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FORMATION OF NONEQUILIBRIUM POPULATION IN A PLASMA UNDER IONIZATION-RESONANCE CONDITIONS

L. I. GUDZENKO and S. I. YAKOVLENKO

P. N. Lebedev Physics Institute, U.S.S.R. Academy of Sciences

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In the case of intense recombination in a dense plasma of complicated chemical composition, the resonant reaction of excitation transfer from the low-lying level of one element to the high-lying levels of another element, and also of ionization in transfer of the excitation, lead to an appreciable change in the character of the nonequilibrium population of the levels of the first element.

1. THE difficulties involved in the analysis of relaxation of a low-temperature dense plasma are due not only to the large number of characteristics of the state of the plasma (the populations of the discrete levels of the atoms and ions, the parameters of the distribution of the free electrons, the density of the radiation at different frequencies) and the nonlinearity of the kinetic equations, but also to the lack of sufficient information concerning the cross sections of a number of important elementary acts. In theoretical estimates of the probabilities of nonradiative transitions one frequently uses such strong simplifying assumptions that it is not clear beforehand whether even the orders of magnitude of the results are correct; at the same time there is practically no reliable procedure for their experimental determination. This is connected, in particular, with the fact that to measure the collision probabilities it is advantageous to use a plasma with a large free-electron density. In such a plasma, the populations of the excited levels assume equilibrium values within short time intervals, in accordance with a law governed by the algebraic equations of the "stationary sink" [11]

$$\frac{dN_\gamma}{dt} = \sum_{\gamma'=1} K_{\gamma\gamma'}\bar{N}_{\gamma'} + C_\gamma = 0, \qquad \gamma = 2, 3, \ldots, n;$$

on the other hand, the populations[1] of the stationary sink follow in a rather unique manner the changes of the temperature $T_e(t)$ and of the density $N_e(t)$ of the free electrons.

At the same time, studies of the relaxation of a low-temperature dense plasma can find a large number of applications, for example: discharge in a strong high-frequency field, the kinetics of plasmo-chemical reactions, development of flares on the sun, action of a focused laser pulse on matter, the diagnostics of a self-contracting discharge, etc. An urgent question is the discussion of the possibilities of creating effective plasma lasers in which the amplifying medium is a pulsed recombining plasma with a large density of free electrons ($N_e \gtrsim 10^{13}$ cm^{-3}). There are already published mentions of the advantages of such lasers when used at high energies and short wavelengths. The urgency for analyzing nonequilibrium processes thus points to the need of speeding up the development of procedures for the corresponding experiments.

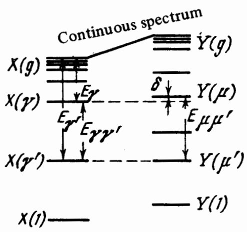

FIG. 1

In a plasma with a complex composition of heavy particles, one can attempt to change noticeably the populations of selected discrete levels by means of different inelastic collisions with heavy particles of other types. From this point of view, the most interesting reactions are resonant excitation transfer (Fig. 1)

$$X(\gamma) + Y(\mu') \rightleftarrows X(\gamma') + Y(\mu), \tag{1}$$

direct ionization with excitation transfer

$$X(\gamma) + Y(\mu') \rightleftarrows X(\gamma') + Y^{+} + e, \tag{2}$$

and various chemical reactions

$$X(\gamma) + Y(\mu') + Z \rightleftarrows (XY)^{*} + Z. \tag{3}$$

Situations in which an important role is played by acts wherein atoms X absorb optical quanta emitted by atoms Y,

$$Y(\mu) \rightleftarrows Y(\mu') + \hbar\omega, \qquad X(\gamma') + \hbar\omega \rightleftarrows X(\gamma), \tag{4}$$

are rarely encountered, for this calls for an energy defect $\delta \equiv |E^{(X)}_{\gamma\gamma'} - E^{(Y)}_{\mu\mu'}|$ of the order of the width of the corresponding lines, i.e., $\delta \lesssim 10^{-4}$ eV. For the reaction (1) to be resonant it suffices in most cases to have $\delta \lesssim 10^{-2}$ eV. In view of the practical absence of information concerning the effectiveness of reactions of the type (3), we shall likewise exclude them from consideration, although many of the conclusions that follow are applicable to them.

Assume, for example, that we are interested in the populations of the levels γ and γ' of the atoms X. Then, in order to be able to form these populations with

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[1] In this paper, the population of a level γ of the atom (ion) Z is defined throughout as the concentration of the atoms in the state γ, divided by the statistical weight of this state. The concentration of the free electrons is divided by two.

the aid of reactions (1) and (2) in some manner, it is necessary that the initial levels of the atom Y be sufficiently populated. For simplicity we assume henceforth $Y(\mu')$ to be the ground state of the atom $Y(1)$. The results remain valid also in the case when $Y(\mu')$ is a strongly populated (say, metastable) state. Let us consider a process that consists of two acts—the reaction (1) and rapid ionization of the atom $Y(\mu)$ by electron impact:

$$Y(\mu)+e\to Y^{+}+e+e.$$

In a plasma with large density of the free electrons, the sufficiently highly excited states of the atoms Z (with "energy depth" $E_{\mu}^{(Z)}$ smaller than several tenths of an electron volt) come into equilibrium with the continuum within times comparable with the free path time of the electron and are therefore well described by the Saha-Boltzmann formula during the course of the relaxation. Such states will be designated by a single symbol Z(g) and called quasi-equilibrium.

Let the relaxation (1) use the state $Y(\mu)$ pertaining to the quasiequilibrium spectrum

$$X(\gamma)+Y(1)\rightleftarrows X(\gamma')+Y(g); \tag{5}$$

Then it leads practically instantaneously to the result

$$X(\gamma)+Y(1)\rightleftarrows X(\gamma')+Y^{+}+e. \tag{6}$$

Unlike the reaction (2), the process (6) can be regarded as resonant ionization. The resonant character and accordingly the large probability of the reaction (5), the high density of the free electrons, and a sufficient concentration of the Y atoms in the initial state make up the conditions that ensure the effectiveness of such a process; we shall call them the ionization-resonance conditions. The process (6), as a rule, is more effective than the direct-ionization reaction (2), but we know of no reliable results of a calculation of the cross sections of reaction (5). The use of the known formulas for excitation transfer in remote collisions[2,3] is not justified in the present case. These formulas were derived under the assumption that the characteristic dimensions of the colliding particles are much smaller than the distances between them. In our case one of the atoms is excited to a very high level and the inverse relation is satisfied. As expected, under such conditions a formal calculation of the cross section for the transfer of excitation from X to an isolated state of Y does not agree with the experimental data[4] even in order of magnitude. As will be shown later, for a rough estimate of the cross section of the process (6) one can use in the concrete calculations the formula (see [3]) for the reaction (2), interpolated with respect to the energy from the ionization potential of the atom Y.

Since the levels of the atoms near the continuum accumulate, and at large N_e the quasiequilibrium spectrum reaches a depth of 0.2–0.3 eV, the ionization resonance in the dense plasma should be encountered in one form or another quite frequently, and its possibilities for the analysis of relaxation should not be underestimated.

2. Let us discuss briefly the dependence of the cross section σ for the transfer of excitation (1), (6) on the characteristics of the atoms X and Y, and above all on the relation between the excitation energy $X(\gamma)$ and the ionization potential of the atom Y. We confine ourselves to the quasiclassical approximation, putting

$$\sigma=2\pi\int_{0}^{\infty} w(a,u)\,a\,da,$$

where w is the total probability of the transition (6), (2) in collisions with an impact parameter a and a relative velocity u. In calculating w, it is necessary to solve the Schrödinger equation for the quasimolecule XY, with a Hamiltonian that depends explicitly on the time. The square of the expansion coefficient for the wave function of the corresponding final state of the quasimolecule wil yield the sought probability as $t\to\infty$. For brevity we shall use the density-matrix formalism, although we shall be dealing with pure states.

We write the Hamiltonian in the form $\hat{H}(t)=H_0+V(t)$, where H_0 is the Hamiltonian of the non-interacting atoms. The equation for the diagonal element (corresponding to the final state of interest to us) has in the interaction representation the form[2)]

$$\frac{d\rho}{dt}=\frac{2}{\hbar^{2}}\int_{0}^{\infty} gW(t-\tau,t)\,[1-\rho(t-\tau)S(t,t-\tau)]\,d\tau. \tag{7}$$

Here

$$S(t,t')=1+\frac{g'W'(t,t')}{gW(t',t)}$$

g and g′ are the statistical weights of the final and initial states, and gW and g′W′ are the frequencies of the analyzed and inverse transitions. In the cases discussed below we have $S=1$ when $g\gg g'$ and $S=2$ when $g=g'=1$.

We shall estimate the solution by assuming that the terms of the quasimolecules are superpositions of the terms of the colliding atoms. Then Eqs. (7) contain two types of characteristic times: the times ($\sim\tau\equiv a/u$) of variation of the matrix elements of the interaction potential, and the times ($\sim\pi/\omega_{mn}$) characterizing the energy gaps between the states of the quasimolecule. From the formulas

$$\begin{aligned}
[1-\rho(t')]gW(t',t)&=\sum_{n,m,m'} V_{m'n}[R(t)]V_{nm}[R(t')]\rho_{mm'}(t')\\
&\quad\times\exp\{i\omega_{mn}[R(t)](t-t')\},\\
\rho(t')g'W'(t,t')&=\sum_{m,n,n'} V_{nm}[R(t)]V_{mn'}[R(t')]\rho_{n'n}(t)\\
&\quad\times\exp\{i\omega_{mn}[R(t)](t-t')\},
\end{aligned} \tag{8}$$

in which the indices m and m′ pertain to the initial states of the quasimolecule and n and n′ to the final ones, it follows that the main contribution to the integral in the right side of (7) is made by energy intervals for which $\delta_{mn}\equiv\pi a\omega_{mn}/u\lesssim 1$.

In the case of resonant energy transfer from a nondegenerate level of the atom X to a nondegenerate level of the atom Y, we obtain the equation

$$\frac{d\rho_{I}}{dt}=\frac{2}{\hbar^{2}}V(t)\int_{-\infty}^{t} V(t')[1-\rho_{I}(t')]\,dt'.$$

[2)] For details see [5].

Its solution determined by the condition $\rho_{\mathrm{r}}(-\infty) = 0$

$$\rho_{\mathrm{r}}(t) = \left\{ \sin\left[\frac{1}{\hbar} \int_{-\infty}^{t} V(t')\,dt' \right] \right\}^2 \qquad (9)$$

coincides with the known solution (see, for example, [2]). In the other particular case, when the atom Y is ionized by an excited atom X (Auger effect of the quasimolecule), we write, assuming the initial state to be isolated,

$$gW(t', t) = \hbar [V_{1\nu}(t') V_{\nu 1}(t)]_{\nu=\nu_0} \int_{-\omega_0}^{\infty} \exp[i\omega(t - t')]\,d\omega,$$

$$\omega_0 \equiv \frac{1}{\hbar}(E_{\gamma}^{(X)} - E_1^{(X)} - J_Y),$$

where $E_{\gamma}^{(X)}$ and $E_1^{(Y)}$ are the initial and final energies of the atom X, J_Y is the ionization energy of the atom Y. If $\delta_0 \equiv \pi a \omega_0/u \gg 1$, we can replace here $(-\omega_0)$ by $(-\infty)$. Recognizing that $g \gg 1$, we then have

$$\frac{d\rho_{\mathrm{i}}}{dt} = \frac{2\pi}{\hbar} \left\{ |V_{1\nu}[R(t)]|^2 \frac{d\nu}{dE} \right\}_{\nu=\nu_0} [1 - \rho_{\mathrm{i}}(t)], \qquad (10)$$

from which we get for $\rho_{\mathrm{i}}(-\infty) = 0$

$$\rho_{\mathrm{i}}(t) = 1 - \exp\left\{ -\frac{2\pi}{\hbar} \int_{-\infty}^{t} \left[|V_{1\nu}(t')|^2 \frac{d\nu}{dE} \right]_{\nu=\nu_0} dt' \right\}. \qquad (11)$$

In the problem of ionization by a slow atom in the dipole-dipole approximation one usually starts out from formulas analogous to (10) and (11) (see, for example, [3]).

An estimate of the probability $w(a, u) = \rho(\infty)$ is simplified if the main contribution to the cross section is made by long-range paths and we can write

$$\hat{V} = [R(t)]^{-(\varkappa+\varkappa'+1)} \hat{Q}_X^{\varkappa} \hat{Q}_Y^{\varkappa'}, \qquad (12)$$

where $\hat{Q}_X^{\kappa}$ and $\hat{Q}_Y^{\kappa'}$ are the operators of the corresponding multipole moments of the atoms X and Y. Neglecting the bending of the nuclear trajectories, we obtain[3)] for the cross section of the reaction (9)

$$\sigma_{\mathrm{r}} = \pi (Q_X Q_Y)^{1/(\varkappa+\varkappa')} u^{-2/(\varkappa+\varkappa')} I_{\mathrm{r}}(\varkappa + \varkappa' + 1), \qquad (13)$$

and for the cross section of the reaction (11)

$$\sigma_{\mathrm{i}} = \pi (Q_X{}^{\varkappa} Q_Y{}^{\varkappa'} g_e u^{-1})^{1/(\varkappa+\varkappa'+1/2)} I_{\mathrm{i}}(\varkappa + \varkappa' + 1). \qquad (14)$$

Here Q_X^{κ} and $Q_Y^{\kappa'}$ are the matrix elements of the corresponding multipole moments and are averaged in accordance with (8), g_e is the statistical weight of the emitted electron,

$$I_{\mathrm{i}}(n) = (2\pi \alpha_{2n})^{1/(n-1/2)} \Gamma\left(\frac{2n-3}{2n-1}\right); \qquad \alpha_n = \sqrt{\pi}\,\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right),$$

$$I_{\mathrm{r}}(n) = \begin{cases} \pi/2 & \text{for } n = 3, \\ \left(\frac{1}{2}\right)^{2(n-2)/(n-1)} \alpha^{2/(n-1)} \Gamma\left(\frac{n-3}{n-1}\right) \sin\left(\frac{\pi}{2}\frac{n-3}{n-1}\right) & \text{for } n > 3. \end{cases}$$

σ depends differently on u in (9) and (11). The ionization cross section is connected with the width of the resonant region $\Delta\omega$ by the relation $\sigma_{\mathrm{i}} \sim (\Delta\omega) \times \exp[1/(\kappa + \kappa')]\, u \exp[-2/(\kappa + \kappa')]$. Since $\Delta\omega \approx u/\sqrt{\sigma_{\mathrm{i}}}$, it follows that $\sigma_{\mathrm{i}} \sim u \exp[-1/(\kappa + \kappa' + 1/2)]$. On the other hand, in the case of resonant excitation transfer, the oscillator strengths are concentrated at the nondegenerate levels, so that the dependence on the velocity is stronger: $\sigma_{\mathrm{r}} \sim u \exp[-2/(\kappa + \kappa')]$. We see from the foregoing that besides depending on u, σ also depends strongly on the total oscillator strength in the resonance band.

The transfer of excitation from X* to the precontinuum region of the bound states of the atom Y is the limiting case of (9) and (11). In this region, the terms of the atoms are hydrogen-like. When $\delta_{n,n+1} \equiv \pi a/un^3 < 1$ (n is the principal quantum number), we can estimate σ from formula (14). Putting $a = \sqrt{\sigma/\pi}$, $\sigma = 10^{-14}$ cm², we can write for the minimum value $n_{\min} = (u^{-1}\sqrt{\pi\sigma})^{1/3} \approx 20$ and accordingly $E_{\min} \sim (2n_{\min}^2)^{-1} \sim 3.4 \cdot 10^{-2}$ eV. Thus, formula (14) can be interpolated with respect to energy below the ionization potential of the atom Y, to a depth of at least several hundredths of an eV. On going over to smaller n, the distances between the groups of levels increase and the cross sections oscillate about an average value, dropping by several orders of magnitude with increasing energy defect. The averaged value, on the other hand, increases towards smaller n in accordance with the growth of the oscillators. One can therefore speak of a resonance with a group of levels. The cross section for the resonant exitation transfer to a group of levels is accordingly, as a rule, much larger than the cross section for direct ionization from the higher excited state X*.

3. We shall henceforth consider the reactions (6) and (2) jointly. In the case of collision transfer of excitation (1), the populations of the levels γ and γ' of the atom X will be influenced by the entire kinetics of the populations of the atom Y; to analyze the functions $N_{\gamma}^{(X)}(t)$ and $N_{\gamma'}^{(X)}(t)$, it is necessary to solve the equations for for the relaxation of the atoms X and Y simultaneously with allowance for the reaction (1). In the case of reactions (2) and (6), the details of the picture of the relaxation of the atoms Y are not reflected in the populations $N_{\gamma}^{(X)}(t)$ and $N_{\gamma'}^{(X)}(t)$, and the quantities $N_e(t)$, $T_e(t)$ and $N_1^{(Y)}(t)$ have a relatively simple time dependence.

Some conclusions concerning the influence of the reactions in question on the populations of the levels of the atom X can be deduced from qualitative considerations. Let us consider two very simple particular cases of pulsed recombination of a plasma: a) when the final state of the atom $X(\gamma')$ is the ground state ($\gamma' = 1$), b) when $X(\gamma)$ is one of the states of the quasiequilibrium spectrum, $\gamma \in g$.

Let us consider the case a). In this case (Fig. 2) we have

$$X_1(\gamma) + Y_1(1) \rightleftarrows X_1(1) + Y_1^{+} + e. \qquad (6')$$

Under conditions of pulsed recombination, the temperature of the free electrons is much lower than the equilibrium value (for the given concentration), and for a

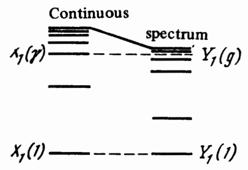

FIG. 2

[3)] In this section we shall henceforth use atomic units m = ħ = e = 1.

number of relatively low levels of the atom X we can neglect the collision transitions from even lower (with respect to energy) states. Let μ be such a level, and let us assume also that the transitions from the states $X(\mu)$ to levels with higher energy have low probability. The stationary-sink equations for the population $\bar{N}_{\beta}^{(X)}$, written without allowance for the reactions (6′), are

$$B_{\beta}^{(x)}N_{+}^{(x)}(N_e)^2+\sum_{\nu>\beta}(V_{\nu\beta}^{(x)}N_e+A_{\nu\beta}^{(x)})\bar{N}_{\nu}^{(x)}=(V_{\beta}^{(x)}N_e+A_{\beta}^{(x)})\bar{N}_{\beta}^{(x)}. \tag{15}$$

The first term here is proportional to the probability of triple recombination (the quasiequilibrium spectrum of the atom X(g) is unified in this case with the continuum) into the state $X(\beta)$, and the sum over ν is proportional to the probabilities of the collision and spontaneous transitions to this state from the higher discrete levels: the right-hand side is proportional to the probability of the departures (collisional and spontaneous) from the state $X(\beta)$ to all others.

Taking the reactions (6′) into account, we have for the series of levels for which the previously-made assumptions are valid

$$B_{\gamma}^{(x)}N_{+}^{(x)}(N_e)^2+\sum_{\nu>\gamma}(V_{\nu\gamma}^{(x)}N_e+A_{\nu\gamma})N_{\nu}^{(x)}$$
$$+q_{1\gamma}N_{+}^{(Y)}N_eN_1^{(x)}=[(V_{\gamma}^{(x)}N_e+A_{\gamma}^{(x)})+q_{\gamma 1}N_1^{(Y)}]N_{\gamma}^{(x)}. \tag{16}$$

Here $q_{\gamma 1}=\langle\sigma_{\gamma 1}u\rangle$, where $\sigma_{\gamma 1}$ are the cross sections of the reactions (6′) and u is the relative velocity of the colliding particles; in addition, in accordance with the detailed balancing principle,

$$\frac{q_{\gamma 1}}{q_{1\gamma}}\equiv\left(\frac{mT_e}{2\pi\hbar^2}\right)^{3/2}\exp\left(\frac{\Delta E_{\gamma}}{T_e}\right), \tag{17}$$

$$\Delta E_{\gamma}\equiv J_X-J_Y-E_{\gamma}\gtrsim 0. \tag{18}$$

We introduce the concept of the effective ionization temperature $T^{(Z)}$ of the element Z and of the equilibrium population $N_{E\gamma}^{(Z)}(T)$ of the level γ, with the aid of the relations

$$\frac{N_1^{(z)}}{N_eN_{+}^{(z)}}\equiv\left(\frac{2\pi\hbar^2}{mT^{(z)}}\right)^{3/2}\exp\left(\frac{J_z}{T^{(z)}}\right), \tag{19}$$

$$N_{E_{\gamma}}^{(z)}(T)\equiv N_1^{(z)}\exp\left(\frac{E_{\gamma}-J_z}{T}\right). \tag{20}$$

Obviously, the effective ionization temperature corresponds to the equilibrium temperature at which the degree of ionization of the element Z is equal to the actual one; $N_{E\gamma}^{Z}(T)$ corresponds to the equilibrium population at a given temperature T and to the concentration of the ground state $N^{(Z)}$.

From (17), (19), and (20) we obtain

$$\frac{q_{1\gamma}N_{+}^{(Y)}N_eN_1^{(X)}}{q_{\gamma 1}N_1^{(Y)}N_{\gamma}^{(X)}}=\left(\frac{T^{(Y)}}{T_e}\right)^{3/2}\exp\left[-\Delta E_{\gamma}\left(\frac{1}{T_e}-\frac{1}{T^{(Y)}}\right)\right]\frac{N_{E_{\gamma}}^{(X)}(T^{(Y)})}{N_{\gamma}^{(X)}}. \tag{21}$$

In the recombination regime $T_e<T^{(Y)}$. In the regime of intense recombination we have $N_{E\gamma}^{(X)}\ll N_{\gamma}^{(X)}$, and therefore the third term in the left side of (16) can be neglected.

The reactions (6′) exert a strong influence on those levels for which the coefficients proportional to $N_1^{(Y)}$ are sufficiently large, namely

$$\alpha_{\gamma}=\frac{q_{\gamma 1}N_1^{(Y)}}{V_{\gamma}^{(X)}N_e+A_{\gamma}^{(X)}}. \tag{22}$$

At the same time, the value of α_{γ} for higher levels is, as a rule, smaller than for the lower ones. This is mainly connected with the rapid increase of $V_{\gamma}^{(X)}$ on going over to the upper levels. Thus, with increasing $N_1^{(Y)}$ the populations $N_{\gamma}^{(X)}$ decrease not in a uniform manner, but, as it were, in an upward sequence. If at a given $N_1^{(Y)}$ the process (6′) is already important for the level γ, but exerts no noticeable influence on the population of the higher levels, then we obtain, using (15),

$$N_{\gamma}=\bar{N}_{\gamma}/(1+\alpha_{\gamma}). \tag{23}$$

The foregoing can also be useful for the measurement of the probabilities of nonradiative transitions.

Frequently, inversion may occur at sufficiently large concentrations $N_1^{(Y)}$. Let us consider the ratio of the populations of the levels γ_2 (upper) and γ_1 (lower). We assume for simplicity that there are no other levels between them. We denote by j_{γ_2} the rate of arrival of electrons at the level γ_2 from all the levels lying above γ_2, and by j_{γ_1} the rate of arrival of electrons from the same levels at the level γ_1. Then, using (16), we have

$$\frac{N_{\gamma_2}}{N_{\gamma_1}}=\frac{1+(\alpha_{\gamma_1})^{-1}}{1+(\alpha_{\gamma_2})^{-1}}\frac{q_{\gamma_1 1}}{q_{\gamma_2 1}}j_{\gamma_2}\left(j_{\gamma_1}+\frac{j_{\gamma_2}}{\alpha_{\gamma_2}}\frac{V_{\gamma_2\gamma_1}N_e+A_{\gamma_2\gamma_1}}{V_{\gamma_2}N_e+A_{\gamma_2}}\right)^{-1}. \tag{24}$$

At sufficiently large α_{γ_1} and α_{γ_2} we have

$$\frac{N_{\gamma_2}}{N_{\gamma_1}}\approx\frac{q_{\gamma_1 1}}{q_{\gamma_2 1}}\frac{j_{\gamma_2}}{j_{\gamma_1}}. \tag{25}$$

Frequently $j_{\gamma_2}\gg j_{\gamma_1}$, and the right-hand side of (25) is then larger than unity.

We present two other simplified variants of the formula (24). The first pertains to the case when the so-called single-quantum approximation[6] is valid, wherein the transitions occur only between the close levels:

$$\frac{N_{\gamma_2}}{N_{\gamma_1}}=(1+\alpha_{\gamma_1})\frac{V_{\gamma_1}N_e+A_{\gamma_1}}{V_{\gamma_2\gamma_1}N_e+A_{\gamma_2\gamma_1}}, \tag{26}$$

When $\alpha_{\gamma_1}\gg 1$ we have

$$\frac{N_{\gamma_2}}{N_{\gamma_1}}=\frac{q_{\gamma_1 1}N_1^{(Y)}}{V_{\gamma_2\gamma_1}N_e+A_{\gamma_2\gamma_1}}. \tag{26′}$$

In the other case, when the reaction (6′) can be neglected for the levels above γ_1, we have

$$\frac{N_{\gamma_2}}{N_{\gamma_1}}=\frac{\bar{N}_{\gamma_2}}{\bar{N}_{\gamma_1}}(1+\alpha_{\gamma_1}). \tag{27}$$

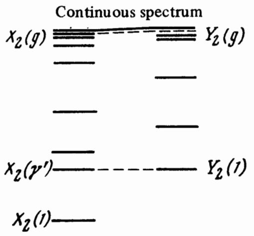

FIG. 3

It follows from (26), in particular, that inversion (in the single-quantum approximation) can be attained even when $\alpha_{\gamma_1} < \alpha_{\gamma_2}$.

So far we have dealt with atoms, but nothing has prevented us from regarding the particles X or Y as ions. However, in the case when the particles Y are ions, the effectiveness of the reactions (6′) is strongly limited: $\alpha_\gamma < q_{\gamma_1}/V_\gamma^{(X)}$; then the limiting value of α_γ ceases to be dependent on $N_1^{(Y)}$.

Let us consider the case b). In this case (Fig. 3) the reaction discussed here has the form

$$X_2(g) + Y_2(1) \rightleftharpoons X_2(\gamma') + Y_2^+ + e. \qquad (6'')$$

For simplicity let us consider only the case of ionization resonance, and assume that the reactions analyzed in case (a) can be neglected. Under the same assumptions as were made in writing down formula (16), we have for the resonant level γ':

$$(V_{\gamma'}^{(x)} + N_e + A_{\gamma'}^{(x)})\bar{N}_{\gamma'}^{(x)} + q_{g\gamma'}N_g^{(x)}N_1^{(x)}$$

$$= [(V_{\gamma'}^{(x)}N_e + A_{\gamma'}^{(x)}) + q_{\gamma' g}N_eN_1^{(Y)}]N_{\gamma'}^{(x)}. \qquad (28)$$

Here, as before, $\bar{N}_{\gamma'}^{(X)}$ is the population of the level γ' in the course of the stationary-sink obtained without allowance for the reaction (6″), and $N_{\gamma'}^{(X)}$ is the same population with (6″) taken into account. From (28) it follows that

$$N_{\gamma'}^{(x)} - \bar{N}_{\gamma'}^{(x)} = \alpha_{\gamma'}'\left[N_1^{(x)}\exp\left(-\frac{J_X}{T^{(X)}}\right) - N_{\gamma'}^{(x)}\exp\left(-\frac{J_Y}{T^{(Y)}}\right)\right], \qquad (29)$$

$$\alpha_{\gamma'}' \equiv q_{g\gamma'}N_1^{(Y)}/(V_{\gamma'}^{(x)}N_e + A_{\gamma'}).$$

It is natural to assume that $N_1^{(X)} \gg N_{\gamma'}^{(X)}$, and that $J_X/T^{(X)}$ and $J_Y/T^{(Y)}$ differ little from each other; then

$$N_{\gamma'}^{(x)} = \bar{N}_{\gamma'}^{(x)} + \alpha_{\gamma'}'\bar{N}_{\gamma'}^{(x)},$$

i.e., in this case the ionization resonance leads to a certain increase of the population of the resonating level.

By way of illustration we present the results of the calculation in [7], where the element X was hydrogen and Y was mercury or xenon. Such a choice was due primarily to the fact that the kinetics of the populations of the discrete levels of recombining atomic hydrogen has been analyzed by now most fully (see, for example, [1]). This analysis can be regarded as sufficiently reliable: the probabilities of the significant elementary acts are known here with good accuracy, and many consequences have already been quantitatively confirmed in experiments. The estimates have shown that in a dense plasma ($N_e \gtrsim 10^{14}$ cm^{-3}) at an effective ionization temperature of each of the mixtures (hydrogen-xenon or hydrogen-mercury) exceeding 0.7 eV, and at an electron temperature $T_e \lesssim 2.2$ eV, an appreciable inversion is realized, making it possible to amplify effectively radiation such as produced by the atomic-hydrogen lines (of the Paschen and Balmer series, respectively). It is shown that the rather large theoretical values of the unsaturated gain obtained in this case ($\kappa \sim 1$ cm^{-1} and $\kappa \sim 100$ cm^{-1} in the hydrogen-xenon and hydrogen-mercury pulsed-recombining plasmas, respectively) are limited not by the rate of depletion of the lower working level, but by the technical possibilities of obtaining the required parameters N_e and T_e. Similar estimates show that amplification can be obtained on a number of lines of the Balmer series by using, say, sulfur, phosphorus, iodine, or lithium in place of mercury.

In conclusion we emphasize once more that the questions considered here are of interest not only from the point of view of developing procedures for the measurement of probabilities of nonradiative transitions, and of developing plasma lasers or selective absorbers. The reactions in question must be taken into account in the most general problems on plasma relaxation.

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