STATISTICAL CHARACTERISTICS OF A MULTIPHOTON PHOTOCURRENT

P. P. BARASHEV

Moscow Physico-technical Institute

Submitted April 16, 1970

Zh. Eksp. Teor. Fiz. 59, 1318–1326 (October, 1970)

The main statistical multiphoton-photocurrent characteristics governed by the non-quadratic character of the photodetector are analyzed within the framework of the phenomenological theory of photoelectric registration of radiation. It is shown that the use of multiquantum photodetectors makes it possible to increase the order of the experimentally investigated correlation functions of the radiation field by at least k times (k is the order of the photoeffect), and by the same token to trace the difference between higher-order correlation functions of the investigated radiation field and the fully-coherent radiation of an ideal single-mode laser or incoherent ("random") thermal radiation. It is shown also that the dispersion of the number of photoelectrons can always be represented in the form of a sum of two terms, the first of which takes into account the particle (photo-electron) fluctuations, and the second the fluctuations of the classical wave fields. Relations are obtained between the initial and factorial moments of the distribution of the classical integral intensity of the radiation and the distribution of the recorded number of the photoelectrons produced in the course of a multiphoton photoeffect of arbitrary order.

IT is known that the correlation properties of optical radiation fields are analyzed, in the main, by using data on the probability of the distribution of the photoelectric counts (see, e.g.,[1,2]). The heretofore performed theoretical and experimental investigations concerning the statistics of the radiation field and the statistics of the photocurrent dealt, however only with the case of the single-photon photoeffect, when the probability of appearance of the photoelectron is proportional to the square of the instantaneous value of the radiation-field amplitude (the case of the so-called quadratic detection). Recent theoretical and experimental investigations (see the review[3]) have shown that the elementary act of external multiphoton photoeffect can, in principle, serve as a detector of the order and of the degree of coherence of the radiation, since the amplitude values of the photocurrent depend on the correlation properties of the electromagnetic field producing this current[1)].

Much more important information concerning the coherent properties of the radiation field is contained, nonetheless, not in the amplitude values of the photocurrent, but in its statistical characteristics, theoretical and experimental investigations of which were initiated recently. In particular, the calculations by Teich and Diament[4] of the statistical characteristics of the two-photon photocurrent have shown that the distribution functions of the two-photon photocurrent not only differ greatly from the distribution functions of the single-photon photocurrent, generated by the same radiation with a single-photon photodetector, but also contain information on the properties of the higher-order correlation functions of the field. In connection with the fact that, generally speaking, it is possible to reconstruct uniquely the distribution of the intensity of the radiation field from the known distribution of the number of photoelectrons, it is of interest to determine how the fluctuation properties of the radiation field and the photocurrent produced by it are connected in multiphoton photodetection of higher order than the second. The present paper is devoted to an estimate of some of the main features of the statistics of the multiphoton photocurrent, features connected with the "non-quadratic character" of the detection process, and unlike[4], where the particular case of the two-photon photoeffect was considered, the present calculations are made for a multiphoton photoeffect of arbitrary order k. The multiphoton photoelectric detection is analyzed within the framework of the phenomenological (semiclassical) theory of radiation registration, when the field is treated classically, and the elementary act of the photoeffect is treated quantum-mechanically (a quantum-mechanical procedure is used to calculate the quantum yield, which we assume to be specified)[2)].

Let $p_k(n, T, t)$ denote the probability of the appearance of n photoelectrons over the surface of multiphoton detector of k-th order in the time interval from t to t + T. Then the probability $p_k(n, T + \Delta T, t)$ is connected with the probability $p_k(n, T, t)$ by the following relations:

$$\begin{aligned} p_k(0, T+\Delta T, t) &= p_k(0, T, t)\,[1-p_k(1, \Delta T, t+T)],\\ p_k(n, T+\Delta T, t) &= p_k(n-1, T, t)\,p_k(1, \Delta T, t+T)\\ &\quad + p_k(n, T, t)\,[1-p_k(1, \Delta T, t+T)]. \end{aligned} \tag{1}$$

[1)]It should be stated that the rapid progress in the procedures for registering weak photocurrents and improvements in photocathodes have brought about a situation, wherein, for example, the two-photon effect is observed at an incident-radiation intensity $I > 10^3$ W/cm^2, corresponding to illumination of a target by focused radiation from a source having a radiation power ≈ 5 mW in the spectral line.

[2)]It is shown in [5] that the results of the semi-classical approach coincide with the results of the purely quantum-mechanical analysis so long as it is possible to neglect the influence of the measuring devices on the radiation field.

For the case of a k-quantum detector characterized by a quantum yield α_k, we can write

$$p_k(1, \Delta T, t+T) = \alpha_k[I(t+T)]^k \Delta T, \tag{2}$$

where I is the intensity of the incident radiation.

It is easy to show that the solution of the differential equation for the distribution function $p_k(n, T, t)$ obtained from relations (1) after taking in them the limiting transition $\Delta T \to 0$, with allowance for expression (2), is of the form

$$p_k(n, T, t) = \frac{1}{n!}\left[\int_t^{t+T} \alpha_k [I(t')]^k dt'\right]^n \exp\left[-\int_t^{t+T} \alpha_k [I(t')]^k dt'\right]. \tag{3}$$

The experimentally-observed probability of counting n photoelectrons in the time interval from t to t + T is determined, as is well known[1], by the mean value of the Poisson distribution (3a), and the averaging must be carried out over the ensemble of the distribution of integral classical intensity of radiation

$$W(T, t) = \int_t^{t+T} I(t')\,dt'.$$

For the stationary and ergodic radiation fields in which we shall henceforth be interested, the probability $p_k(n, T, t)$ does not depend on t, and can therefore be written in the form

$$\langle p_k(n, T, t)\rangle \equiv p_k(n, T) = \int_{(W)} \frac{(\alpha_k W_k)^n}{n!} \exp[-\alpha_k W_k] P(W)\,dW, \tag{4}$$

where

$$W_k \equiv W_k(T) = \int_0^T [I(t')]^k dt', \quad W \equiv W(T) = \int_0^T I(t')\,dt'. \tag{5}$$

In expression (4), which in fact is a generalization of the wellknown Mandel formula[6] to the case of multiphoton photoemission, P(W)dW denotes the probability that during the time interval ΔT the integral intensity of the incident radiation W(T) changes from W to W + dW. The operation of averaging over the ensemble P(W) leads thus to a deviation of the photoelectron distribution $p_k(n, T)$ from the Poisson distribution, and it is precisely this deviation which is a measure of the fluctuation (correlation) properties of the radiation field (the bunching effect etc.).

It is known[7,8] that from the experimentally obtained distribution of the photoelectrons $p_k(n, T)$ we can reconstruct uniquely the distribution function of the classical integral intensity of radiation P(W). Indeed, if we take $p_k(n, T)$ to stand for the expression

$$p_k(n, T) = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0}} \int_\varepsilon^\infty \frac{(\alpha_k W_k)^n}{n!} \exp[-\alpha_k W_k] P(W)\,dW, \tag{6}$$

then, using the properties of the direct and inverse Fourier transforms for the functions F(x) and $\Phi(u)$

$$F(x) = \int_0^\infty e^{ixu}\,\Phi(u) e^{-u}\,du, \quad \Phi(u) = \frac{e^u}{2\pi}\int_{-\infty}^{+\infty} F(x) e^{-ixu}\,dx,$$

we find that

$$P(W) = \frac{e^{\alpha_k W_k}}{2\pi} \sum_{n=0}^\infty p_k(n, T)(i)^n \int_{-\infty}^{+\infty} x^n e^{-i\alpha_k W_k x}\,dx, \tag{7}$$

since

$$F(x) = \sum_{n=0}^\infty (ix)^n p_k(n, T).$$

The integral in (7) can be expressed in terms of a δ function, since

$$\int_{-\infty}^{+\infty} x^n e^{-i\alpha_k W_k x}\,dx = (i)^n \frac{d^n}{d(\alpha_k W_k)^n}(2\pi)\,\delta(\alpha_k W_k),$$

and therefore we have finally for expression (7), which is the inverse of (6), in the case of multiphoton detection

$$P(W) = \exp(\alpha_k W_k) \sum_{n=0}^\infty (-1)^n p_k(n, T)\,\delta^{(n)}(\alpha_k W_k), \tag{8}$$

where $\delta^{(n)}(\alpha_k W_k)$ denotes the n-th derivative of the δ function.

The relations between the initial moments of the distributions $p_k(n, T)$ and P(W) can be obtained after introducing the characteristic functions of these distributions in the form[9]

$$\sum_{f=1}^\infty \langle n^f\rangle \frac{s^f}{f!} = \sum_{f=1}^\infty \langle(\alpha_k W_k)^f\rangle \frac{(e^s - 1)^f}{f!}. \tag{9}$$

Equating the coefficients of equal powers of the expansion in the parameter $s(s < 1)$, we get

$$\langle n^m\rangle = a_1^{(m)}\langle\alpha_k W_k\rangle + a_2^{(m)}\langle(\alpha_k W_k)^2\rangle + \ldots + a_m^{(m)}\langle(\alpha_k W_k)^m\rangle, \tag{10}$$

where

$$a_j^{(m)} = \sum_{i_1=1}^{m-1}\sum_{i_2=1}^{i_1-1}\cdots\sum_{i_{j-1}=1}^{i_{j-2}-1}\binom{m-1}{i_1}\binom{i_1-1}{i_2}\cdots\binom{i_{j-2}-1}{i_{j-1}}, \quad j=2,3,\ldots,m;\ a_1^{m}=1 \tag{11}$$

At the same time we can derive from (8) for the initial moments $\langle(\alpha_k W_k)^m\rangle$ of the distribution P(W) the expression

$$\langle(\alpha_k W_k)^m\rangle = \sum_{n=0}^\infty (-1)^n p_k(n, T)\int_{(W)} \exp[\alpha_k W_k][(\alpha_k W_k)^m]\,\delta^{(n)}(\alpha_k W_k)\,dW$$
$$= \sum_{n=0}^\infty p_k(n+m)\frac{(n+m)!}{n!} = \sum_{n=0}^\infty p_k(n, T)\,n(n-1)\ldots(n-m+1) \equiv M_k^{(m)}, \tag{12}$$

from which we see that the initial moment $\langle(\alpha_k W_k)^m\rangle$ is equal to the m-th factorial moment $M_k^{(m)}$ of the distribution $p_k(n, T)$. Then

$$M_k^{(m)} = \langle(\alpha_k W_k)^m\rangle = b_1^{(m)}\langle n\rangle + b_2^{(m)}\langle n^2\rangle + \ldots + b_m^{(m)}\langle n^m\rangle, \tag{13}$$

where

$$b_j^{(m)} = (-1)^{m-j}\sum(x_1 x_2 \ldots x_{m-j}), \quad j = 1, 2, \ldots, m-1; \quad b_m^{(m)} = 1 \tag{14}$$

(the summation in (14) is carried out over the entire set of successively possible combinations of the products $x_1 x_2 \ldots x_{m-j}$, taken only once each).

The possibility of investigating higher correlation functions of radiation fields with the aid of multiphoton photodetectors becomes clear if we rewrite (12), using (5), in the form

$$M_k^{(m)} = \alpha_k^m \int_0^T \ldots \int_0^T \langle[I(t_1)]^k \ldots [I(t_m)]^k\rangle\,dt_1 \ldots dt_m, \tag{15}$$

from which it is clear that the functional dependence of the factorial moments $M_k^{(m)}$ is not only determined uniquely by the form of the distribution function of the intensity of the incident radiation P(W), but also remains similar for all cases of detection when the product km = const. Since the integrand in (15) is, as is well known[10], the correlation function $\Gamma^{(km)}$ of order km, it is clear that the use of multiphoton photodetectors makes it possible to increase the order of the experimentally investigated correlation functions[3)]. In particular, the use of a k-quantum photodetector makes it possible to increase the order of the investigated correlation functions by k times compared with the case of the single-photon detections, if one deals with measurement of the amplitude photocurrent values corresponding to the first moment $M_k^{(1)} = \langle n_k \rangle$. On the other hand, a study of the fluctuation properties of the multiphoton photocurrent (e.g., its dispersion), uncovers in turn one more possibility of investigating correlation functions of order 2k. Indeed, we have for the fluctuations of the multiphoton photocurrent (see (10))

$$\langle (\Delta n_k)^2 \rangle = \alpha_k W_k + \alpha_k^2 [\langle W_k^2 \rangle - \langle W_k \rangle^2]. \tag{16}$$

It is seen from (16) that the fluctuation of the photoelectric counts can always be represented in the form of a sum of contributions from the fluctuations of the classical particles (photoelectrons) and the fluctuation of the classical wave fields taken into account by the first and second terms in expression (16), respectively. The first term corresponds to dispersion of the Poisson process, and the second takes into account the deviation from it (in particular, for a δ-function distribution P(W) the quantity in the parentheses of (16) vanishes, and for a Gaussian distribution it is equal to $\langle W_k \rangle^2$). Since $(W_k)^2 \sim I^{2k}$, the dispersion of the number of photoelectrons characterizes the form of the correlation function of order 2k, and, as will be seen from what follows, the fluctuations of the particle number (of the photocurrent), due to radiation with known statistical properties, increase noticeably with increasing order of the photoeffect k.

Let us illustrate the possibility of using multiquantum photodetectors for the study of the statistical properties of the radiation field, for the case when $\Delta T < \tau_c$, where $\tau_c \lesssim 1/\Delta\nu$ is the coherence time of the incident radiation. In this case (see (5))

$$W_k(T) \approx [I(t)]^k T, \quad W(T) \approx IT, \tag{17}$$

and therefore the derivative of the function $G_k(r)$, defined by the relation

$$G_k(r) = \sum_{n=0}^{\infty} p_k(n, T) r^n = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} (1-r)^m M_k^{(m)}, \tag{18}$$

is given by

$$G_k(r) = \int_{(I)} \exp[-\alpha_k T(1-r) I^k] P(I)\, dI, \tag{19}$$

from which, as can be readily seen,

$$p_k(n, T) = \frac{1}{n!} \frac{d^n}{dr^n} G_k(r) \Big|_{r=0}, \tag{20}$$

and

$$M_k^{(m)} = (\alpha_k T)^m \int_{(I)} I^{km} P(I)\, dI = (\alpha_k T)^m \langle I^{km} \rangle. \tag{21}$$

Let us consider first the statistical characteristics of the multiphoton photocurrent corresponding to the model of the radiation field generated by a nonlinear Van der Pol oscillator[11], when

$$P(I) = \frac{2}{\pi I_0} (1 + \operatorname{erf} w)^{-1} \exp\left[-\left(w - \frac{I}{\sqrt{\pi} I_0}\right)^2\right] \tag{22}$$

(in (22) w is a numerical parameter characterizing the operation of the laser in the "sub-threshold" ($w < 0$) and "above-threshold" ($w > 0$) regimes, and I_0 is the intensity of the laser radiation at the generation threshold when $w = 0$). In this case, as shown by calculations, we obtain the following relations for the mean value of the radiation intensity $\langle I^k \rangle$, for the factorial moments $M_k^{(m)}$ ($M_k^{(1)} = \langle n_k \rangle$), for the fluctuations of the particle number $\langle (\Delta n_k)^2 \rangle$ and of the generating functions $G_k(r)$:

$$\langle I^k \rangle = k! \langle I \rangle^k \exp\left[\frac{(k-1) w^2}{2}\right] (1 + \operatorname{erf} w)^{k-1} \left(\frac{\pi}{2}\right)^{(k-1)/2} \frac{D_{-k-1}(-\sqrt{2} w)}{[D_{-2}(-\sqrt{2} w)]^k},$$

$$M_k^{(m)} = \frac{\exp(-w^2/2)}{1 + \operatorname{erf} w} [\alpha_k T I_0^k]^m (km)! \left(\frac{\pi}{2}\right)^{(km-1)/2} D_{-km-1}(-\sqrt{2} w),$$

$$\langle (\Delta n_k)^2 \rangle = \langle n_k \rangle$$

$$+ \langle n_k \rangle^2 \left[\frac{(2k)!}{(k!)^2} \left(\frac{\pi}{2}\right)^{1/2} \exp\left(\frac{w^2}{2}\right) [1 + \operatorname{erf} w] \frac{D_{-2k-1}(-\sqrt{2} w)}{[D_{-k-1}(-\sqrt{2} w)]^2} - 1\right],$$

$$G_k(r) = \frac{\exp(-w^2/2)}{1 + \operatorname{erf} w}$$

$$\times \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} (ks)! (\alpha_k T I_0^k)^s (1-r)^s \left(\frac{\pi}{2}\right)^{(ks-1)/2} D_{-ks-1}(-\sqrt{2} w), \tag{23}$$

where $D_s(z)$ is the parabolic-cylinder function, and erf w is the probability integral. The excess of the fluctuations compared with the Poisson function denotes, as already mentioned, the deviation of the initial distribution of intensity P(I) from the distribution of an ideal single-mode laser ($P(I) = \delta(I - \langle I \rangle)$, $\langle I^k \rangle = \langle I \rangle^k$, $M_k^{(m)} = [\alpha_k T \langle I \rangle^k]^m$; $\langle (\Delta n_k)^2 \rangle = \langle n_k \rangle$, $G_k(r) = \exp[-\alpha_k T(1-r) \langle I \rangle^k]$).

It can be shown that the generalization of the intensity distribution P(I), introduced in[12] for the case of two-mode radiation against the background of thermal noise, leads in the case of radiation containing N unphased modes to the expression

$$P(I) = \frac{N}{\langle I_n \rangle} \left(\frac{I}{\langle I_c \rangle}\right)^{(N-1)/2} \exp\left[-\frac{I + \langle I_c \rangle}{\langle I_n \rangle} N\right] J_{N-1}\left(2N \frac{\sqrt{\langle I_c \rangle I}}{\langle I_n \rangle}\right), \tag{24}$$

where J_{N-1} is the modified Bessel function of order $N - 1$, and $\langle I_n \rangle$ and $\langle I_c \rangle$ are the mean intensities of the thermal and coherent radiations of the individual mode, respectively. The statistical characteristics of a k-quantum photocurrent are then determined by the expressions

$$\langle I^k \rangle = \frac{\Gamma(k+N)}{\Gamma(N) N^k} \langle I_n \rangle^k \exp\left[-N \frac{\langle I_c \rangle}{\langle I_n \rangle}\right] \Phi\left(k+N, N, N \frac{\langle I_c \rangle}{\langle I_n \rangle}\right),$$

$$M_k^{(m)} = \frac{\Gamma(km+N)}{\Gamma(N)} \exp\left[-N \frac{\langle I_c \rangle}{\langle I_n \rangle}\right] \left(\frac{\alpha_k T \langle I_n \rangle^k}{N^k}\right)^m$$

$$\times \Phi\left(km+N, N, N \frac{\langle I_c \rangle}{\langle I_n \rangle}\right),$$

$$\langle (\Delta n_k)^2 \rangle = \langle n_k \rangle + \langle n_k \rangle^2 \left[\exp\left(N \frac{\langle I_c \rangle}{\langle I_n \rangle}\right) \frac{\Gamma(2k+N)\Gamma(N)}{[\Gamma(k+N)]^2}\right. \tag{25}$$

[3)]This is important in the case when the statistical properties of the investigated radiation differ from thermal radiation, when the correlation function of all the higher orders ($km > 1$) cannot be reduced to the correlation functions of first order, as in the case of thermal radiation.

$$\times\frac{\Phi(2k+N,N,N\langle I_c\rangle/\langle I_n\rangle)}{[\Phi(k+N,N,N\langle I_c\rangle/\langle I_n\rangle)]^2}-1\Big],$$

$$G_k(r)=\frac{\exp(-N\langle I_c\rangle/\langle I_n\rangle)}{\Gamma(N)}\sum_{s=0}^{\infty}\frac{(-1)^s}{s!}(1-r)^s\Gamma(ks+N)(\alpha_kT)^s\cdot$$
$$\times\left[\frac{\langle I_n\rangle}{N}\right]^{ks}\Phi\left(ks+N,N,N\frac{\langle I_c\rangle}{\langle I_n\rangle}\right),$$

where $\Phi(a, b, x)$ is the confluent hypergeometric function and $\Gamma(z)$ is the gamma function.

Expressions (25) with $k = 1$ and $N = 1$ go over into the well known relations obtained for the case of single-photon detection in[2,13], and as $N \to \infty$ they go over into the expressions

$$\langle I^k\rangle=k!\langle I\rangle^k,$$
$$M_k^{(m)}=(km)![\alpha_kT\langle I\rangle^k]^m,$$
$$\langle(\Delta n_k)^2\rangle=\langle n_k\rangle+\langle n_k\rangle^2\left[\frac{(2k)!}{(k!)^2}-1\right],$$
$$G_k(r)=\sum_{s=0}^{\infty}\frac{(-1)^s}{s!}(ks)!(1-r)^s(\alpha_kT\langle I\rangle^k)^s, \qquad (26)$$

which are typical of the case of k-quantum detection of radiation with an exponential distribution of the intensity $P(I) = (1/\langle I\rangle)\exp(-I/\langle I\rangle)$, typical of a thermal source whose radiation is fully polarized.

In the case when the degree of polarization of the thermal radiation is $0 < \gamma < 1$, i.e., when the distribution is given by[1,2]

$$P(I)=\frac{1}{\gamma\langle I\rangle}\left\{\exp\left[-\frac{2I}{(1+\gamma)\langle I\rangle}\right]-\exp\left[-\frac{2I}{(1-\gamma)\langle I\rangle}\right]\right\}, \qquad (27)$$

the statistical characteristics of the photocount of the particles is given by

$$\langle I^k\rangle=k!\langle I\rangle^k\frac{1}{\gamma2^{k+1}}[(1+\gamma)^{k+1}-(1-\gamma)^{k+1}],$$
$$M_k^{(m)}=\frac{(km)!}{2^{km}}[\alpha_kT\langle I\rangle^k]^m\frac{1}{2\gamma}[(1+\gamma)^{km+1}-(1-\gamma)^{km+1}],$$
$$\langle(\Delta n_k)^2\rangle=\langle n_k\rangle+\langle n_k\rangle^2\left[\frac{(2k)!}{(k!)^2}2\gamma\frac{[(1+\gamma)^{2k+1}-(1-\gamma)^{2k+1}]}{[(1+\gamma)^{k+1}-(1-\gamma)^{k+1}]^2}-1\right],$$
$$G_k(r)=\frac{1}{2\gamma}\sum_{s=0}^{\infty}\frac{(-1)^s}{s!}\frac{(ks)!}{2^{ks}}[\alpha_kT\langle I\rangle^k]^s(1-r)^s\{(1+\gamma)^{ks+1}-(1-\gamma)^{ks+1}\} \qquad (28)$$

(when $\gamma = 1$ the expressions in (28) go over into those of (26)).

It is seen from (23), (25), (26), and (28) that, independently of the form of the intensity-distribution function of the incident radiation $P(I)$, the fluctuation of the photocurrent increases with increasing order k of the photoeffect $(\sim(2k)!/(k!)^2)$; this, in principle, facilitates experimental investigations of the correlation functions of higher orders[4)]. Preliminary experimental data obtained by a group of Hungarian physicists[5)], who investigated the fluctuation characteristics of two-, three-, and four-photon photocurrents, show that the fluctuations of the four-photon photocurrent exceed by almost one order of magnitude the fluctuations of the two-photon photocurrent; this is in satisfactory agreement with the foregoing estimates.

Finally, for the case of a Gaussian distribution of the radiation intensity, written in the form[2]

$$P(I)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(I-\langle I\rangle)^2}{2\sigma^2}\right], \qquad (29)$$

where σ^2 is the variance of the intensity fluctuations, we have

$$\langle I^k\rangle=k!\langle I\rangle^k(\sqrt{2\pi})^{k-1}\exp\left[\frac{\langle I\rangle^2}{4\sigma^2}(k-1)\right]\frac{D_{-k-1}(-\langle I\rangle/\sigma)}{[D_{-2}(-\langle I\rangle/\sigma)]^2},$$
$$M_k^{(m)}=\frac{(km)!}{\sqrt{2\pi}}(\alpha_kT)^m\sigma^{km}\exp\left[-\frac{\langle I\rangle^2}{4\sigma^2}\right]D_{-km-1}\left(-\frac{\langle I\rangle}{\sigma}\right),$$
$$\langle(\Delta n_k)^2\rangle=\langle n_k\rangle+\langle n_k\rangle^2\left[\sqrt{2\pi}\frac{(2k)!}{(k!)^2}\exp\left(\frac{\langle I\rangle^2}{4\sigma^2}\right)\frac{D_{-2k-1}(-\langle I\rangle/\sigma)}{[D_{-k-1}(-\langle I\rangle/\sigma)]^2}-1\right],$$
$$G_k(r)= \qquad (30)$$
$$=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\langle I\rangle^2}{4\sigma^2}\right)\sum_{s=0}^{\infty}\frac{(-1)^s}{s!}\sigma^{ks}(ks)![\alpha_kT(1-r)]^sD_{-ks-1}\left(-\frac{\langle I\rangle}{\sigma}\right).$$

Expressions (30) with $k = 2$ coincide formally with the relations obtained in[15], characterizing the change of the statistical properties of the radiation field in second-harmonic generation, when the first harmonic has a distribution (29). Finally, it should be stated that relations (23), (26), and (30) coincide in the particular case $k = 2$ with the expressions obtained in[4], where the statistical characteristics of the two-quantum photocurrent were investigated.

In conclusion the author is grateful to V. P. Kuznetsov for valuable advice and remarks.

[1] E. Wolf and L. Mandel, Reviews of Modern Physics, 37, 231 (1965).

[2] J. A. Armstrong and A. W. Smith, Progress in Optics, 6, ed. E. Wolf, North-Holland Publ. Co., Amsterdam, 1967, p. 211.

[3] A. D. Gladun and P. P. Barashev, Usp. Fiz. Nauk 98, 493 (1969) [Sov. Phys.-Usp.].

[4] M. C. Teich and P. Diament, J. Appl. Phys. 40, 625 (1969).

[5] L. Mandel, E. C. G. Sudarshan, and E. Wolf, Proc. Phys. Soc., 84, 435 (1964).

[6] L. Mandel, Proc. Phys. Soc. 72, 1037 (1955).

[7] E. Wolf and L. Mehta, Phys. Rev. Lett. 13, 705 (1964).

[8] G. J. Troup, Proc. Phys. Soc., 86, 39 (1965).

[9] B. R. Levin, Teoreticheskie osnovy statisticheskoĭ radiotekhniki (Theoretical Principles of Statistical Radio Engineering), Soviet Radio, 1969.

[10] R. Glauber, in: Kvantovaya optika i kvantovaya radiofizika (Quantum Optics and Quantum Radiophysics) (Russian Translation), Mir, 1966, p. 91.

[11] H. Risken, Zs. Phys. 186, 85 (1965).

[12] G. Lachs, J. Appl. Phys. 38, 3439 (1967).

[13] P. J. Magill and R. P. Soni, Phys. Rev. Lett. 16, 911 (1966).

[14] V. A. Kovarksiĭ, Zh. Eksp. Teor. Fiz. 57, 1613 (1969) [Sov. Phys.-JETP 30, 872 (1970)].

[15] V. G. Tunkin and A. S. Chirkin, ibid. 58, 191 (1970) [31, 107 (1970)].

[4)]The same relations lead to the result obtained in [14], namely that the probability of the elementary act of k-photon absorption following illumination by a "random" source is larger by k! times than following illumination by an ideal single-mode laser.

[5)]Z. Horvath, private communication.

Translated by J. G. Adashko
151