HELICON-ACOUSTIC NUCLEAR RESONANCE IN CONDUCTORS

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It is shown that a system of nuclear spins in a conductor, excited by a coherent field of helicons of resonance frequency, may generate sound.

1. The properties of helicons in conductors have been considered in detail in a number of review articles.[1-3] It was proposed to utilize the ability of helicons to penetrate deep into the volume of a conductor without any significant attenuation in order to observe nuclear magnetic resonance (NMR) in conductors.[4,5] The advantage of observing NMR with helicons, in contrast to ordinary NMR in conductors, is the possibility of investigating nuclei and their interactions inside the volume of the sample, and not just in the skin layer where surface effects may substantially influence the experimental results.

The conditions under which the observation of NMR with helicons is possible were investigated in the article by Maxfield.[3] It is shown that in typical metals having free-electron concentrations $n \approx 10^{22}$ to 10^{23} cm^{-3} the propagation of helicons with the NMR frequency (we shall call such helicons resonant helicons (RH)) is possible only in strong magnetic fields ($H > 45$ to 65 kG) since for smaller values of the magnetic field the RH is rapidly damped as a consequence of the Doppler shifted cyclotron resonance.[3] For conductors having a smaller concentration of free charge carriers, the condition for the weakness of the damping of the RH will be satisfied for smaller values of the magnetic field intensity. For example, for InSb with a free-electron concentration $n = 10^{17}$ cm^{-3}, in order to observe NMR in In^{115} the magnitude of the magnetic field must satisfy the condition $H > 2$ G.

A helicon wave propagating through a sample is always coupled to an acoustic wave of the same frequency as a consequence of the interaction of the conduction electrons with the acoustic lattice vibrations. It was proposed to use this direct mechanism of interaction for either the generation of sound by helicons[6,7] or else for the excitation of helicons by acoustic waves.[8] Kaner and Skobov[8] showed that helicons interact only with transverse sound waves having the same direction of propagation as the helicons. The interaction is resonant when the phase velocities of the electromagnetic and sound waves coincide.

A system of nuclear spins, excited by a coherent electromagnetic field, emits coherent free (in connection with pulsed excitation) or stimulated (in connection with steady-state excitation) induction electromagnetic signals.[9] If a sound field of the resonance frequency is used as the excitation, then phonon induction signals are emitted.[10,11] In articles [10-12] the possibility was indicated that a system of spins excited by a coherent field can emit coherent response signals whose physical nature is different from the nature of the exciting field (for example, excitation by a sound field and emission of an electromagnetic response, or vice versa).

Let us consider the possibility of generating phonon induction signals by a system of nuclear spins which is excited by a coherent RH field.

2. For the observation of NMR with helicons the RH frequency ω_{RH} is determined by the well-known expression:[9]

$$\omega_{RH} = \gamma H, \tag{1}$$

where γ is the gyromagnetic ratio and H is the intensity of the magnetic field at the nucleus. The dispersion law for a helicon having a wave vector **k** parallel to the direction of the constant magnetic field has the following form:[6,13]

$$k^2 = \omega\omega_p^2 / c^2\omega_c, \tag{2}$$

where $\omega_p^2 = 4\pi ne^2/m^*$ is the plasma frequency, n denotes the concentration of free charge carriers, e is the charge of the carriers, m* is the effective mass of the carriers, and $\omega_c = eH/m^*c$ is the cyclotron frequency. For a RH the expression for the wave vector has the following form:

$$k_{RH}^2 = 4\pi\gamma ne / c. \tag{3}$$

From Eq. (3) it follows that for a RH in a given substance the wave vector (wavelength) is a constant quantity. Since γ for different substances differs by no more than an order of magnitude, the wavelength of the RH primarily depends on the concentration of free electrons. In the range of concentrations from 10^{23} to 10^{17} cm^{-3} the wavelength of the RH will be within the limits from 10^{-4} to 10^{-1} cm. Therefore, in order to investigate the effects of coherence associated with the resonance[9] (free and stimulated induction, echo) it is necessary to use a formalism developed for a description of large systems (the dimensions of the sample are much larger than the wavelength of the radiation).[14]

Let us use the results of the work on the investigation of phonon[10,11] and photon[12-15] induction and echo for large systems. In these articles it is shown that the coherent induction signals (free and stimulated) of a system of two-level particles, excited by a coherent field of resonant frequency, have an appreciable magnitude only in the case when the wave vector of the emitted radiation is equal to the wave vector characterizing the exciting field (for echo signals, this dependence is somewhat more complicated). In our case the system of nuclear spins is excited by the coherent RH field with the wave vector determined by expression (3). A coherent phonon induction signal will be generated by the system when the wave vectors of the RH and of the resonant sound (RS) are equal:

$$k_{RH} = k_{RS}. \tag{4}$$

In the range of NMR frequencies $\omega < 10^9\ \mathrm{sec}^{-1}$ under consideration (for the values of magnetic fields $H < 10^5$ G being used) the RS wavelengths are considerably larger than the lattice constant; therefore the linear dispersion law is valid for the sound wave:

$$k_{RS} = \omega / v, \tag{5}$$

where v denotes the velocity of the sound-wave mode under consideration. Using expressions (3) and (5) we obtain the result that in order to fulfil condition (4) H must satisfy the equation

$$H = v(4\pi ne/\gamma c)^{1/2}. \tag{6}$$

The paramagnetic resonance conversion of helicons into sound (or vice versa) occurs at such values of the constant magnetic field. For example, for InSb with a free-electron concentration $n = 10^{17}\ \mathrm{cm}^{-3}$ in connection with the observation of NMR in In^{115} longitudinal sound will be generated for H = 476 G, ω = 0.56 MHz (a transition between Zeeman levels with $\Delta m = 2$).

We note that condition (4) is simultaneously also the condition which is necessary for the direct conversion of helicons into sound[6,7] (or vice versa[8]). From an experimental point of view, this circumstance is inconvenient since it hinders isolation of the signal associated with the paramagnetic resonance. In principle this difficulty may be eliminated by using longitudinal sound waves for detection (or for excitation). It was mentioned earlier that the direct conversion of helicons into longitudinal sound waves (or vice versa) is impossible. The longitudinal and transverse sound waves propagate independently in the directions corresponding to the pure sound modes of the crystal. Therefore, in connection with the propagation of RH in the direction corresponding to a pure sound mode of the crystal, one can with a great deal of confidence ascribe the appearance of longitudinal sound waves to the mechanism suggested by us. Since the velocities of the longitudinal and transverse sound waves are different, the resonance curves for the longitudinal and transverse modes will be shifted by an amount

$$\Delta H = (4\pi ne/\gamma c)^{1/2}\Delta v, \quad \Delta\omega = \gamma\Delta H, \tag{7}$$

where $\Delta v = v_l - v_t$ (v_l denotes the velocity of longitudinal sound, v_t is the velocity of transverse sound). This effect may also be utilized in order to isolate a useful signal.

In impurity semiconductors it is possible to change the concentration of the free charge carriers by using photo-ionization of the impurity atoms. In this case one can achieve fulfilment of condition (6) by varying the intensity of the illumination.

3. In the calculation of the intensities of the phonon and photon induction and echo signals in articles [10-12,15], the damping of the stimulating and emitted waves associated with propagation in the sample was neglected. In our case the resonant helicon wave will always be attenuated, owing to direct conversion into sound. Let us consider a sample in the form of a parallelepiped of length L and cross section S, placed in a constant magnetic field H (0, 0, H). The direction of the field, which is taken as the z axis, is parallel to the larger edge (L) of the parallelepiped. The nuclear spin $I = 1/2$.

Let a pulse of resonant helicon wave of duration $\Delta t \ll T_1, T_2, T_2^*$ (where T_1, T_2, and T_2^* are, respectively, the longitudinal, irreversible transverse, and reversible transverse relaxation times of the nuclear spins) with wave vector **k** (0, 0, k) be excited at the face z = 0 of the sample at the moment of time t = 0. Let us further assume that condition (6) is satisfied for the longitudinal sound mode in the z direction. By virtue of the difference in the velocities of the longitudinal and transverse sound modes, we assume that the resonance condition for the transverse mode is not fulfilled. In this case the transverse sound wave, which is excited due to the direct mechanism for the interaction of helicons with sound,[6,7] will be weak and we shall neglect its effect on the nuclear spins. The RH propagating in the sample is damped according to the law

$$H_{RH}(z) = H_{RH}(0)\exp\{-\alpha z\}, \tag{8}$$

where α is the attenuation coefficient due to all possible mechanisms of helicon damping, and $H_{RH}(z)$ is the amplitude of the magnetic field of the RH in the z plane.

We measure the intensity of the induction signal (IS) after its emergence from the sample. Under the assumption that the (IS) corresponds to the sum of the contributions from each nucleus, it is necessary to take into consideration that the wave radiated by a nucleus is again attenuated during its passage through the sample. One can write the intensity of an induction signal of type ξ in mode s with wave vector **k**, per unit solid angle in the direction **k**, in the following way (see [12], formula (3)):

$$\begin{aligned} w_\xi^{IS}(\mathbf{k}_s) = w_{\xi 0}(\mathbf{k}_s) \sum_{j\neq l}^{N} (\mathrm{Sp}\,\rho^j(\vartheta^j) R^j_{\mathbf{k}_{s+}} \exp\{-\beta_\xi(L - z^j)\}) \\ \times (\mathrm{Sp}\,\rho^l(\vartheta^l) R^l_{\mathbf{k}_{s-}} \exp\{-\beta_\xi(L - z^l)\}), \end{aligned} \tag{9}$$

where $w_{\xi 0}(\mathbf{k}_S)$ is the intensity of the spontaneous radiation of type ξ with wave vector $\mathbf{k}_S$ emitted from an isolated nucleus per unit solid angle in the direction **k**, $\rho^j(\vartheta^j)$ is the density matrix of the j-th excited nucleus, $\vartheta^j = \vartheta(z^j) = \vartheta(0)\exp\{-\alpha z^j\}$ determines the degree of phase ordering in the motion of the spins,

$R^j_{\mathbf{k}_S} = R^j_\pm \exp\{\pm i\mathbf{k}_S \cdot \mathbf{r}^j\}$, $R^j_\pm = R^j_x \pm iR^j_y$, **R** is the effective spin operator, β_ξ is the attenuation coefficient for a wave of type ξ propagating through the sample, and N is the number of nuclei in the sample.

For a two-level system (spin 1/2) the expression for the intensity of the IS with wave vector **k**, equal to the wave vector of the stimulating field, has the following form:

$$\begin{aligned} w_\xi^{IS}(\mathbf{k}) = w_{\xi 0}(\mathbf{k}) C(T) \sum_{j\neq l}^{N} \sin(\vartheta\exp\{-\alpha z^j\})\exp\{-\beta_\xi(L - z^j)\} \\ \times \sin(\vartheta\exp\{-\alpha z^l\})\exp\{-\beta_\xi(L - z^l)\} = w_{\xi 0}(\mathbf{k}) C(T) f(\vartheta, \alpha, \beta_\xi, L), \end{aligned} \tag{10}$$

where $C(T) = \frac{1}{4}\tanh^2(\hbar\omega/2k_B T)$, k_B is the Boltzmann constant and T is the temperature in degrees Kelvin. In order to estimate the function f (ϑ, α, β_ξ, L) we change from the summation over nuclei to an integration over the length of the sample:

$$f(\vartheta, \alpha, \beta_\xi, L) = \left[\frac{N}{L}\int_0^L \sin(\vartheta\exp\{-\alpha z\})\exp\{-\beta_\xi(L-z)\}dz\right]^2$$

$$-\frac{N}{L}\int_0^L \sin^2(\vartheta\exp\{-\alpha z\})\exp\{-2\beta_\xi(L-z)\}dz. \qquad (11)$$

If the IS is of the same physical nature as the stimulating pulse (in our case a helicon), then $\beta_\xi = \alpha$. In this, the simplest case, $f(\vartheta, \alpha, \alpha, L)$ has the following form:

$$f(\vartheta, \alpha, \alpha, L) = \frac{N^2\vartheta'^2}{L^2\alpha^2}\left[-\frac{\sin\vartheta}{\vartheta} + \frac{\sin\vartheta'}{\vartheta'} + \operatorname{ci}\vartheta - \operatorname{ci}\vartheta'\right]^2$$

$$-\frac{N\vartheta'^2}{L\alpha}\left[-\frac{\sin\vartheta(\sin\vartheta + 2\vartheta\cos\vartheta)}{2\vartheta^2} + \frac{\sin\vartheta'(\sin\vartheta' + 2\vartheta'\cos\vartheta')}{2\vartheta'^2}\right.$$

$$\left. + \operatorname{ci}2\vartheta - \operatorname{ci}2\vartheta'\right], \qquad (12)$$

where $\vartheta' = \vartheta\exp\{-\alpha L\}$.

Estimates of the power of the phonon induction signals coming from a system of nuclear spins, excited by sound or electromagnetic pulses, are given in articles.[10,11] Since the probability for the spontaneous emission of a phonon by a nuclear spin does not depend on the way in which excitation of the spin occurred, these estimates are also valid in our case. Upon fulfilment of condition (6) for helicon-acoustic nuclear resonance, the sound power generated in the direction **k** is proportional to

$$W_{ac}(\mathbf{k}) \sim \frac{\omega^2 G^2 N^2}{Sv^3 m_0}\operatorname{th}^2\left(\frac{\hbar\omega}{2k_B T}\right)\varphi(\vartheta, \alpha, \beta_{ac}, L), \qquad (13)$$

where G is the coupling constant for the spin-phonon interaction, S is the area of a transverse cross-section of the sample, m_0 is the density, and $\varphi(\vartheta, \alpha, \beta_{ac}, L) = N^{-2}f(\vartheta, \alpha, \beta_{ac}, L)$.

Let us estimate the intensity of the longitudinal sound wave for observation of helicon-acoustic NMR in InSb with $n = 10^{17}\ \text{cm}^{-3}$ on In^{115} nuclei ($H = 476$ G, $\omega = 0.56$ MHz). Having taken $N = 10^{23}$, $L = 1$ cm, $S = 1\ \text{cm}^2$, $G = 2\pi \times 10^{-18}$ erg, $T = 4.2°$K, $\vartheta = 90°$, and $\alpha \approx \beta_{ac} \approx 0.1$, we obtain $W \approx 3.7 \times 10^{-3}$ erg/sec. This corresponds to a deformation $\epsilon \approx 2.37 \times 10^{-7}$ and may be detected by present-day sound receivers.

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