ROLE OF BOUNDARY SINGULARITIES IN THE OCCURRENCE OF SUPERCONDUCTIVITY

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Submitted April 13, 1970

Zh. Eksp. Teor. Fiz. 59, 957–964 (September, 1970)

The boundary singularities of the self-energy of an electron for the absolute temperature T equal to zero and their influence on the nature of the one-electron excitations of the metal at low temperatures are studied in a model of a metal with an electron-phonon interaction. In this connection it is not assumed that the metal is necessarily a superconductor at low temperatures. It is shown that even in the zero-order approximation (second-order perturbation theory) the real part of the electron's self-energy $\Sigma^{(0)}$ at $T = 0$ may contain boundary singularities of the type $x \ln x$ which lead to a deviation of the analytic properties of the one-electron Green's function from the properties resulting from the Lehmann expansion. Taking account of the sum of the main sequence for Σ does not change this result. The observed anomalies in the analytic properties of the one-electron Green's function are explained by a violation of the adiabatic hypothesis. It is assumed that in the case under consideration the violation of the adiabatic hypothesis is a consequence of the presence of a tendency for the metal to undergo a phase transition into the superconducting state.

As is well known, at low temperatures certain metals and alloys pass into the superconducting state. Many articles have been devoted to an investigation of the properties of superconductors and to a deduction of the fundamental relations in the theory of superconductivity. However there are a number of questions which the existing theory in its present state of development does not satisfactorily explain. In particular, the reasons obstructing the transition of good metals into the superconducting state are not clear.

In order to clarify this question physically, a systematic investigation is carried out in which it is not assumed that the metal necessarily undergoes a transition into the superconducting state at low temperatures. In particular, an investigation of the spectrum of the electronic excitations of a metal at low temperatures is of interest, for the purpose of finding certain indications that in a number of cases a change of the ground state of the electronic liquid is possible. It is natural that the results obtained in this connection can be valid only in the capacity of criteria for the occurrence of superconductivity.

Below the question involves the spectrum of the electronic excitations of a metal in a model having an electron-phonon interaction. The electron-phonon interaction in metals has been investigated earlier, for example, in[1] under the assumption that the metals are not superconductors. However, in this connection the possibility of the emergence of the so-called boundary singularities in the self-energy of the electron was not taken into consideration, and the results of[1] do not reach the conclusion that in a number of cases the analytic properties of the one-electron Green's function at $T = 0$ contain an indication of the violation of the adiabatic hypothesis.

It is well known that a number of different types of singularities are possible in the Feynman integrals which arise in the usual way in perturbation theory; these are a consequence of a singularity of the integrands.[2] Below the question involves one class of singularities of Feynman integrals—the boundary singularities.[2,3] Here boundary singularities are to be understood as the singularities of a multiple integral, which depend on external parameters for values of the latter at which the regions of singularity of the integrand occupy a "critical" position relative to the boundary of the region of integration. Besides the usual boundary singularities[3] such a definition also includes singularities associated with contact of the regions of singularity of the integrand with the boundaries of the region of integration. The singularities which lead to the well-known Kohn effect[4] are an example of the second type of boundary singularities. In fact the singularities of Feynman integrals need not be purely boundary with regard to their origin; however, here it is precisely boundary effects which are of interest.

In the case of a system of nonrelativistic interacting particles the boundary singularities are a reflection of the characteristic features of the interactions of the particles, due to such common factors for many systems as the periodic nature of the structure of the crystals and the presence of a minimum wavelength of the lattice vibrations in crystals, and the Pauli exclusion principle for particles obeying Fermi-Dirac statistics.

Since the nature of the occurrence of boundary singularities is common for many physical systems, boundary singularities must be associated with certain general properties of the latter. In particular, an investigation of their role in the occurrence of superconductivity is of interest. Investigation of this role is possible in principle in a model with any interaction which leads to Cooper pairing of the electrons. However, since an isotope effect exists for the majority of pure superconductors, apparently it is advisable to study the connection between boundary singularities and the occurrence of superconductivity in a model of a metal with an electron-phonon interaction.

The real part of an electron's self-energy Σ at $T = 0$ already contains boundary singularities in the zero-order approximation (second-order perturbation theory). In spite of the fact that singularities in the electron's self-energy for $T = 0$ are also possible in

principle in higher orders of perturbation theory, a separate examination of the boundary singularities of $\Sigma^{(0)}$ makes sense in connection with the possibility of a direct calculation of $\Sigma^{(0)}$ under certain assumptions. An estimate indicates that the effects, which the presence of boundary singularities in $\Sigma^{(0)}$ lead to, remain even upon taking account of the sum of the main sequence for Σ.

1. An ideal, homogeneous metal is considered with the absolute temperature T = 0. Only the interaction of the electrons in the metal with longitudinal phonons is considered. The real part of the electron's mass operator has the following form in the zero-order approximation:

$$\Sigma^{(0)}(\mathbf{k}, \varepsilon) = g^2 \int \frac{d^3 f}{(2\pi)^3} \frac{\omega_t}{2} \left\{ \frac{1 - n_{\mathbf{k}-t}}{\varepsilon - \varepsilon_{\mathbf{k}-t} - \omega_t} + \frac{n_{\mathbf{k}-t}}{\varepsilon - \varepsilon_{\mathbf{k}-t} + \omega_t} \right\}, \quad (1)$$

where the usual notation is used: g is the coupling constant for the interaction of the electrons with longitudinal phonons in the metal, $\epsilon_{\mathbf{k}}$ and $\omega_{\mathbf{f}}$ are the dispersion laws for electrons and phonons, $n_{\mathbf{k}}$ is the occupation number of the states with a given wave vector for the electrons with T = 0 in the absence of any interaction with the phonons, and $\mathbf{k}$ and $\mathbf{f}$ are wave vectors.

In the isotropic case with a quadratic dispersion law for the electrons, $\epsilon_{\mathbf{k}} = \alpha k^2$ and in the Debye model for the phonons, the integral in Eq. (1) can be evaluated in closed form. Evaluation shows that the real part of the electron's mass operator (1) contains singularities of the type x ln x. The equations for the lines in the plane of the variables k and ϵ on which these singularities occur have the following form in the most interesting region $|k - k_F| < f_D$:

1. $\varepsilon - \omega_D - \alpha(f_D + k)^2 = 0$,
2. $\varepsilon \pm \omega_D - \alpha(f_D - k)^2 = 0$, $k + k_F \geqslant f_D$
   $k + k_F \leqslant f_D$,
3. $\varepsilon \pm \omega_D - \varepsilon_F = 0$, $k + k_F \geqslant f_D$,
4. $\varepsilon \pm s(k + k_F) - \varepsilon_F = 0$, $k + k_F \leqslant f_D$,
5. $\varepsilon \pm s(k - k_F) - \varepsilon_F = 0$.

Here ϵ_F and k_F denote the energy and modulus of the wave vector of the electron on the Fermi surface in the absence of any interaction with the phonons ($\epsilon_F = \alpha k_F^2$), ω_D and f_D are the limiting frequency and wave vector of the phonons in the Debye model ($\omega_D = s f_D$). The derivatives of $\Sigma^{(0)}(k, \epsilon)$ with respect to ϵ and with respect to k (except lines 3) have singularities of the type ln x on lines 1 through 5.

The singularities of $\Sigma^{(0)}(k, \epsilon)$ on lines of the type 1 through 5 are possible not only in the isotropic case in the Debye model. In the general case the singularities of of $\Sigma^{(0)}$ appear at those values of ϵ and k at which the region of singularity of the integrand (1) touches the boundary of the region of integration, determined by the Brillouin zone and the Fermi surface, displaced by the vector $\mathbf{k}$ in momentum space, or it passes through one of the corner points of the Brillouin zone or the intersection of the Brillouin zone with the Fermi surface, shifted by the vector $\mathbf{k}$ in momentum space. It should however be noted that in the general case the form x^2 ln x is more characteristic for the boundary singularities of $\Sigma^{(0)}$ associated with the corner points of the boundaries of the region of integration. The form x ln x for this type of boundary singularity in the case under consideration (on the lines 3)—is a consequence of isotropy.

2. Near the Fermi surface in the quasi-particle approximation the spectrum of the elementary excitations of the electronic liquid is determined from the equation:

$$\varepsilon - \varepsilon_k - \Sigma^{(0)}(k, \varepsilon) = 0. \quad (2)$$

Since $\omega_D \ll \epsilon_F$, under reasonable assumptions with regard to the interaction constants the line 1 does not intersect the mass surface, the lines 3—5 intersect the mass surface near the Fermi surface, and line 2 intersects it near $k_2 \approx f_D/2$.

Near the singularities on lines 2—4

$$\partial\Sigma^{(0)}(k, \varepsilon) / \partial\varepsilon > 0,$$

which leads to a nonunique dependence of ϵ on k. As an example the form of the dependence of ϵ on k near the points ϵ', k' of intersection of the lines 3, 4 with the mass surface (without the fine details due to the anomalies of $\partial\Sigma^{(0)}/\partial k$ on lines 4) is shown in Fig. 1. Here for definiteness it is assumed that the branches of $\epsilon(k)$, containing the points of intersection of the lines of singularity of $\Sigma^{(0)}$ with the mass surface, do not intersect. Near $k'_{3\pm}$, $\epsilon'_{3\pm}$

$$\frac{\partial \Sigma^{(0)}(k, \varepsilon)}{\partial \varepsilon} \approx -\frac{g^2}{8\pi^2} \frac{f_D^2}{v_F} \ln \left| \frac{\varepsilon - \varepsilon_F \pm \omega_D}{\omega_D} \right|,$$

$$\frac{\partial \varepsilon}{\partial k}\Big|_{k=k'_{3\pm}} = 0. \quad (3)$$

Near $k'_{4\pm}$, $\epsilon'_{4\pm}$

$$\frac{\partial \Sigma^{(0)}(k, \varepsilon)}{\partial k} \approx \mp \frac{g^2}{8\pi^2} \frac{s}{v_F} (2k_F)^2 \ln \left| \frac{\varepsilon - \varepsilon_F \pm s(k + k_F)}{\omega_D} \right|,$$

$$\frac{\partial \Sigma^{(0)}(k, \varepsilon)}{\partial \varepsilon} \approx -\frac{g^2}{8\pi^2} \frac{(2k_F)^2}{v_F} \ln \left| \frac{\varepsilon - \varepsilon_F \pm s(k + k_F)}{\omega_D} \right|,$$

$$\frac{\partial \varepsilon}{\partial k}\Big|_{k=k'_{4\pm}} = \mp s. \quad (4)$$

Values of the derivatives of ϵ with respect to k are presented for the branches containing points of intersection of lines 3—4 with the mass surface. These branches occupy an interval Δ in the energy

$$\Delta^{(3)} \sim 2\omega_D \exp\left\{ -\frac{8\pi^2 v_F}{g^2 f_D^2} \right\},$$

$$\Delta^{(4)} \sim 2\omega_D \exp\left\{ -\frac{8\pi^2 v_F}{g^2 (2k_F)^2} \right\}. \quad (5)$$

Since $\partial\Sigma^{(0)}/\partial\epsilon > 1$ for the branches under consideration, the poles of the Fourier components of the electronic Green's function corresponding to these branches are located in the upper half of the complex energy plane for $\epsilon \approx \epsilon_F + \omega_D$ ($\epsilon \approx \epsilon_F + s(k + k_F)$) and in the lower half plane for $\epsilon \approx \epsilon_F - \omega_D$ ($\epsilon \approx \epsilon_F - s(k + k_F)$).

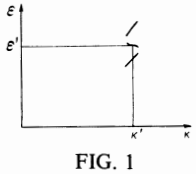

FIG. 1

As will be shown below, a similar situation (the presence of segments of the mass surface on which $\partial \operatorname{Re}\Sigma/\partial\epsilon > 1$) remain even upon taking account of the sum of the main sequence for Σ. This is in contradiction with the analytic properties which follow from the Lehmann expansion for the one-electron Green's function (this assertion is valid for $\partial \operatorname{Re}\Sigma/\partial\epsilon > 1$ irrespective of the dependence of the behavior of the imaginary part of Σ, since here either the position of the poles is anomalous or the behavior of the imaginary part of the Green's function is anomalous), and this is an indication of a violation of the adiabatic hypothesis. The state obtained from the ground state of an ideal electron-phonon system without interaction with the aid of adiabatic switching-on of the interaction is not the ground state, and therefore is not stable.

The nature of the changes introduced into the spectrum of the electronic excitations by the singularities on lines 2 is of a much finer scale due to the presence of the factor s/v_F which appears simultaneously in the derivatives of $\Sigma^{(0)}$ with respect to ϵ and with respect to k. The same also pertains to the singularities on lines 5. Taking account of the higher-order terms in the perturbation-theoretic series for Σ leads to a smoothing out of these changes.

3. As already mentioned, the electron's self-energy Σ also has boundary singularities in the higher order terms of perturbation theory. The main contribution to the anomalies on lines of the type 3, 4 for the terms of Σ is contained in the sequence of diagrams having polarization loops (see Fig. 2). The term of interest here, the real part of the sum of this sequence, has the following form:

$$\operatorname{Re}\Sigma_F(k,\varepsilon) = -g^2\int\frac{d^3f}{(2\pi)^3}n_{k-f}\,\omega_f^2 \times\{(\varepsilon-\varepsilon_{k-f})^2-\omega_f^2(1+\operatorname{Re}\Pi^{(0)}(\varepsilon-\varepsilon_{k-f},f))\} \times\{[(\varepsilon-\varepsilon_{k-f})^2-\omega_f^2(1+\operatorname{Re}\Pi^{(0)}(\varepsilon-\varepsilon_{k-f},f))]^2 + \omega_f^4[\operatorname{Im}\Pi^{(0)}(\varepsilon-\varepsilon_{k-f},f)]^2\}^{-1},$$

where $\pi^0(\omega, f)$ is the polarization operator in the zero-order approximation; the remaining notation is the same as in Eq. (1). In the isotropic case and for the Debye model one can represent $\operatorname{Re}\Sigma_F(k, \epsilon)$ in the form

$$\operatorname{Re}\Sigma_F(k,\varepsilon) = \frac{g^2s^2}{4\pi^2 v_k}\int_{k-k_F}^{\min\{f_D,\,k+k_F\}} f^3df \times\int_{\varepsilon-\varepsilon_{k-f}}^{\varepsilon-\varepsilon_F} dy\,\frac{y^2-s^2f^2(1+\operatorname{Re}\Pi^{(0)}(y,f))}{[y^2-s^2f^2(1+\operatorname{Re}\Pi^{(0)}(y,f))]^2+s^4f^4[\operatorname{Im}\Pi^{(0)}(y,f)]^2}. \quad (7)$$

Since the quantity $\operatorname{Im}\pi^{(0)} \sim s/v_F$ for $\omega \sim \omega_D$ and for $f \sim f_D$, one would expect that taking account of the sum of the sequence of diagrams for Σ with polarization loops does not significantly change the results of Sec. 2. In fact, this is obvious for large interaction constants. For small interaction constants (such that $1 + \operatorname{Re}\pi^{(0)}(\omega, f)$ does not differ appreciably from unity for $\omega \sim \omega_D$, $f \sim k_F$), an estimate with the replacement of $\operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f)$ by $\operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f_D)$ and without taking account of the cutoff factor in $\operatorname{Im}\pi^{(0)}(\omega, f)$ leads to the following expression for the derivative of $\operatorname{Re}\Sigma_F(k, \epsilon)$ with respect to ϵ near $\epsilon_F \pm \tilde{s}f_D$ for $k + k_F > f_D$:

$$\frac{\partial\operatorname{Re}\Sigma_F(k,\varepsilon)}{\partial\varepsilon} \approx -\frac{g^2f_D^2}{16\pi^2v_k}\frac{1}{1+\operatorname{Re}\Pi^{(0)}(\tilde{s}f_D,f_D)} \times\left\{\ln\left[\left[\left(\frac{\varepsilon-\varepsilon_F}{\omega_D}\right)^2-(1+\operatorname{Re}\Pi^{(0)}(\tilde{s}f_D,f_D))\right]^2+[\operatorname{Im}\Pi^{(0)}(\tilde{s}f_D,f_D)]^2\right]+2\right\}, \quad (8)$$

where $\tilde{s} = s\sqrt{1 + \operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f_D)}$.

If $k + k_F < f_D$, then f_D is replaced everywhere by $k + k_F$ and ω_D by $s(k + k_F)$. Under the assumption that $f_D > k + k_F \approx 2k_F$, the condition under which $\partial \operatorname{Re}\Sigma_F/\partial\epsilon$ may exceed unity can be represented in the following form:

$$\frac{s}{v_F} < \frac{1}{\pi\gamma}\exp\left\{-\frac{1}{\gamma}\right\}, \quad (9)$$

where $\gamma = g^2k_F^2/2\pi^2v_F$. For $f_D < k + k_F$ the corresponding condition is obtained in analogous fashion. For $f_D \approx 2k_F$ it agrees with (9). A condition of the type (9) determines the lower bound on the interaction constant, the value at which violation of the adiabatic hypothesis occurs for the system under consideration. From (9) it follows that $\gamma_{min} \approx 0.195$ for $s/v_F = 10^{-2}$ and $\gamma_{min} \approx 0.128$ for $s/v_F = 10^{-3}$. An estimate involving the replacement of $\operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f)$ by $\operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f_D)$ leads to an overestimated (by 30 to 40% for $s/v_F = 10^{-2}$, $\gamma = 1/5$) value of $\partial \operatorname{Re}\Sigma_F/\partial\epsilon$. Out of the simple estimates, the estimate involving the replacement of $\operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f)$ by $\operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f_D)(f_D/f)^2$ is more successful for $k + k_F > f_D \approx 2k_F$ or $k + k_F < f_D$ (here f_D is replaced everywhere by $k + k_F$). In this connection, for large values of f the values of the integrand in Eq. (7) are not overestimated so much (at the upper limit with respect to y), and for average values of f the values are underestimated, which partially cancels the error for large values of f. Such an estimate leads to an expression for $\partial \operatorname{Re}\Sigma_F/\partial\epsilon$ which differs from (8) by the absence of the pre-logarithmic factor $(1 + \operatorname{Re}\pi^{(0)}(\tilde{s}f_D, f_D))^{-1}$. Here the criterion for violation of the adiabatic hypothesis takes the form

$$\frac{s}{v_F} < \frac{1}{\pi\gamma}\exp\left\{-\frac{1}{\gamma}-1\right\}, \quad (10)$$

$\gamma_{min} \approx 0.264$ for $s/v_F = 10^{-2}$ and $\gamma_{min} \approx 0.15$ for $s/v_F = 10^{-3}$.

It is clear that more accurate estimates do not introduce anything essentially new. At the same time the considered model of a metal is overly simplified, and there is no need for greater accuracy. Taking account of the cutoff factor in $\operatorname{Im}\pi^{(0)}$ would only lead to the vanishing of γ_{min}. In connection with this, condition (10) should be replaced by the following:

$$|\operatorname{Im}\Pi^{(0)}(\varepsilon-\varepsilon_F, f')| < \exp\{-1/\gamma-1\}, \quad (11)$$

where

$$f' = \begin{cases} k+k_F & \text{for } k+k_F < f_D, \\ f_D & \text{for } k+k_F > f_D \approx 2k_F, \end{cases}$$

ϵ and k are evaluated on the mass surface near the maximum of $\partial \operatorname{Re}\Sigma_F/\partial\epsilon$.

4. In the temperature technique at low temperatures one can also find an indication of the presence of

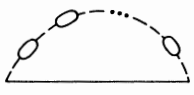

FIG. 2

boundary singularities in the electron's self-energy Σ at $T = 0$. At a finite temperature the real part of the electron's self-energy has the following form in second-order perturbation theory:[5]

$$\Sigma^{(0)}(k, \varepsilon, T) = g^2 \int \frac{d^3 f}{(2\pi)^3} \frac{\omega_f}{2} \left\{ \frac{1 - n_{k-f} + \nu_f}{\varepsilon - \varepsilon_{k-f} - \omega_f} + \frac{n_{k-f} + \nu_f}{\varepsilon - \varepsilon_{k-f} + \omega_f} \right\}, \quad (12)$$

where n_k and ν_f are the occupation numbers of the states with a given wave vector for noninteracting electrons and phonons, respectively, at a temperature T; the remaining notation is the same as in Eq. (1). The part $\Sigma_F^{(0)}(k, \epsilon, T)$ of the mass operator (12) which leads to the singularities 3–5 for $T = 0$ can be represented in the form

$$\Sigma_F^{(0)}(k, \varepsilon, T) = \int_0^\infty d\xi \left(-\frac{\partial n(\xi - \mu)}{\partial \xi} \right) \Sigma_F^{(0)}(k, \varepsilon, T = 0, \xi), \quad (13)$$

where

$$\Sigma_F^{(0)}(k, \varepsilon, T = 0, \xi) = g^2 \int \frac{d^3 f}{(2\pi)^3} \theta(\xi - \varepsilon_{k-f}) \frac{\omega_f^2}{\omega_f^2 - (\varepsilon - \varepsilon_{k-f})^2},$$

$$n(\xi - \mu) = \left\{ 1 + \exp\left(\frac{\xi - \mu}{kT} \right) \right\}^{-1}, \quad \theta(\xi) = \begin{cases} 1 & \text{for } \xi > 0, \\ 0 & \text{for } \xi < 0, \end{cases}$$

μ denotes the chemical potential, and k is Boltzmann's constant.

From (13) it is seen at once that at finite temperatures there are no singularities on lines of the types 3–5. However an estimate indicates that as $T \to 0$ the derivatives of $\Sigma_F^{(0)}(k, \epsilon, T)$ with respect to ϵ and with respect to k (except for the lines of type 3) behave like ln T on lines 3–5. At low temperatures this leads to a rearrangement of the spectrum of the electronic excitations near the Fermi surface (Fig. 1) and to the appearance of anomalous analytic properties for the one-electron Green's function. The temperature T_0 at which the anomalous analytic properties appear is determined by the condition

$$\max \left\{ \frac{\partial \Sigma^{(0)}(k, \varepsilon, T_0)}{\partial \varepsilon} \Big|_{\varepsilon = \varepsilon(k)} \right\} = 1. \quad (14)$$

From Eqs. (3), (4), (13), and (14) it follows that

$$T_0^{(3)} \sim \omega_D \exp\left\{ -\frac{8\pi^2 v_F}{g^2 f_D^2} \right\}, \quad T_0^{(5)} \sim \omega_D \exp\left\{ -\frac{2\pi^2 v_F}{g^2 k_F^2} \right\}. \quad (15)$$

The changes introduced into the spectrum of the electronic excitations by the singularities on lines of the type 5 would appear only at temperatures which are much lower by a factor of $\sim \exp\{\lambda^{-1}(v_F/s)^2\}$, i.e., since $v_F \gg s$ and $\lambda \sim 1$ practically at a temperature of absolute zero. Singularities of $\Sigma^{(0)}(k, \epsilon, T)$ on lines of the type 1–2 also exist at a finite temperature.

In the temperature technique the anomalies of the analytic properties of the one-electron Green's function due to the boundary singularities of $\Sigma^{(0)}$ on the lines 2–4 for $T = 0$ are associated with the fact that the considered approximation (12) becomes invalid at low temperatures $T \sim T_0$. In fact the considered approximation (12) is invalid even at rather high temperatures because of the singularities of $\Sigma^{(0)}$ on the lines 2. However, in the considered isotropic model of a metal for $s \ll v_F$ these singularities do not lead to any physical effects for reasons analogous to the ones cited at the end of Sec. 2. Therefore, in fact the approximation (12) can be used down to temperatures $T \sim T_0$ for examination of the circle of questions for which the behavior of Σ near line 2 is not essential. Apparently the very same is also valid in regard to utilization of the approximation (12) at temperatures $T \lesssim T_0$ if the parameters of the system are such that violation of the adiabatic hypothesis does not occur.

It is natural to conjecture that the violation of the adiabatic hypothesis which arises in a number of cases in the system under consideration is associated with the presence of a tendency toward a phase transition of the metal into the superconducting state at low temperatures. In connection with the validity of this conjecture the violation of the adiabatic hypothesis and, as a consequence, the violation of the analytic properties of the one-electron Green's function would be a convenient criterion for a theoretical investigation of the tendency toward a phase transition into a superconducting state for real metals.

It is well-known that in a system which exhibits such a tendency the scattering amplitude of the electrons, calculated in the usual diagram technique without taking account of the presence of the condensate, has a pole in the complex energy plane.[6] At the same time it is obvious that the terms of the perturbation-theoretic series for the scattering amplitude of the electrons in a phonon model have boundary singularities which are analogous in origin to the ones considered here. In this connection an investigation of the conditions under which the presence of these boundary singularities leads to the appearance of a pole in the total amplitude for the scattering of electrons is of interest.

The presence in the temperature technique of indications concerning the possibility of a violation of the adiabatic hypothesis in the system under consideration makes it possible to hope for the possibility of finding certain rules for the selection of diagrams or for decoupling the system of equations for the Green's function, which reduce to a correct description of the actual state of the system at low temperatures.

In the model of a metal under consideration here, the Coulomb interaction between the electrons is taken into account only in the screening of the electron-phonon interaction. It is clear that taking account of the direct electron-electron interaction cannot cancel the anomalies of the electron's self-energy for $\epsilon \approx \epsilon_F \pm \omega_D$. However, taking account of this interaction may lead to a smoothing out of such anomalies. In connection with this it is of interest to estimate the influence of the screened Coulomb interaction between electrons on the criterion for the violation of the adiabatic hypothesis in the system under consideration.

[1] A. B. Migdal, Zh. Eksp. Teor. Fiz. 34, 1438 (1958) [Sov. Phys.-JETP 7, 996 (1958)].

[2] L. D. Landau, Zh. Eksp. Teor. Fiz. 37, 62 (1959) [Sov. Phys.-JETP 10, 45 (1960)].

[3] Rudolph C. Hwa and Vigdor L. Teplitz, Homology and Feynman Integrals, W. A. Benjamin, Inc., 1966 (Russ. Transl., Mir, 1969).

[4] E. J. Woll, Jr. and W. Kohn, Phys. Rev. 126, 1693 (1962).

[5] D. N. Zubarev, Uspekhi Fiz. Nauk 71, 71 (1960) [Sov. Phys.-Uspekhi 3, 320 (1960)].

[6] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, 1962 (English Transl., Prentice-Hall, Inc., 1963).

Translated by H. H. Nickle
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