STIMULATED SUPERCONDUCITIVITY OF THIN FILMS

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The behavior of a thin superconducting film with paramagnetic impurities in an external microwave field is investigated by the method developed by Gor'kov and Éliashberg for nonstationary effects in superconducting alloys. It is found that in the region of gapless superconductivity there exists an interval amplitudes and external-field frequencies such that the temperature of the transition to the superconducting alloys. It is found that in the region of gapless superconductivity there exists an interval of amplitudes and external-field frequencies such that the temperature of the transition to the superconducting state and the critical current are larger than in the absence of an external alternat-

1. INTRODUCTION

WE consider in this paper the superconductivity induced by an external microwave field in a thin film with paramagnetic impurities. This means that there exists a region of amplitudes and external alternating field frequencies at which the superconductivity "improves," i.e., the ordering parameter, the temperature of the transition to the superconducting state, and the critical current all increase with increasing field. Such a situation is, in a certain sense, the inverse of the usual case when the external field can only weaken the superconducting properties.

The effect of stimulated superconductivity was already considered by Éliashberg[1] in first order in the field intensity, for alloys with nonmagnetic impurities. In the case of gapless superconductivity, which can occur in alloys with paramagnetic impurities, it is possible to take into account the field more accurately, and by the same token observe the saturation of the effect with respect to the field and the vanishing of the effect at excessively large fields.

Equations for the ordering parameter Δ in the case of a high concentration of the paramagnetic impurities[2] and in the case of low concentrations[3] were derived. It follows from these equations that at external-field frequencies exceeding a certain characteristic value Ω_0, the value of $\Delta(t)$ coincides with its timed-averaged value. We shall show below that these equations remain in force at external-field frequencies lower than a definite value ω_1 ($\omega_1 \gg \Omega_0$). In other words, stimulation of superconductivity occurs at sufficiently high frequencies exceeding ω_1.

It was shown in[1] that in a superconductor having an energy gap the effect is brought about by the characteristic behavior of the density of states as a function of the energy, namely, by the decrease of the density of states with increasing distance from the threshold. In the case of gapless superconductivity the root singularity possessed by a superconductor with a gap becomes smoothed out, but the maximum in the density of states remains. It must be assumed that this is the cause of the stimulated superconductivity for the gapless situation.

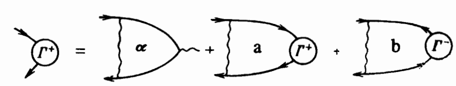

FIG. 1

2. DERIVATION OF EQUATIONS

We shall consider a thin film with paramagnetic impurities. A plane electromagnetic wave is incident normal to its surface. The film thickness is smaller than both the coherence length and the depth of penetration of the electromagnetic field. At frequencies lower than Δ^2/T, the role of the penetration depth is played by the London depth, and at higher frequencies by the skin depth. It can thus be assumed that the ordering parameter Δ and the field A do not vary over the thickness of the film.

According to the method developed by Gor'kov and Éliashberg[2,3], the equations for Δ and A are derived with the aid of the Feynman diagram technique. We are interested in the anomalous term in the equation for Δ in the case when the frequencies of the external field exceed Ω_0, and Δ is equal to its time-averaged value. Then, as follows from[3], the anomalous term is

$$U = \frac{\tau_1}{4i}\int \frac{\varepsilon\, d\varepsilon}{\varepsilon^2 + \tau_s^{-2}}(\Gamma^+ - \Gamma^-), \tag{1}$$

where τ_1 and τ_S are the path times introduced by Abrokosov and Gor'kov[4], and $\Gamma^\pm$ are the vertex parts.

These vertex parts contain in the denominator the Fourier transform of the diffusion Green's function $(-i\omega + Dk^2)$. Since the problem is spatially homogeneous, we must set k equal to zero, and replace ω, in accord with[1], by $(\omega + i\gamma)$, where γ is the reciprocal of the relaxation time of the excitation energy and is determined by the electron-phonon or by the electron-electron interaction.

For the vertex parts we can write an equation of the ladder type (Fig. 1), each blocks of which (α, a, and b) contains a diffusion line, designated by a wavy line. This diffusion line should be set in correspondence with the expression

$$I = \frac{1}{2\pi\tau_1}\frac{1}{\gamma\tau_1}.$$

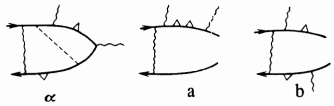

FIG. 2

Examples of diagrams for the bare vertex α and for the separating diagrams a and b are shown respectively in Fig. 2. They contain each two gaps and two field vertices with frequencies of opposite sign; in all these diagrams the two field vertices or on one side relative to the impurity line, which is shown dotted. Otherwise we should obtain terms containing div **A**, which we assume to be equal to zero. In addition in these diagrams at least one gap vertex lies between the field vertices, so that the parameter in the expansion in terms of ω can be $\omega\tau_s \ll 1$ and not $\omega\tau_1$, which is much smaller than $\omega\tau_s$ (the smallness of τ_1 compared with τ_s means that the concentration of the ordinary impurities exceeds the concentration of the paramagnetic ones).

The separating diagrams, which contain four gap or four field vertices, make no contribution.

Summing the aforementioned diagrams, we obtain for the bare vertex the expression

$$\alpha = \frac{4i}{\gamma\tau_1}\frac{\omega}{T}\,\mathrm{ch}^{-2}\left(\frac{\varepsilon}{2T}\right) D\left(\frac{e}{c}\right)^2 A_\omega A_{-\omega}\Delta^2 \frac{\omega\varepsilon}{\tau_s^2(\varepsilon^2+\tau_s^{-2})^3},$$

where ϵ is the frequency of the outer ends and ω is the field frequency.

The separating diagrams a and b differ from zero only in the second order of the expansion in $\omega\tau_s$. In addition, diagrams with field vertices, which lie on the upper and lower lines (an example is Fig. 2c), are particularly important here. They lead to the appearance of terms with the derivatives $\partial\Gamma^\pm/\partial\epsilon$ and $\partial^2\Gamma^\pm/\partial\epsilon^2$. Indeed, the expression for the diagram in $\Gamma^\pm$ is

$$\Gamma^\pm(\varepsilon-\omega)\left(b+\omega\frac{\partial b}{\partial\varepsilon}+\frac{\omega^2}{2}\frac{\partial^2 b}{\partial\varepsilon^2}\right)+\Gamma^\pm(\varepsilon+\omega)\left(b-\omega\frac{\partial b}{\partial\varepsilon}+\frac{\omega^2}{2}\frac{\partial^2 b}{\partial\varepsilon^2}\right)$$
$$= b\omega^2\frac{\partial^2\Gamma^\pm}{\partial\varepsilon^2}-2\omega^2\frac{\partial b}{\partial\varepsilon}\frac{\partial\Gamma^\pm}{\partial\varepsilon}+\omega^2\frac{\partial^2 b}{\partial\varepsilon^2}\Gamma^\pm.$$

The dependence of the diagram b on the frequencies is obtained by summing the impurity ladder of dashed lines around the vertex Δ:

$$b = -\frac{2\Delta^2}{\gamma}D\left(\frac{e}{c}\right)^2 A_\omega A_{-\omega}\,\mathrm{Re}\left[\left(\varepsilon-\frac{i}{\tau_s}\right)\left(\varepsilon-\omega+\frac{i}{\tau_s}\right)\right]^{-1}.$$

The summation of the diagrams, when all the foregoing is taken into account, yields an equation for the quantity $\Gamma = \tau_1(\Gamma^+ - \Gamma^-)/4i$:

$$\Gamma = \frac{2}{T\tau_s}\frac{p}{(1+x^2)^3}x\,\mathrm{ch}^{-2}\frac{x}{2T\tau_s} - p\left[\frac{2x^2}{(1+x^2)^2}\Gamma'' + \frac{4x(1-x^2)}{(1+x^2)^3}\Gamma'\right]$$
$$- 2p\frac{5x^4-24x^2+3}{(1+x^2)^4}\Gamma,$$

where we have changed over to the dimensionless variable $x = \epsilon\tau$,

$$p = \omega\tau_s\frac{\omega}{2\gamma}(\Delta\tau_s)^2\tau_s D\left(\frac{e}{c}\right)^2 A^2. \qquad (2)$$

We use here the time-averaged $A^2 = 2A_\omega A_{-\omega}$.

It is now convenient to change over to a new function $y(x)$ by means of the formula

$$\Gamma = \frac{1}{T\tau_s}\frac{1+x^2}{x}y.$$

The equation for this function is

$$\frac{d^2y}{dx^2}+y\left[\frac{3(x^4-6x^2+1)}{x^2(1+x^2)^2}+\frac{(1+x^2)^2}{2x^2}\frac{1}{p}\right]=\frac{\mathrm{ch}^{-2}(x/2T\tau_s)}{(1+x^2)^2}. \qquad (3)$$

The anomalous term U in the equation for Δ, defined by formula (1), is expressed in terms of the function y:

$$U = \frac{2}{T\tau_s}\int_0^\infty y\,dx. \qquad (4)$$

In the case of a large paramagnetic-impurity concentration, such that $T\tau_s \ll 1$, the equation for Δ becomes

$$\frac{\pi^2}{6}\tau_s^2(T_c^2-T^2)-\frac{(\Delta\tau_s)^2}{12}-2\tau_s\left(\frac{e}{c}\right)^2 DA^2+U=0,$$
$$\tau_s^2T_c^2 = 6\pi^{-2}\ln(\pi T_{c0}\tau_s/2\gamma), \qquad (5)$$

T_{c0} is the transition temperature in the absence of paramagnetic impurities.

At low paramagnetic-impurity concentrations ($T\tau_s \gg 1$) we have

$$\frac{T_c-T}{T_c}-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T^2}-\frac{\pi}{2}\frac{D}{T}\left(\frac{e}{c}\right)^2 A^2+U=0. \qquad (6)$$

The equation for the field does not contain an anomalous such as U. In the limit, say, when $T\tau_s \gg 1$, we have

$$\mathbf{j} = \sigma\mathbf{E} - D\frac{mpe^2}{2\pi c}\frac{\Delta^2}{T}\mathbf{A}. \qquad (7)$$

Thus, allowance for the effect of superconductivity stimulation by an alternating field reduces to the addition of an anomalous term U to the normal part of the equation for Δ. As will be shown below, its sign is opposite that of the already present field-containing term. In other words, generally speaking, a situation is possible where an increase of the field leads not to the customary weakening of the temperature factor $(T_c - T)/T_c$, but to its enhancement.

Using the derived equations (3), (4), and (6), and also (7), we can consider the question of the critical temperature of the transition of the film into the superconducting state, the critical current, and the critical field destroying the superconductivity.

3. TRANSITION TEMPERATURE

We are interested in obtaining for the inhomogeneous equation (3) a solution that decreases at infinity and is integrable. Its integral as a function of one parameter p (we consider now the case $T\tau_s \gg 1$) can be approximated as follows:

$$\int_0^\infty y\,dx \approx \frac{0{,}2p}{1+p}.$$

In weak fields, U increases linearly with the field and enters the equation with a plus sign. In sufficiently strong fields, saturation sets in.

At a given value of the external field, Eq. (6) determines the ordering parameter Δ as a function of the temperature. If we regard this equation as the dependence of the temperature on Δ, then such a function will have a maximum at a certain point Δ_0. This means that the temperature of the superconductor can be in-

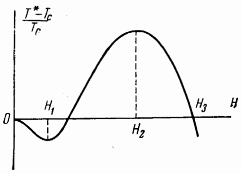

FIG. 3

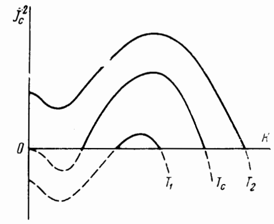

FIG. 4

creased only up to a certain value, beyond which the superconductivity is abruptly destroyed.

The quantity Δ_0, which determines the new transition temperature, is given by

$$(\Delta_0\tau_s)^2 = 4{,}4T\tau_s\frac{A_1}{A}\left(1-\frac{A_1}{A}\right)\theta\left(1-\frac{A_1}{A}\right),$$
$$\theta(x)=\begin{cases}0, & x<0\\ 1, & x>0\end{cases};\qquad \tau_s D\left(\frac{e}{c}\right)^2 A_1^2=\frac{0{,}4}{(\omega\tau_s)^2T/\gamma}. \tag{8}$$

The relation obtained for the field dependence of the transition temperature T^* is

$$T_c\tau_s\frac{T^*-T_c}{T_c}=0{,}4\left(1-\frac{A_1}{A}\right)^2\theta\left(1-\frac{A_1}{A}\right)-\frac{\pi}{2}A^2. \tag{9}$$

The quantity A here is the modulus of the vector potential of the field inside the film. In order to connect it with the intensity of the field on the surface, it is necessary to solve the concrete electromagnetic problem.

We shall assume the frequency of the incident field to be sufficiently large, $\omega \gg \Delta^2/T$, so that the depth of penetration of the field into a bulky sample is determined by the skin depth. As already mentioned, we are considering the normal incidence of an electromagnetic wave of amplitude H on a film whose thickness d is much smaller than the skin penetration depth δ_S. Then, provided $\delta_S \ll \lambda$, the wave is completely reflected from the film, and for the time-averaged value of A^2 we obtain

$$A^2=\delta_s^4H^2/2d^2.$$

The dependence of $(T^* - T_C)/T_C$ on H is shown in Fig. 3. We see that if the field is smaller than a certain value H_1, then a rapid lowering of the transition temperature takes place with increasing field, so that $(T^* - T_C)/T_C$ reaches its local minimum $-0.6\gamma/\omega^2\tau$ in a field

$$H_1^2=\frac{0{,}8}{(\omega\tau_s)^2T/\gamma}\frac{d^2}{\delta_s^4}\frac{1}{\tau_sD}\left(\frac{c}{e}\right)^2. \tag{10}$$

With further increase of the field, the transition temperature increases and reaches a maximum

$$\left(\frac{T^*-T_c}{T_c}\right)_{max}=\frac{0{,}4}{T\tau_s} \tag{11}$$

at a field value

$$H_2^2=\frac{0{,}6}{((\omega\tau_s)^2T/\gamma)^{1/3}}\frac{d^2}{\delta_s^4}\frac{1}{\tau_sD}\left(\frac{c}{e}\right)^2. \tag{12}$$

Finally, further increase of the field leads to a lowering of the critical temperature, which becomes equal to T_C in a field given by

$$H_3^2=0{,}5\frac{d^2}{\delta_s^4}\frac{1}{\tau_sD}\left(\frac{c}{e}\right)^2. \tag{13}$$

The quantity $\omega^2\tau_S/\gamma$ contained here is large compared with unity, since γ is small, of the order of T^3/Θ^2 in the case of electron-phonon interaction. We can estimate the scales of the fields H_1, H_2, and H_3 by taking typical values of the parameters contained in the formulas. At $T \sim 10°K$, $\omega \sim 10^{11}\ sec^{-1}$, $\tau_S \sim 10^2\tau_1 \sim 10^{-12}$ sec, $d \sim 10^{-1}$, and $\delta \sim 10^{-4}$ cm we obtain $H_1 \sim 1$ Oe, $H_2 \sim 10$ Oe, and $H_3 \sim 10^2$ Oe. Thus, the situation obtained is experimentally feasible.

4. THE CRITICAL CURRENT

In a thin film, in the presence of an alternating field, a time-constant field current can flow. The expression for its density can be obtained from (7)

$$6\pi^3\frac{T}{p^4e^2\tau_1}\frac{j^2}{\Delta^4}=\frac{T_c-T}{T_c}-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T^2}-\frac{\pi}{2}\frac{D}{T}\left(\frac{e}{c}\right)^2A^2+U. \tag{14}$$

We see therefore that the dependence of the current on the ordering parameter Δ has, in analogy with the preceding case, a maximum at a certain point Δ_0, and at currents exceeding the critical value at a given amplitude of the external alternating field, the superconducting state is destroyed jumpwise. Having an equation for the current, (14), and an equation for Δ_0:

$$\frac{T_c-T}{T_c}-\frac{21\zeta(3)}{16\pi^2}\frac{\Delta_0^2}{T^2}-\frac{\pi}{2}\frac{D}{T}\left(\frac{e}{c}\right)^2A^2+\frac{0{,}2}{T\tau_s}\frac{2p^2+3p}{(1+p)^2}=0, \tag{15}$$

we can find the dependence of the critical current on the amplitude of the external field. This dependence, at different temperatures, can be shown in Fig. 4, where only the solid sections of the curves have a physical meaning. The central curve corresponds to the temperature T_C ($T_1 > T_C$, $T_2 < T_C$). The value of the critical current at a zero field is determined by the usual expression without allowance for the anomalous term:

$$6\pi^3\frac{T}{p^4e^2\tau_1}j_c^2=\frac{1}{3}\left(\frac{16\pi^2T^2}{21\zeta(3)}\right)^2\left(\frac{T_c-T}{T_c}\right)^3. \tag{16}$$

On approaching the maximum temperature

$$(T^*-T_c)/T_c|_{max}=0{,}4/T\tau_s$$

The plot of the critical current against the field contracts to a point, and the critical current vanishes at a field H_2. At temperatures close to the maximum, we obtain for this current the expression

$$6\pi^3\frac{T}{p^4e^2\tau_1}j_c^2=\frac{28}{\tau_s^4}\left(\frac{T^2\tau_s\gamma}{\omega^2}\right)^{2/3}\left(\frac{0{,}4}{T\tau_s}-\frac{T-T_c}{T_c}\right). \tag{17}$$

5. REMARKS AND COMPARISON WITH EXPERIMENT

It is necessary to determine the frequency interval in which the effects indicated above take place. The point is that in the theory we expanded in the parameter $\Delta\tau_S \ll 1$ corresponding to gapless superconductivity. Therefore the entire analysis remains valid of Δ is sufficiently small. Thus, for example, on the plot of the transition temperature vs. Δ, the maximum should be reached at $\Delta = \Delta_0$, such that $\Delta\tau_S \ll 1$, with Δ_0 determined as a function of the field by Eq. (8). Since the increase of the gap comes into play at $H \sim H_1$, when $(\Delta_0\tau_S)^2 \sim \gamma/\omega^2\tau$, it is necessary to stipulate $\omega \gg \omega_1 \sim (\gamma/\tau_S)^{1/2}$. (It should be noted here that in fields $H_1 \ll H \ll H_2$ the value of $\Delta_0\tau_S$ becomes larger than unity at all frequencies, so that such fields need not be taken into consideration.)

All the foregoing calculations were made for the case $\omega\tau_S \ll 1$. It can be shown that when $\omega \gg \omega_2 \sim \tau_S^{-1}$ there is no stimulation of superconductivity, provided we remain all the time in the region $\Delta\tau_S \ll 1$, i.e., the anomalous term U enters the equation with a minus sign at all field values. The effect will thus take place in the frequency interval $\omega_1 < \omega < \omega_2$.

In addition, since we take Δ to be equal to its time-averaged value, it follows, as shown in[3], that this calls for the condition $\omega \gg D(e/c)^2A^2$ in addition to the condition $\omega \ll \gamma$. The former will take place in fields of the order of H_1, provided only $\omega \gg (\gamma/T)^{1/3}\tau_S$.

It should be noted that the dependence of Δ on A at a fixed temperature, as follows from (6), has a hysteresis character.

Finally, it is meaningful to compare our results with the existing experimental data[5-7]. Dayem and Wiegand[6], in an experiment with a superconducting film in the presence of a high-frequency field, obtained the dependence of the critical current on the radiation power at various frequencies. This dependence had a maximum at a finite value of the power, and the critical current increased with increasing frequency. However, this experiment was performed at temperatures below critical. Above the critical temperature T_c, the picture obtained[7] for the dependence of the critical current on the radiation power is similar to Fig. 4. The current existed only in a definite power interval, which contracted with increasing temperature about a certain finite value, and the current vanished completely at a certain temperature.

In conclusion, the author is grateful to G. M. Éliashberg for suggesting the problem and continuous direction of the work, and to L. P. Gor'kov for valuable discussions.

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