve 3. There were no measurements made in variant (1).

Thus, we can state that the results of the calculation agree with the available experimental data. In addition



Plot of $\sigma(H)$ in accordance with formula (33): curves: 1—σ_{22}, $\mathbf{H} \perp C_6$, 2—σ_{11}, $\mathbf{H} \perp C_6$, 3—$\sigma_{11} = \sigma_{22}$, $\mathbf{H} \parallel C_6$. Insert: solid line—theory, dashed—experiment.

to the experimentally observed relatively strong anisotropy of the directions of the magnetic field $\mathbf{H} \parallel C_6$ and $\mathbf{H} \perp C_6$, calculation predicts a weaker anisotropy at a fixed magnetic field $\mathbf{H} \perp C_6$, and in an electric field $\mathbf{E} \parallel C_6$ and $\mathbf{E} \perp C_6$ (curves 1 and 2).

In conclusion, we call attention to certain oscillatory effects in a longitudinal alternating electric field. At large values $\nu\tau_{33} > 1$, according to (28), we have $\sigma_{33} \approx M_{33}\nu^2\tau_{33}$. Calculating $1/\tau_{33}$ with allowance for the acoustic scattering, we can obtain a formula analogous to (55), but with an argument $\nu - N\omega_c$ in place of $\omega - N\omega_c$:

$$\frac{1}{\tau_{33}}=\left(\frac{2}{\pi}\right)^{3/2}\frac{m^{3/2}a_0{}^2\omega_c\mathrm{sh}(\lambda\hbar\nu/2)}{\rho\nu w^2\hbar^4\lambda^{3/2}}\,\mathrm{th}\,\frac{\lambda\hbar\omega_c}{2}$$
$$\times\left\{\frac{\lambda\hbar\nu}{2}K_1\left(\frac{\lambda\hbar\nu}{2}\right)+\frac{1}{2\mathrm{ch}(\lambda\hbar\omega_c/2)}\frac{\lambda\hbar|\nu-\omega_c|}{2}\right.$$
$$\times K_1\left(\frac{\lambda\hbar|\nu-\omega_c|}{2}\right)+\frac{1}{4\mathrm{ch}^2(\lambda\hbar\omega_c/2)}$$
$$\left.\times\frac{\lambda\hbar|\nu-2\omega_c|}{2}K_1\left(\frac{\lambda\hbar|\nu-2\omega_c|}{2}\right)+\ldots\right\}. \tag{64}$$

Formula (64) was obtained under the assumption of a relatively strong field $\lambda\hbar\omega_c/2 \gtrsim 1$, and high temperatures of the active phonons $\lambda\hbar\omega_{\kappa j} < 1$. It gives oscillation peaks σ_{33} at $\nu = N\omega_c$. A similar calculation with scattering by optical phonons leads to the appearance of oscillation maxima at the points $\nu \mp N\omega_c \pm \omega = 0$. In the limit as $\nu \to 0$ we obtain the oscillation of the longitudinal magnetoresistance, obtained by Gurevich and Firsov[11] and referred to in connection with formula (55).

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