CONNECTION BETWEEN THE RENORMALIZED MASS AND THE RENORMALIZED CHARGE OF THE ELECTRON WITH THE NON-RENORMALIZED VALUES IN THE $\alpha(\alpha L)^n$ APPROXIMATION

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The connection between the renormalized mass and the renormalized charge of the electron, m and α, the non-renormalized values m_0 and α_0, and the cutoff parameter Λ is obtained with allowance for the contribution of the $\alpha(\alpha L)^n$ approximation that follows from the "senior-logarithmic" or $(\alpha L)^n$ approximation of Landau, Abrikosov, and Khalatnikov. The derived relations are used to analyze the question of the character of the divergences of the self-energy of the electron and the polarization of the vacuum, the question of the "superconducting" solution in quantum electrodynamics, and the question of the fictitious pole of the photon Green's function.

INTRODUCTION

THE question of the connection of the renormalized mass and charge of the electron m and e and the non-renormalized values m_0 and e_0 with the cutoff parameter Λ resulting from the formalism of modern quantum electrodynamics is of considerable interest in a number of respects. It is related, in particular, to the question of the true character (outside the scope of perturbation theory) of the divergences in quantum electrodynamics[1-4], the problem of the internal self-consistence of the theory[1-4], the question of the existence of "superconducting" solution in electrodynamics[5,4], the question of the possible electrodynamic spectrum of the leptonic masses, etc.

In the well known papers of Landau, Abrikosov, and Khalatnikov[6], this connection was obtained assuming a weak non-renormalized coupling

$$e_0^2 = \alpha_0 \ll 1, \quad \frac{\alpha_0}{\pi}\ln\frac{\Lambda^2}{m^2} \equiv \alpha_0 L \lesssim 1 \tag{1}$$

in the so-called logarithmic or $(\alpha_0 L)^n$ approximation, when all the terms of the non-renormalized series of the form $(\alpha_0 L)^n$ are summed and the terms $\alpha_0(\alpha_0 L)^n$ and the smaller ones are neglected[1)]. This yielded

$$\frac{m_0}{m} = \left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right)^{9/4}, \tag{2}$$

$$\frac{\alpha}{\alpha_0} = 1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}. \tag{3}$$

It can be shown[7,4], by using the renormalizability of the theory, that relations (2) and (3) are valid also outside the region (1) at arbitrarily large $\alpha_0 > 0$, provided the following conditions are satisfied:

$$e^2 = \alpha \ll 1, \quad \alpha L \lesssim 1. \tag{4}$$

Formulas (2) and (3), in particular, can be obtained[7,4] by summing all the terms of the type $(\alpha L)^n$ of the renormalized series for the self-energy of the electron and the polarization of the vacuum, neglecting the terms $\alpha(\alpha L)^n$ and smaller; no limitations on α_0 arise in this case. This means, in other words, that the relations (2) and (3) admit of an analytic continuation into the region of large α_0 if the conditions (4) are satisfied, corresponding to a transition from the $(\alpha_0 L)^n$ approximation to the $(\alpha L)^n$ approximation.

This circumstance makes it possible to prove on the basis of (2) and (3) (within the framework of the $(\alpha L)^n$ approximation) the existence of a "superconducting" solution $m = \Lambda\exp(-3\pi/2\alpha)$ in electrodynamics with $m_0 = 0$, and to demonstrate that on going outside the framework of the standard perturbation theory the divergence of the self-energy of the electron is transformed from logarithmic to linear, as a result of which the parameter Λ drops out from the expressions for the renormalization constants Z_2 and Z_3[4]. Allowance for these facts casts new light on the question of the vanishing of the charge in the unphysical pole of the photon Green's function[2,7].

As noted in[4], an urgent problem is the investigation of the stability of the indicated results of the $(\alpha L)^n$ approximation with respect to allowance for the higher-order approximations. In this paper we find and analyze the connection between m, α, m_0, α_0, and Λ with allowance for the contribution of the $\alpha(\alpha L)^n$ approximation. We shall show that in the case of sufficiently small m_0, particularly when $m_0 = 0$, this contribution turns out to be appreciable.

The problem reduces to finding in the $\alpha(\alpha L)^n$ approximation the mass operator and the constant Z_3. This problem is solved below by using the $\alpha(\alpha L)^n$ asymptotic forms, determined in[8,9], of the renormalized Green's function and vertex function. The final result is

$$\frac{m_0}{m} = \left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right)^{9/4}\left\{1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2} - \frac{3\alpha}{8\pi} + \frac{97}{96}\frac{\alpha^2}{\pi^2}\ln\frac{\Lambda^2}{m^2} + \frac{27}{16}\frac{\alpha}{\pi}\ln\left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right)\right\}, \tag{5}$$

$$\frac{\alpha}{\alpha_0} = 1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2} + \frac{\alpha}{18\pi} + \frac{3\alpha}{4\pi}\ln\left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right). \tag{6}$$

1)In the general case we denote by L all the large logarithmic parameters such as $\ln(\Lambda^2/m^2)$, $\ln(p^2/m^2)$, etc.

2. MASS OPERATOR

We are interested in the value of the mass operator on the mass shell, $\hat{p}_0 = im$, $p_0^2 = -m^2$. It is determined by the integral

$$i\Sigma^*(m, \Lambda, \alpha) = \frac{i\alpha}{4\pi^3}\int \gamma_\mu G(p_0 - k)\Gamma_\nu(p_0 - k, p_0)D_{\mu\nu}(k)\,d^4k, \tag{7}$$

where G, Γ, and D are renormalized functions. The $\alpha(\alpha L)^n$ contribution of interest to us comes from the following parts of the integral (7):

$$I = \frac{i\alpha}{4\pi^3}\int \gamma_\mu G_0(p_0 - k)\Gamma_0^\nu(p_0 - k, p_0)D_0^{\mu\nu}(k)\,d^4k, \tag{8}$$

$$I_1 = \frac{i\alpha}{4\pi^3}\int \gamma_\mu[\tilde{G}(p_0 - k)\Gamma_0^\nu(p_0 - k, p_0)D_0^{\mu\nu}(k) + G_0(p_0 - k)\tilde{\Gamma}_0^\nu(p_0 - k, p_0)D_0^{\mu\nu}(k) + G_0(p_0 - k)\Gamma_0^\nu(p_0 - k, p_0)\tilde{D}_{\mu\nu}(k)]\,d^4k; \tag{9}$$

where the zero subscript denote the values of the corresponding functions in the $(\alpha L)^n$ approximation[6,7,4] [2)], and the tilde denotes the values in the $(\alpha(\alpha L)^n$ approximation[8,9].

The integral I gives both the previously obtained $(\alpha L)^n$ contribution I_0[4]

$$I_0 = m(1 - \xi^{9/4}), \quad \xi \equiv 1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}, \tag{10}$$

and the sought $\alpha(\alpha L)^n$ contribution, which we denote by $\tilde{I}$. The quantity I_0 occurs upon "logarithmic" integration in (8), whereas to determine $\tilde{I}$ it is necessary to carry out a more accurate integration. This integration greatly simplifies when account is taken of the following remarks.

The functions G_0, Γ_0^ν, and $D_0^{\mu\nu}$ which enter in the integral (8) coincide in the region $k^2 \lesssim m^2$, with the required degree of accuracy, with the corresponding free functions, whereas in the region $k^2 \gg m^2$ they take the form[6,7,4] [3)]

$$\begin{gathered}
D_0^{\mu\nu}(k) = \frac{\eta^{-1}}{ik^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + d_l\frac{k_\mu k_\nu}{ik^4}, \quad \eta = 1 - \frac{\alpha}{3\pi}\ln\frac{k^2}{m^2},\\
G_0(p_0 - k) = [(\hat{k} - \hat{p}_0)A_0(k) + imB_0(k)]^{-1},\\
A_0(k) = \exp\left\{-\frac{\alpha}{4\pi}d_l\ln\frac{k^2}{m^2}\right\}, \quad B_0(k) = A_0(k)\eta^{9/4},\\
\Gamma_0^\nu(p_0 - k, p_0) = \gamma_\nu A_0(k) - \frac{k_\nu}{k^2}\{\hat{p}_0[A_0(k) - 1] - im[B_0(k) - 1]\}\\
-\frac{\hat{p}_0(\hat{k}\gamma_\nu - \gamma_\nu\hat{k})}{2k^2}A_0(k)(1 - \eta^{9/4})\\
(\Lambda^2 \gg k^2 \gg m^2).
\end{gathered} \tag{11}$$

In integrating the logarithmic terms (those containing $\ln(k^2/m^2)$) the $\alpha(\alpha L)^n$ contribution can come only from the region $k^2 \gg m^2$. This gives rise to integrals of the type

$$\alpha\int_m^\Lambda\left(\alpha\ln\frac{k^2}{m^2}\right)^n\left(\frac{m^2}{k^2}\right)^l\frac{d^4k}{k^4}, \quad n, l \geq 1.$$

Integration by parts readily shows that such integrals make no $\alpha(\alpha L)^n$ contribution.

Thus, in (8) it is possible to effectively replace the functions G_0, Γ_0^ν, and $D_0^{\mu\nu}$ by the corresponding free functions in the entire region of integration. As a result, the integral $\tilde{I}$ takes the form

$$\tilde{I} = \frac{i\alpha}{4\pi^3}\int \gamma_\mu[\hat{k} - \hat{p}_0 + im]^{-1}\gamma_\nu\left[\delta_{\mu\nu} + (d_l - 1)\frac{k_\mu k_\nu}{k^2}\right]\frac{d^4k}{ik^2} = \frac{3\alpha}{8\pi}m. \tag{12}$$

Let us consider now I_1. Owing to the presence of the functions with the tilde, this integral contains one α more than I, and therefore the $\alpha(\alpha L)^n$ contribution to it is made by the logarithmic integration that occurs in the region $\Lambda^2 \gg k^2 \gg m^2$. The functions $\tilde{G}$ and $\tilde{D}_{\mu\nu}$ then take the form

$$\tilde{D}_{\mu\nu}(k) = \tilde{d}(k)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{ik^2}, \tag{13}$$

$$\begin{gathered}
\tilde{G}(p_0 - k) = [(\hat{k} - \hat{p}_0)(A_0 + \tilde{A}) + im(B_0 + \tilde{B})]^{-1}\\
-[(\hat{k} - \hat{p}_0)A_0 + imB_0]^{-1} \approx -\frac{A_0^{-2}(k)}{k^2}\left\{\left[\hat{k} - \hat{p}_0 + 2\frac{\hat{k}(kp_0)}{k^2}\right]\tilde{A}(k)\right.\\
\left. + im[\tilde{B}(k) - 2\tilde{A}(k)B_0(k)A_0^{-1}(k)]\right\},
\end{gathered} \tag{14}$$

where $\tilde{d}$, $\tilde{A}$, and $\tilde{B}$ are determined respectively by formulas (8), (62), and (63) of the article[8]. The function $\tilde{\Gamma}_\nu(p_0 - k, p_0)$ is given by formulas (29) and (38) and (39) of[9]. Substituting the corresponding expression in (9) and integrating, we obtain with logarithmic accuracy, after rather cumbersome calculations, which we omit,

$$I_1 = -m\frac{\alpha}{\pi}\left[\frac{27}{16}\xi^{5/4}\ln\xi - \frac{97}{32}\xi^{9/4} + \frac{85}{32}\xi^{5/4} + \frac{3}{8}\right]. \tag{15}$$

Adding expressions (10), (12), and (15) we arrive at the following results for the self energy of the electron with allowance for the $(\alpha L)^n$ and $\alpha(\alpha L)^n$ approximations:

$$\begin{gathered}
i\Sigma^*(m, \Lambda, \alpha) = I_0 + \tilde{I} + I_1 =\\
= m\left\{1 - \xi^{5/4}\left[\xi + \frac{27\alpha}{16\pi}\ln\xi - \frac{97\alpha}{32\pi}\xi + \frac{85\alpha}{32\pi}\right]\right\}.
\end{gathered} \tag{16}$$

Taking (16) into account and the relation

$$m_0 + i\Sigma^*(m, \Lambda, \alpha) = m \tag{17}$$

we obtain the following connection between the quantities m, m_0, Λ, and α:

$$\frac{m_0}{m} = \xi^{5/4}\left[\xi + \frac{27\alpha}{16\pi}\ln\xi - \frac{97\alpha}{32\pi}\xi + \frac{85\alpha}{32\pi}\right], \tag{18}$$

where ξ is defined in (10).

3. THE CONSTANT Z_3

Let us derive now in the $\alpha(\alpha L)^n$ approximation an expression for the renormalization constant Z_3, which determines the connection between α_0 and α:

$$\alpha/\alpha_0 = Z_3(\Lambda/m, \alpha). \tag{19}$$

Z_3 is determined in terms of the polarization operator Π^* in accordance with the formula (see, for example, formulas (16.100) and (16.129) of [11])

$$Z_3 = 1 - i\frac{\delta_{\mu\nu}}{8}\frac{\partial^2}{\partial k_\mu\partial k_\nu}\Pi^*(k)\big|_{k=0} \equiv 1 + C. \tag{20}$$

It is easy to show that in the $\alpha(\alpha L)^n$ approximation the following three integrals contribute to the quantity C:

$$C = C_1 + C_2 + C_3, \tag{21}$$

$$\begin{gathered}
C_1 = \frac{i}{12}\frac{\alpha}{4\pi^3}\mathrm{Sp}\int T_\mu(p)T_\nu(p)T_\nu(p)T_\mu(p)\,d^4p,\\
C_2 = \frac{i}{12}\left(\frac{\alpha}{4\pi^3}\right)^2\mathrm{Sp}\int T_\nu(p)T_\mu(p)T_\sigma(p,k)T_\mu(k)T_\nu(k)T_\tau(k,p)\cdot
\end{gathered} \tag{22}$$

[2)] We write the vector indices of these functions in the form of superscripts only for convenience, making no distinction between superior and inferior indices. The metric and the γ matrix are the same as in [10].

[3)] We use a gauge in which d_l is an arbitrary number.

$$\times D_{\sigma\tau}(p-k)\, d^4p\, d^4k, \tag{23}$$

$$C_3 = \frac{i}{12}\left(\frac{\alpha}{4\pi^3}\right)^2 \mathrm{Sp}\int T_\nu(p) T_\nu(p) T_\sigma(p,k) T_\mu(k) T_\mu(k) T_\tau(k,p) \times D_{\sigma\tau}(p-k)\, d^4p\, d^4k, \tag{24}$$

where we have introduced for brevity

$$T_\mu(p,k) \equiv \Gamma_\mu(p,k) G(k), \quad T_\mu(p) \equiv T_\mu(p,p). \tag{25}$$

We consider first the integral C_1. It is easy to estimate that the terms proportional to m in (22) in the functions

$$G(p) = A^{-1}(p)\,[-\hat{p} + imB(p)A^{-1}(p)]^{-1}$$

and

$$\Gamma_\mu(p,p) = -\frac{\partial}{\partial p_\mu} G^{-1}(p) = \frac{\partial}{\partial p_\mu}[\hat{p}A(p) - imB(p)]$$

make an $\alpha(\alpha L)^n$ contribution only in the region $p^2 \lesssim m^2$, in which it is possible to replace $B(p)$ and $A(p)$ by unity with sufficient accuracy. Therefore it is possible to write effectively

$$G(p) \approx A^{-1}(p)\,(im - \hat{p})^{-1}, \tag{26}$$

$$\Gamma_\mu(p,p) \approx \frac{\partial}{\partial p_\mu}[\hat{p}A(p)] = \gamma_\mu A(p) + \hat{p}\frac{\partial}{\partial p_\mu}A(p) \approx \approx A(p)\left(\gamma_\mu - \frac{\alpha}{2\pi} d_l \frac{\hat{p}p_\mu}{p^2}\right), \tag{27}$$

$$T_\mu(p) = \left(\gamma_\mu - \frac{\alpha}{2\pi} d_l \frac{\hat{p}p_\mu}{p^2}\right)(im - \hat{p})^{-1}. \tag{28}$$

Substituting (28) in (22) and integrating, we readily obtain

$$C_1 = -\frac{\alpha}{3\pi}\left[\ln\frac{\Lambda^2}{m^2} - \frac{1}{6} + \frac{\alpha}{\pi} d_l \ln\frac{\Lambda^2}{m^2}\right]. \tag{29}$$

In the integrals C_2 and C_3 it suffices to take all the functions in the $(\alpha L)^n$ approximation, since one of the integrations has obviously a nonlogarithmic character, while the second should be logarithmic. In these integrals, only the region of large momenta ($p^2, k^2 \gg m^2$) is significant, for none of the integrations is logarithmic in the regions where one of the momenta is small. This makes it possible to neglect everywhere the mass m compared with the integration momenta. We can consequently write the functions G and Γ_μ in (23) and (24) in the following form[4,9]:

$$G_0(p) = -\frac{1}{\hat{p}A_0(p)}, \quad \Gamma_0^\mu(p,p) = \gamma_\mu A_0(p), \quad T_\mu(p,p) = -\gamma_\mu\frac{1}{\hat{p}}, \quad p^2 \gg m^2. \tag{30}$$

$$\Gamma_0^\mu(p,k) = \gamma_\mu A_0(p) + \frac{p_\mu\hat{k}}{p^2}[A_0(p) - A_0(k)] + + \frac{\hat{k}(\hat{p}\gamma_\mu - \gamma_\mu\hat{p})}{2p^2} A_0(p)\left(1 - \frac{\xi^{3/4}}{\eta^{3/4}}\right), \quad p^2 \gg k^2 \gg m^2, \tag{31}$$

$$\Gamma_0^\mu(p,k) = \gamma_\mu A_0(k) + \frac{k_\mu\hat{p}}{k^2}[A_0(k) - A_0(p)] + + \frac{(\gamma_\mu\hat{k} - \hat{k}\gamma_\mu)\hat{p}}{2k^2} A_0(k)\left(1 - \frac{\eta^{3/4}}{\xi^{3/4}}\right), \quad k^2 \gg p^2 \gg m^2. \tag{32}$$

$$D_0^{\sigma\tau}(p-k) = \frac{1}{i(p-k)^2}\left[\delta_{\sigma\tau} - \frac{(p-k)_\sigma(p-k)_\tau}{(p-k)^2}\right] \times \left[1 - \frac{\alpha}{3\pi}\ln\frac{(p-k)^2}{m^2}\right]^{-1} + d_l\frac{(p-k)_\sigma(p-k)_\tau}{i(p-k)^4}. \tag{33}$$

Let us arbitrarily integrate first with respect to k_μ. In integrating the terms containing $\ln k^2$ or $\ln(p-k)^2$, the required $\alpha(\alpha L)^n$ contribution is made, as can be readily estimated, by the regions $k^2 \gg p^2$ and $k^2 \ll p^2$. As shown in[8,9], the integration of the logarithmic terms in these regions reduces effectively to the substitutions $\ln k^2, \ln(p-k)^2 \to \ln p^2$ under the integral sign, making it possible to simplify greatly the corresponding functions, and in particular to make the substitutions

$$T_\mu(p,k) \to -\gamma_\mu\frac{1}{\hat{k}}, \quad T_\mu(k,p) \to -\gamma_\mu\frac{1}{\hat{p}}. \tag{34}$$

Substituting (30), (34), and (33) with the replacements $\ln(p-k)^2 \to \ln p^2$ in (23) and (24), we obtain relatively simple integrals, the calculation of which yields

$$C_2 + C_3 = \frac{\alpha}{\pi}\left(\frac{\alpha}{3\pi} d_l \ln\frac{\Lambda^2}{m^2} + \frac{3}{4}\ln\xi\right). \tag{35}$$

Adding (29) with (35) and substituting in (20), we obtain the following expression for Z_3 with allowance for the $(\alpha L)^n$ and $\alpha(\alpha L)^n$ approximations:

$$Z_3 = \xi + \frac{3\alpha}{4\pi}\ln\xi + \frac{\alpha}{18\pi}. \tag{36}$$

Taking into account the definition of ξ in (10) and (19), we arrive at relation (6).

4. ANALYSIS OF RESULTS

Relation (5) (or (18)) is an equation defining the function $m(m_0, \Lambda, \alpha)$. This equation does not admit of an explicit solution in analytic form; its analysis, however, is facilitated by regarding the ration m_0/m as a parameter, which according to (5) lies in the range $0 \le m_0/m \le 1$.

Under the condition

$$(m_0/m)^{4/9} \gg \alpha\ln(m/m_0) \tag{37}$$

the term $\alpha\ln\xi$ is small compared with ξ, and the solution can be sought in the form of iterations. With good accuracy, it is given by the formula

$$m = \Lambda\exp\left\{-\frac{3\pi}{2\alpha}\left[1 - \left(1 + \frac{97\alpha}{72\pi}\right)\left(\frac{m_0}{m}\right)^{4/9}\right] - \frac{1}{2}\ln\frac{m_0}{m} - \frac{85}{48} + O(\alpha)\right\}. \tag{38}$$

Expression (36) for Z_3 with allowance for (38) takes the form

$$Z_3 = \left(1 + \frac{97\alpha}{72\pi}\right)\left(\frac{m_0}{m}\right)^{4/9} - \frac{81\alpha}{72\pi}. \tag{39}$$

This expression in the region of its applicability (37) satisfies the necessary inequalities

$$0 \leqslant Z_3 \leqslant 1. \tag{40}$$

At a sufficiently small value of the ratio m_0/m, when condition (37) is violated, the role of the term $\alpha\ln\xi$ in (18) becomes significant. We shall consider below the case of greatest physical interest, $m_0 = 0$. In this case we arrive at the equation

$$\xi + \frac{27\alpha}{16\pi}\ln\xi + \frac{85\alpha}{32\pi} - \frac{97\alpha}{32\pi}\xi = 0. \tag{41}$$

When $\alpha \ll 1$ it has a solution $\xi(\alpha)$ lying in the interval $0 < \xi(\alpha) \ll 1$. Writing this solution in the form

$$\xi(\alpha) \equiv \frac{\alpha}{\pi}\zeta(\alpha), \tag{42}$$

and taking (10) into account, we arrive at the relation

$$1 - \frac{2\alpha}{3\pi}\ln\frac{\Lambda}{m} = \frac{\alpha}{\pi}\zeta(\alpha),$$

from which we get the following expression for the mass:

$$m(0, \Lambda, \alpha) = \Lambda \exp\left\{-\frac{3\pi}{2\alpha} + \frac{3}{2}\zeta(\alpha)\right\}. \quad (43)$$

The function $\zeta(\alpha)$ is defined by the equation

$$\left(1 - \frac{97\alpha}{32\pi}\right)\zeta + \frac{27}{16}\ln\zeta = \frac{27}{16}\ln\frac{\pi}{\alpha} - \frac{85}{32}, \quad (44)$$

from which it follows that $\zeta(\alpha)$ has a logarithmic singularity at the point $\alpha = 0$. Thus, allowance for the $\alpha(\alpha L)^n$ approximation greatly influences the solution $m(0, \Lambda, \alpha)$ for the renormalized mass in electrodynamics with $m_0 = 0$, but retains its "superconducting" character. This raises the question of the influence exerted on the "superconducting" solution by all higher $\alpha^k(\alpha L)^n$ approximations. This question will be considered in a separate article.

Substituting (43) in (36), and taking (44) into account, we get

$$Z_3 = \frac{\alpha}{\pi}\left(\frac{5}{9}\zeta - \frac{9}{8}\right). \quad (45)$$

Taking (44) and the condition $\alpha \ll 1$ into account, we can easily verify that (45) is satisfied by the inequalities in (40).

Let us consider in conclusion the question of the unphysical pole of the photon Green's function. Taking the $(\alpha L)^n$ and $\alpha(\alpha L)^n$ approximations into account, the transverse part of the renormalized photon Green's function can be written in the form[8]

$$D_{\mu\nu}(k) = \frac{d(k^2)}{ik^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right),$$
$$d^{-1}(k^2) = \eta + \frac{3\alpha}{4\pi}\ln\eta + \frac{5\alpha}{9\pi}, \quad (46)$$
$$\eta = 1 - \frac{\alpha}{3\pi}\ln\frac{k^2}{m^2}, \quad \Lambda^2 \gg k^2 \gg m^2.$$

Using the relation

$$\eta = \xi + \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{k^2}$$

and (36), we can transform (46) into

$$d^{-1}(k^2) = Z_3 + \frac{\alpha}{2\pi}\left[\frac{2}{3}\ln\frac{\Lambda^2}{k^2} + \frac{3}{2}\ln\left(1 + \frac{\alpha}{3\pi\xi}\ln\frac{\Lambda^2}{k^2}\right) + 1\right]. \quad (47)$$

From this and from (40) it follows that

$$d^{-1}(k^2) > Z_3 > 0, \quad \Lambda^2 \gtrsim k^2 \gg m^2 \quad (48)$$

and consequently no "unphysical pole" arises in the region of applicability of the formulas in question.

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