SCATTERING OF CONDUCTION ELECTRONS BY SPIN WAVES IN TERBIUM AND DYSPROSIUM SINGLE CRYSTALS

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Measurements were made of the temperature dependence of the electrical resistivity of terbium and dysprosium single crystals along the [0001] (c axis) and [1$\bar{2}$10] (a axis) directions. These measurements were carried out in the temperature range 4.2–370°K. The component of the resistivity due to the scattering of conduction electrons by spin waves was isolated. It was shown that this magnetic component of the resistivity was anisotropic throughout the investigated range of temperatures: in the magnetically ordered state, we found that $\rho_m^c(T) > \rho_m^a(T)$ and in the paramagnetic region, $\rho_m^c < \rho_m^a$. The anisotropy of the scattering by disordered spins, ρ_m^a/ρ_m^c, was 1.2 for dysprosium and 1.1 for terbium. The dependence $\rho_m(T)$ was almost quadratic between 15 and 40°K.

THE present paper describes an investigation of the temperature dependence of the electrical resistivity $\rho(T)$ of Tb and Dy single crystals along the principal directions [0001] (c axis) and [1$\bar{2}$10] (a axis). The purpose of this investigation was to isolate the magnetic component of the resistivity $\rho_m(T)$. Until now, the results of measurements of $\rho(T)$ of rare-earth metals have been analyzed making some a priori assumptions about the nature of $\rho_m(T)$.[1,2] In contrast to this procedure, our method of analysis made it possible to determine the nature of the temperature dependence, as well as its numerical parameters, without making any a priori assumptions about the function $\rho_m(T)$.

When the temperature dependence of the resistivity is separated into its components, it is usual to assume that the different mechanisms by which conduction electrons are scattered make additive contributions to the total resistivity:

$$\rho(T) = \rho_0 + \rho_e(T) + \rho_m(T) + \rho_p(T), \qquad (1)$$

where ρ_0 is the residual resistivity; $\rho_e(T)$ is the resistivity associated with electron-electron collisions; $\rho_m(T)$ and $\rho_p(T)$ are the components of the resistivity due to the scattering of electrons by magnons (spin waves) and phonons, respectively. Generally speaking, we must make allowance for the influence of the internal magnetic field on the motion of electrons in a metal, which affects the electrical resistivity. In the case of nonmagnetic metals, this influence is manifested by the ordinary magnetoresistance and it is a function of the ratio l/r (l is the mean free path of the electrons and r is the radius of curvature of electrons in the magnetic field). However, since samples are usually not very pure (Table I), we can ignore this contribution to $\rho(T)$. In effective fields such that $l/r \ll 1$, we find that $\Delta\rho/\rho \propto (l/r)^2$. For metals with $\rho(300°K)/\rho(4.2°K) \approx 10$ subjected to a magnetic field $H \approx 3 \times 10^4$ Oe, the relevant ratio is $l/r \lesssim 0.1$, i.e., the contribution of the magnetoresistance is less than 1% of the total resistivity $\rho(T)$.

Curves 1 and 2 in Fig. 1 represent the temperature-dependent components of the resistivity $\rho_i(T) = \rho(T) - \rho_0$ for Tb and Dy. In order to determine the dependence $\rho_m(T)$, we must subtract the electron-electron and electron-phonon components of the resistivity from the temperature dependence of the ideal resistivity $\rho_i(T)$. In earlier investigations[1,2], the $\rho_m(T)$ dependence of polycrystalline and single-crystal samples of Dy[3] was usually determined neglecting the value of ρ_e compared with ρ_p. However, this procedure is not generally justified because, at temperatures of the order of 10°K or less, the value of ρ_e is comparable with the other temperature-dependent contributions to the resistivity.

Sample	Orientation	$\rho_0 \cdot 10^6$, Ω·cm	ρ (300° K)·10^6, Ω·cm	ρ (300 °K)/ρ (4,2° K)
Tb	[0001]	6.2	100.2	16.2
	[1$\bar{2}$10]	7.8	136.6	17.5
Dy	[0001]	5.6	76.8	13.7
	[1$\bar{2}$10]	6.4	114.7	17.9

A recent calculation of the Fermi surfaces of heavy rare-earth metals,[4] carried out using the augmented plane wave method and making allowance for the relativistic effects, has demonstrated that the Fermi surfaces of Tb, Dy, Er, and Lu are similar. Moreover, these calculations show that the four metals have similar densities of states of electrons on the Fermi surface $N(E_F)$. This has been confirmed experimentally: the coefficient of the electronic specific heat γ is practically the same for all four metals and equal to 10.5 mJ-deg^{-1} mole^{-1}.[5] Thus, the electronic structure of Tb, Dy, Er, and Lu is such that we may assume that the contributions $\rho_e(T)$ of these rare-earth metals should, generally, be similar.

The phonon component of the electrical resistivity $\rho_p(T)$ can be eliminated by means of the Bloch-Grüneisen function. For this purpose, it is sufficient to know the characteristic temperature Θ_D and the phonon component of the resistivity at this temperature $\rho_p(\Theta_D)$. The validity of the Bloch-Grüneisen description of the temperature dependence of the electrical resistivity of rare-earth metals has been checked

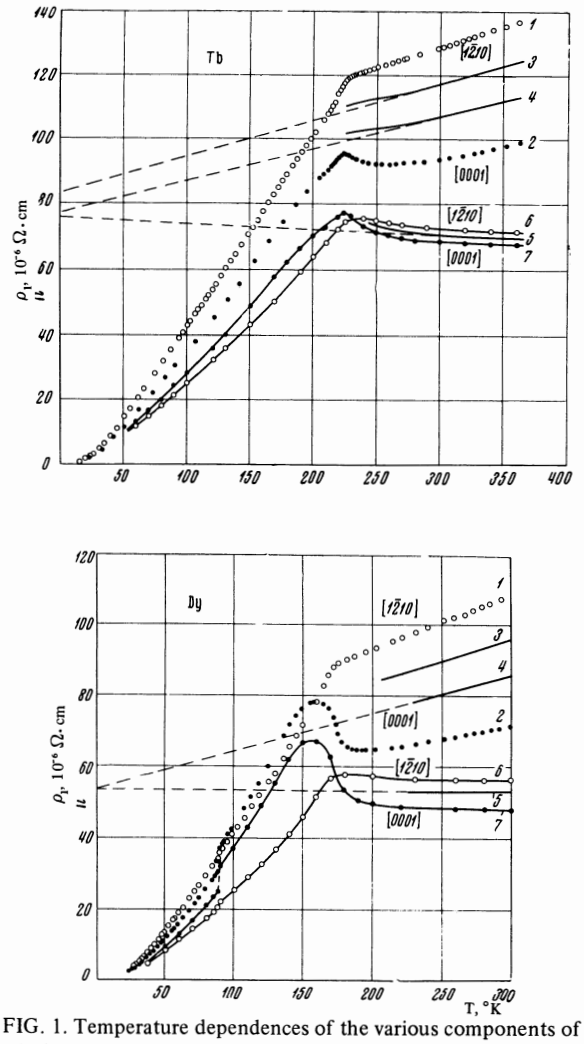

FIG. 1. Temperature dependences of the various components of the electrical resistivity of terbium and dysprosium single crystals: 1), 2) ideal resistivity $\rho_i(T) = \rho(T) - \rho_0$ along the directions $[1\bar{2}10]$ and [0001]; 3) ideal resistivity of polycrystalline samples; 4) phonon and magnetic components of the resistivity of polycrystalline samples ($\rho_p + \rho_m$); 5) magnetic component of the resistivity of polycrystalline samples $\rho_m(T)$; 6) $\rho_m(T)$ along the $[1\bar{2}10]$ direction; 7) $\rho_m(T)$ along the [0001] direction.

for Lu, which is a paramagnetic metal.[1] The characteristic temperature Θ_D of Lu can be found from the resistivity by the standard technique described in[6] but in the case of Tb and Dy this temperature should be deduced from the elastic constants[7] (Θ_D = 175°K for Tb and 180°K for Dy but no value is available for Lu). This is because the value of Θ_D of terbium and dysprosium cannot be found from the specific heat or the electrical resistivity due to the indeterminacy of the magnetic contributions (if these contributions are neglected, the value of Θ_D is underestimated).

The dependences $\rho_e(T)$ and $\rho_p(T)$ for Lu were determined using the electrical resistivity data given in[8]. The average value of Θ_D for Lu, deduced from $\rho(T)$ determined over a wide range of temperatures, is 140°K, which is in agreement with the results reported earlier.[1] At temperatures $T < 0.1\ \Theta_D$, i.e., below 15°K, an analysis of the temperature-dependent resistivity in accordance with the formula

$$\rho_i(T) = \rho_e(T) + \rho_p(T) = aT^2 + bT^5, \qquad (2)$$

gives the low-temperature value of $\rho_p(T)$. We can then find quite easily the other parameters of Lu which are of interest to us: $\rho_p(\Theta_D)$; $\rho_p(T)$ throughout the investigated range of temperatures; and then $\rho_e(T)$. It is found that the room-temperature value of ρ_e is quite large and represents about $0.2\rho_p$.

We shall now describe the method used in the determination of $\rho_m(T)$ of polycrystalline samples of Tb and Dy. Assuming that $\rho_e(T)$ is the same for heavy rare-earth metals, we can write the magnetic component of the electrical resistivity of Tb (and similarly of Dy) in the following form:

$$\rho_m(T)_{Tb} = \rho_i(T)_{Tb} - [\rho_p(T)_{Tb} + \rho_e(T)_{Lu}]. \qquad (3)$$

The values of $\rho_p(\Theta_D)$ for Tb and Dy, which are required in the calculation of the phonon component of the resistivity, can be found from $\rho_p(T)_{Lu}$ but with a correction for the atomic weight M and for the characteristic temperature. This correction is made on the assumption that, at temperatures $T > \Theta_D$, the resistivity is given by $\rho \propto kT/M\Theta_D^2$, where k is a characteristic constant proportional to the radius of the Debye zone. However, these constants have similar values for heavy rare-earth metals and the correction for k has little effect on the final results.

The curves denoted by 3 in Fig. 1 represent the temperature-dependent components of the electrical resistivity of polycrystalline samples of Tb and Dy. Subtracting from $\rho_i(T)$ the value of $\rho_e(T)_{Lu}$, we obtain curves 4, which represent only the scattering of electrons by phonons and magnons. Linear extrapolation of curves 4 to 0°K gives the values of ρ_m in the paramagnetic region: 77 μΩ-cm for Tb and 53 μΩ-cm for Dy. Next, if the values of $\rho_p(T)$ are subtracted from curves 4, we find that $\rho_m(T)$ is independent of temperature in the paramagnetic region (curves 5 in Fig. 1) and is 75 μΩ-cm for Tb and 54 μΩ-cm for Dy. These values are in agreement with the magnetic components of the resistivity found by extrapolation of the curves denoted by 4. This agreement confirms the correctness of the procedure used to find the magnetic component of the electrical resistivity of Tb and Dy.

To be able to find $\rho_m(T)$ for various directions in single crystals of Tb and Dy, we have to know the value of $(\rho_e + \rho_p)$ for each of these directions at every temperature. In the case of Lu, we know these quantities as well as the coefficients A_1 and A_2, which relate $(\rho_e + \rho_p)$ of a polycrystalline sample to the resistivities of a single crystal along the c and a axes:

$$(\rho_e + \rho_p)_c = A_1(\rho_e + \rho_p)_{\text{polycryst}},$$
$$(\rho_e + \rho_p)_a = A_2(\rho_e + \rho_p)_{\text{polycryst}}.$$

The coefficients A_1 and A_2 for Lu are practically independent of temperature. Using the similarity of the Fermi surfaces of heavy rare-earth metals, we may assume that in the case of polycrystalline samples of Tb and Dy the sum $(\rho_e + \rho_p)$ is related by the coefficients A_1 and A_2 to the combined phonon and electron components of the resistivity along the appropriate axes in single crystals of these metals. Thus, we can find the required magnetic component of the resistivity by determining the phonon and electron com-

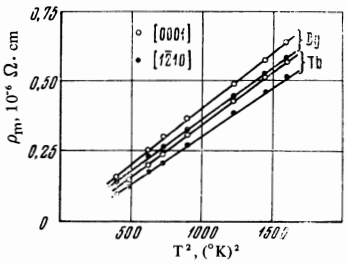

FIG. 2. Dependence of the magnetic component of the resistivity on the square of the temperature.

ponents along the principal directions in single crystals of Tb and Dy and then subtracting $\rho_i(T)$. The curves denoted by 6 in Fig. 1 represent the magnetic component of the resistivity along the a axis and curves 7 represent the corresponding component along the c axis.

The magnetic component of the electrical resistivity is strongly anisotropic throughout the investigated range of temperatures. In the ferromagnetic and antiferromagnetic regions, we have $\rho_m^c(T) > \rho_m^a(T)$. At low temperatures, ranging from 15 to 40°K, the power exponent of $\rho_m(T)$ is close to 2, irrespective of the crystallographic directions (Fig. 2).

In the paramagnetic region, the magnetic component of the resistivity is independent of temperature but is still anisotropic. However, the nature of the anisotropy is different than in the magnetically ordered states: in the paramagnetic region, we have $\rho_m^c < \rho_m^a$. The anisotropy of the scattering of conduction electrons on disordered spins can be understood on the basis of the following considerations. In general, the electrical conductivity depends on the scattering processes (via the relaxation time τ) and on the electron structure parameters:

$$\sigma_{ij} = \frac{e^2\tau}{4\pi^3\hbar}\int v_i\, dS_j,$$

where v_i is the velocity of an electron along an i-th direction; dS_j is an element of the Fermi surface area. If we assume that the relaxation times for the scattering of electrons by magnons (τ_m) and by phonons (τ_p) are isotropic, we find that, in the paramagnetic region[9]

$$\rho_m{}^a/\rho_m{}^c = \alpha_a/\alpha_c = \int v_c\, dS_c \Big/ \int v_a\, dS_a > 1,$$

where α_a and α_c are the coefficients associated with the electron-phonon resistivity component. The results obtained in the present investigation indicate that $\rho_m^a/\rho_m^c = 1.2$ and $\alpha_a/\alpha_c = 2$ for Dy and that $\rho_m^a/\rho_m^c = 1.1$ and $\alpha_a/\alpha_c = 1.5$ for Tb. This difference shows that the anisotropy of ρ_m in the paramagnetic region is not solely due to the anisotropy of the Fermi surface but is also due to the anisotropy of the relaxation times representing the scattering of conduction electrons by magnons and phonons.

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