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EXCITATION OF LANGMUIR OSCILLATIONS IN A PLASMA BY THE FIELD OF A TRANSVERSE WAVE

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The conditions are investigated for the excitation of natural electronic potential and nonpotential plasma oscillations by the field of an external monochromatic pumping wave with a frequency exceeding double the plasma frequency. It is shown that at not too low an electron temperature, the instability against buildup of such oscillations is decisive, since the time of its development is much shorter than the growth time of the perturbations in stimulated Mandel'shtam-Brillouin scattering and in the aperiodic instability.

It is known that a plasma located in the field of a transverse pump wave, whose frequency ω_0 exceeds the plasma frequency $\omega_p = \sqrt{\omega_{Le}^2 + \omega_{Li}^2}$ ($\omega_{L\alpha}^2 = 4\pi e_\alpha^2 n_\alpha / m_\alpha$), can be unstable both against the buildup of nonpotential perturbations with a frequency close to ω_0[1] and against the excitation of ion-acoustic oscillations[2]. The latter process, constituting stimulated Mandel'shtam-Brillouin scattering (SMBS), is possible only in a nonisothermal plasma with an electron temperature exceeding the ion temperature ($e_i T_e \gg e T_i$), with $\omega_0 < \omega_{Li} c / 2v_{Ti}$ (c is the speed of light and $v_{T\alpha} = \sqrt{T_\alpha / m_\alpha}$ is the thermal velocity of the particles of specials α).

We shall show in this paper that at not too low an electron temperature, in the external-field frequency region

$$2\omega_p < \omega_0 < \omega_{Le} c / v_{Te} \tag{1}$$

the decisive role is played by instability against the buildup of Langmuir oscillations. This instability develops at pump-wave electric-field intensities much lower than those at which the SMBS and the aperiodic instabilities investigated by Kiriĭ[1] are possible.

Research on the influence of a given electromagnetic pump wave on the Langmuir oscillations in a plasma wave initiated by Volkov[3]. He has shown that under the resonance conditions corresponding to the decay of the pump wave into transverse and Langmuir waves, one should expect the plasma to be unstable against the buildup of small perturbations. However, the results obtained by him do not hold in the immediate vicinity of resonance, where the growth increment of the perturbations reaches a maximum. In later papers[4,5] the corresponding maximal increment was found without taking into account the finite wavelength of the pump wave and the dissipation of the excited oscillations. In addition, a number of investigations (cf., e.g.,[6]) dealt with the decay of transverse wave into Langmuir and transverse waves in the interaction of wave packets that are broad in wave-number space.

We shall determine below, with account taken of the dissipative effects, the threshold amplitude of the monochromatic pump wave and the stability increment for excitation of a Langmuir and a transverse wave in a plasma.

We consider a homogeneous plasma situated in the field of a plane transverse pump wave with a specified amplitude

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \sin(\omega_0 t - \mathbf{k}_0 \mathbf{r}), \quad \omega_0^2 = c^2 k_0^2 + \omega_p^2, \quad \mathbf{k}_0 \mathbf{E}_0 = 0.$$

Assuming that the perturbations with frequencies $\omega \pm \omega_0$ and wave numbers $\mathbf{k} \pm \mathbf{k}_0$ satisfy approximately the dispersion law for the transverse waves in the plasma, we obtain the following dispersion equation for the most rapidly growing small perturbations with $\mathbf{k} \parallel \mathbf{k}_0$ (cf.[1,2]):

$$\frac{1}{\delta\varepsilon_e(\omega + i\gamma, k)} + \frac{1}{1 + \delta\varepsilon_i(\omega + i\gamma, k)} + \frac{1}{4}\frac{v_E^2}{c^2} \times \Big[\frac{k^2 c^2/(\omega - \omega_0)^2}{\varepsilon^{tr}(\omega + i\gamma - \omega_0) - c^2(k - k_0)^2/(\omega - \omega_0)^2} + \frac{k^2 c^2/(\omega + \omega_0)^2}{\varepsilon^{tr}(\omega + i\gamma + \omega_0) - c^2(k + k_0)^2/(\omega + \omega_0)^2}\Big] = 0. \tag{2}$$

Here $\delta\epsilon_\alpha(\omega, k)$ is the contribution of the particles of species α to the linear longitudinal dielectric constant of the plasma $\epsilon(\omega, k) = 1 + \delta\epsilon_e(\omega, k) + \delta\epsilon_i(\omega, k)$, $v_E = eE_0/m_e\omega_0$ is the velocity of the electron oscillations in the pump-wave electric field and is assumed to be much smaller than the velocity of light ($v_E \ll c$). The transverse dielectric constant is given by

$$\varepsilon^{tr}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}\Big(1 - i\frac{\nu_{eff}}{\omega}\Big),$$

where the effective frequency of the electron-ion collisions in the external HF field, for an oscillation velocity v_E much smaller than the electron thermal velocity ($v_E \ll v_{Te}$), is equal to[7]

$$\nu_{eff} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\frac{e^2 e_i^2 n_i}{T_e^{3/2}} \ln\frac{v_{Te}}{\omega_0 r_{min}}.$$

Under conditions when perturbations with frequency $\omega - \omega_0$ and wave number $k - k_0$ are the natural oscillations in the plasma, i.e., when

$$k^2 c^2 = 2kk_0 c^2 - \omega(2\omega_0 - \omega), \tag{3}$$

and perturbations with frequency ω and wave number k are the natural longitudinal wave in the plasma, the growth increment of the perturbations γ reaches a maximum and is determined by the equation

$$(\gamma+\gamma_0)(\gamma+\tilde{\gamma})=\frac{1}{16}\frac{v_E^2}{c^2}\frac{\omega}{\omega_0-\omega}k^2c^2\delta, \tag{4}$$

to which the dispersion equation (2) reduces. Here $\gamma_1\tilde{\gamma} \ll \omega$, and the damping decrements of the transverse and longitudinal waves, $\tilde{\gamma}$ and γ_0, are given by

$$\tilde{\gamma}=\frac{1}{2}\frac{\omega_p^2\nu_{\text{eff}}}{(\omega_0-\omega)^2}\quad \gamma_0=\frac{\omega}{2}\left[\frac{\operatorname{Im}\delta\varepsilon_e(\omega)}{(\operatorname{Re}\delta\varepsilon_e(\omega))^2}+\frac{\operatorname{Im}\delta\varepsilon_i(\omega)}{(1+\operatorname{Re}\delta\varepsilon_i(\omega))^2}\right]\delta,$$

$$\delta=\left(-\frac{\omega}{2}\frac{\partial}{\partial\omega}\left[\frac{1}{\operatorname{Re}\delta\varepsilon_e(\omega)}+\frac{1}{1+\operatorname{Re}\delta\varepsilon_i(\omega)}\right]\right)^{-1}.$$

If one of these decrements is much larger than the other, and also larger than the instability increment γ, i.e.,

$$\Gamma\equiv\max(\gamma_0,\tilde{\gamma})\gg\gamma,\quad \min(\gamma_0,\tilde{\gamma}),$$

then we get from (4)

$$\gamma=\frac{1}{16}\frac{v_E^2-v_{\text{E. thr}}^2}{c^2}\frac{\omega}{\omega_0-\omega}\frac{k^2c^2}{\Gamma}\delta, \tag{5}$$

where the threshold value of the external-field intensity, above which the plasma becomes unstable, is given by

$$\frac{v_{\text{E. thr}}^2}{c^2}=16\frac{\gamma_0\tilde{\gamma}}{k^2c^2\delta}\frac{\omega_0-\omega}{\omega}. \tag{6}$$

If the intensity of the external field is such that $\gamma \gg \Gamma$ (or $|\gamma_0-\tilde{\gamma}| \ll \gamma_0$), then

$$\gamma=\frac{1}{4}\frac{v_E}{c}\sqrt{\frac{\omega\delta}{\omega_0-\omega}}kc-\frac{\gamma_0+\tilde{\gamma}}{2}. \tag{7}$$

When Langmuir oscillations are exited with a frequency

$$\omega=\omega_p\left(1+\frac{3}{2}\frac{k^2v_{Te}^2}{\omega_p^2}\right)>kv_{Te}$$

the relation (3) can be satisfied only in the external-field frequency region bounded by the inequality (1). In this case $\delta = 1$, $\gamma_0 = \gamma_{\text{eff}}/2$, and when $\omega_0 \gg \omega_0 \approx \omega_p$ we find from formulas (3) and (5)-(7) that $k = 2k_0 = 2\omega_0/c$, while the maximum increment and the threshold value of the pump field intensity are determined by

$$\gamma=\frac{1}{2}\frac{v_E^2-v_{\text{E. thr}}^2}{c^2}\omega_p\frac{\omega_0}{\nu_{\text{eff}}}\qquad \gamma\ll\frac{1}{2}\nu_{\text{eff}}, \tag{8}$$

$$\gamma=\frac{1}{2}\frac{v_E}{c}\sqrt{\omega_p\omega_0},\qquad \frac{1}{2}\nu_{\text{eff}}\ll\gamma\ll\omega_p, \tag{9}$$

$$\frac{v_{\text{E. thr}}}{c}=2\frac{\nu_{\text{eff}}}{\omega_0}\sqrt{\frac{\omega_p}{\omega_0}}. \tag{10}$$

Comparing (10) with the SMBS threshold obtained from (6) for

$$\omega=\omega_s=kv_s,\quad \delta=\left(\frac{kv_{Te}}{\omega_{Le}}\right)^{-2},\quad \gamma_0=\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{\omega_{Le}}\omega_s,$$

we find that the instability against the buildup of Langmuir oscillations develops at pump field intensities lower than the threshold value for SMBS, subject to the following condition on the electron temperature:

$$\frac{v_{Te}}{c}>\frac{\omega_{Le}}{\omega_0}\sqrt{\frac{\nu_{\text{eff}}}{\omega_{Li}}}. \tag{11}$$

If in addition $v_{Te}/c > \omega_{Li}/2\omega_0$, then the increment of the buildup of the Langmuir oscillations exceeds the increment for the buildup of the low-frequency ion-acoustic oscillations at all external-field intensities.

At an electron temperature limited by the inequalities

$$\frac{\omega_{Li}}{2\omega_0}>\frac{v_{Te}}{c}>\frac{\omega_{Le}}{\omega_0}\sqrt{\frac{\nu_{\text{eff}}}{\omega_{Li}}}, \tag{12}$$

a comparison of the increments (8) and (9) with the corresponding values for $\omega = \omega_s$ and with the increment

$$\gamma\sim\omega_0\left(\frac{v_E}{c}\frac{\omega_{Li}}{\omega_0}\right)^{2/3}, \tag{13}$$

which is obtained upon excitation of low-frequency oscillations $|\omega + i\gamma| < \omega_{Li}$ and $\gamma > \omega_s$[2], shows that the growth increment of the perturbations in SMBS and the increment (13) exceed expression (9) only at the following values of the external field intensity:

$$\left(\frac{\omega_{Li}}{\omega_{Le}}\right)^2\sqrt{\frac{\omega_{Le}}{\omega_0}}>\frac{v_E}{c}>\sqrt{2\pi}\frac{\omega_{Li}}{\omega_{Le}}\left(\frac{v_{Te}}{c}\right)^2\left(\frac{\omega_0}{\omega_p}\right)^{3/2}. \tag{14}$$

Let us compare now our results with the conditions for the occurrence of the aperiodic instability investigated in[1]. The threshold value of the pump field intensity for the development of such an instability exceeds (10) when

$$\frac{v_{Te}}{c}>\frac{\omega_p}{\omega_0}\sqrt{\frac{\nu_{\text{eff}}}{\omega_p}}. \tag{15}$$

Under the same conditions, the increments (8) and (9) exceed the corresponding values for the aperiodic instability.

Thus, in an isothermal plasma, where the SMBS process is impossible, the here-investigated instability against the buildup of Langmuir oscillations is decisive under the conditions (1) and (15). In a nonisothermal plasma, in the region of the external-field frequencies (1), the growth increment of the perturbations for SMBS exceeds the increment for the buildup of perturbations with frequency $\omega = \omega_p$ only if the condition (11) is violated or if relations (12) and (14) are satisfied.

We note finally that the expression (9) for the nondissipative increment differs from the value obtained earlier[4,5] by a numerical factor, since we took into account here the finite wavelength of the pump field.

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240