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TEMPERATURE DEPENDENCE OF THE COEFFICIENT OF SPECULAR REFLECTION OF CONDUCTION ELECTRONS FROM THE SURFACE OF ZINC OR CADMIUM

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Measurements were made of the dependence of the resistance on the temperature $\rho(T)$ for thin (filamentary whiskers 1.3—3.5 μ thick) and bulky samples of zinc and cadmium. The appreciable difference between the temperature-dependent parts of the resistance of thin and bulky samples is attributed to the dependence of the coefficient of specular reflection p of the electrons (and accordingly of the diffuseness coefficient $q = 1 - p$) on the temperature. The measurement results are used to find the temperature dependence of the coefficients p and q. It is found that the q(T) plot has the form of a curve with saturation: when $T > T_{\rm dif}$ the reflection of the electrons from the surface becomes completely diffuse ($q = 1$, $p = 0$), and when $T \ll T_{\rm dif}$ the result is $q \propto T^{3.7 \pm 0.3}$. The value of $T_{\rm dif}$ for zinc is ~30°K and for cadmium ~20°K. For a complete description of the static size effect, it is proposed to represent the resistance connected with scattering from the surface in the form of a sum of two parts: $\rho^{\rm sur} = \rho_0^{\rm sur} + \rho_{\rm i}^{\rm sur}$, where $\rho_0^{\rm sur}$ is a constant and $\rho_{\rm i}^{\rm sur}$ is a function of the temperature. The resistance $\rho^{\rm sur}_{\rm 0dif}$ is determined by the value p_0 of the specular-reflection coefficient at $T = 0$°K, while $\rho_{\rm i}^{\rm sur} = (\rho_{\rm dif}^{\rm sur} - \rho_0^{\rm sur})F(T)$. With increasing temperature, the function F(T) changes from zero to unity; $\rho_{\rm dif}^{\rm sur}$ is the surface resistance in the case when the coefficient p_0 is equal to zero.

INTRODUCTION

IN the study of the influence of boundaries on the kinetic properties of metallic conductors, the correlation between the distribution functions of the electrons incident on the boundary and those scattered from it is taken into account by assuming that a fraction p of the electrons is reflected specularly, and the rest diffusely. The quantity p is called the specularity coefficient, and its complement $q = 1 - p$ is called the diffuseness coefficient. Such an oversimplified approach to the problem makes it possible to describe the interaction of the electrons with the surface, averaged both over the surface itself and over the angles of incidence. In the case of total diffuse reflection ($p = 0$, $q = 1$) the result in first approximation reduces to the fact that in a thin wire the mean free path connected with the scattering by the surface is equal to the sample diameter; in total specular reflection ($p = 1$, $q = 0$), the kinetic coefficients should not depend on the thickness of the sample[1)].

The specularity coefficient was introduced by Nordheim[2] who postulated the existence of a temperature dependence of p. Since Nordheim's work the question of the temperature dependence of the coefficient p has not been considered to this very day. Moreover, the majority of theoretical and experimental investigations were made under the assumption that there is no total diffuse reflection at any temperature. This was justified, since samples of "good" metals are expected to exhibit specular reflection from the surface only if the dimensions of the inhomogeneities are comparable with the interatomic distances.

Such perfect surfaces are possessed by filamentary single crystals of metals—whiskers. A comparison of the experimental dependences of the resistance on the thickness for whiskers and thin samples obtained by etching shows that for whiskers of copper, zinc, and cadmium at T = 4.2°K the coefficient is $p_{4.2} \approx 0.5$[3-5] [2)].

The nonzero value of p and the results of investigations of singularities in the temperature dependence of the resistance of zinc and cadmium whiskers have raised again the question of the existence of a temperature dependence of the specularity coefficient[5,6]. Such a dependence would explain convincingly the great difference between the temperature dependence of the resistance of bulky and of thin samples, observed in different metals in a number of investigations[4-9].

A temperature dependence of the coefficient p means that the surface of the sample is made non-ideal not only by etch pits, growth steps, emergence of dislocations, and other static defects, but also by the thermal motion of the atoms near the surface. p should reach its maximum value p_0, lying in the range $0 \leq p_0 \leq 1$, at T = 0°K. With increasing temperature, as the amplitude of the thermal oscillations of the atoms increases, p should decrease. At a certain temperature $T_{\rm dif}$ each act of scattering from the surface leads to a change of the electron momentum by a value comparable with this momentum. It can also be assumed that $T_{\rm dif}$ deter-

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[1)] We do not consider here the Price's conclusions [1], according to which, under certain conditions, the electric conductivity depends on the sample thickness even when p = 1.

[2)] To determine p it is necessary to know the product $\rho^\infty \lambda^\infty = A$, which is a constant of the metal (ρ^∞ and λ^∞ are respectively the resistance and the mean free path of the electrons in a bulky sample at the same temperature). The quantity A, in turn, is determined experimentally under the assumption that the electrons are reflected from the surfaces of the investigated samples in pure diffuse manner. Since this is not realistic, the true value of $p_{4.2}$ may be larger than the indicated one.

mines the temperature at which the collision with the surface leads to a total loss of the energy acquired by the electron in the electric field. Above T_{dif}, the scattering is purely diffuse, regardless of the physical and impurity state of the surface. On the basis of the physics of the phenomenon, we can expect T_{dif} of "good" metals to be comparable with the Debye temperature Θ_D.

The p(T) dependence must undoubtedly be taken into account in investigations of the temperature variation of the resistance of thin samples for which $p_0 \neq 0$. For wires this can be done in first approximation with the aid of a formula that is valid for any value of p[10]:

$$\rho^d = \rho^\infty + \frac{A}{d} f(p), \tag{1}$$

where $A = \rho^\infty \lambda^\infty$ and the form of the function f(p) depends on the relation between λ^∞ and d: $f(p) = (3/4)(1 - p)$ when $\lambda^\infty \ll d$ and $f(p) = (1 - p)/(1 + p)$ when $\lambda^\infty \gg d$.

In the case of pure diffuse reflection determined only by the static defects ($p_0 = 0$), we have f = 1 (or 3/4) at any temperature. This makes the temperature parts of the resistance of thin and bulky samples equal: $\rho_i^d(T) = \rho_i^\infty(T)$. On the other hand, if $p_0 = 1$, then the maximum possible difference between the quantities $\rho_i^d(T)$ and $\rho_i^\infty(T)$ should be observed. Then the difference $\Delta\rho_i^d = \rho_i^d - \rho_i^\infty$ reaches a maximum value at $T = T_{dif}$. Above T_{dif}, the difference $\Delta\rho_i^d$ should remain constant if the condition $\lambda^\infty \ll d$ is attained at $T = T_{dif}$. On the other hand, if the sample thickness is such that the condition $\lambda^\infty \gg d$ is nevertheless satisfied at $T = T_{dif}$, then the difference $\Delta\rho_i^d$ should decrease from the maximum value A/d to (3/4)A/d.

The foregoing considerations were the basis of an experimental determination of the temperature dependences of p and q.

SAMPLES AND MEASUREMENT PROCEDURES

The zinc and cadmium whiskers were grown from the vapor phase by the method described in the paper of Coleman and Sears[11]. The chemical purity of the initial materials for the growing of the whiskers and of the bulky samples was characterized by a resistance ratio[3)] $\rho(293°K)/\rho(4.2°K)$ equal to 10,000 and 7000 for zinc and cadmium, respectively. At T = 4.2°K, these quantities correspond to electron mean free paths $\lambda_{Zn}^\infty \approx 300\ \mu$ and $\lambda_{Cd}^\infty \approx 200\ \mu$.

We used for the measurements only filamentary whiskers, which have a much larger size effect than ribbon whiskers grown in the same ampoules as the filamentary whiskers. No special determination of the orientation of the sample axes was carried out. According to the published data[12-14], filamentary whiskers of zinc and cadmium grow in the directions of the axes $[11\bar{2}3]$, $[11\bar{2}2]$, and $[11\bar{2}1]$. This corresponds approximately to angles ϑ equal to 30, 40, and 60° between the axis [001] and the axes of the samples. The whisker thickness was defined as $d = \sqrt{S}$, where S is the cross section area of the whisker determined from the room-temperature resistance, from the distance between the potential contacts of the samples, and from the resistivity of bulk samples averaged over the three angles indicated above. The thickness of the measured whiskers ranged from 1.3 to 3.5 μ. The data on the samples are given in the table.

The electric mounting of the whiskers were made by the "clamping contact" method: the whisker was placed on four copper contacts covered with a thin layer of indium and secured on a Fiberglas plate. Another special plate pressed the whiskers to the contacts[15]. Experience with such a mounting found it to be very convenient and to guarantee electric and mechanical reliability of the contact.

To obtain temperatures in the interval 4.2–293°K, we used the temperature gradient inside a Dewar with liquid helium. The mounted sample was placed inside a thick-wall chamber of copper. The chamber, fastened to the end of a monel rod, could be moved inside the Dewar. The temperature corresponding to a certain position of the chamber was measured with a thermocouple made of gold with 0.03° iron and chromel. At low temperatures, its sensitivity was approximately 15 μV/deg. The thermocouple was calibrated with a gas thermometer. The accuracy with which the temperature was determined was not worse than 0.1 deg.

MEASUREMENT RESULTS

To find p = p(T) it is necessary to separate the function f(p) from expression (1). To this end it is necessary to know the product $\rho^\infty \lambda^\infty = A$ for a given orientation of the whisker axis and to measure the temperature dependence of the resistivities ρ^d and ρ^∞. The values of $A_\parallel$ and $A_\perp$ were taken by us from Aleksandrov's paper[16], where it was found that $A_\parallel = 0.9$ and $A_\perp = 2.3$ for zinc and $A_\parallel = 1.6$ and $A_\perp = 2.4$ for cadmium (the values are given in units of 10^{-11} ohm-cm^2; the indices correspond to the directions parallel and perpendicular to the [0001] axis. A_ϑ for an arbitrary angle ϑ was calculated from these values. As a result we obtained $A_{30°} = 1.24$, $A_{40°} = 1.47$ and $A_{60°} = 1.94$ for zinc and $A_{30°} = 1.8$, $A_{40°} = 1.9$ and $A_{60°} = 2.1$ for cadmium. The ideal resistivity $\rho_{i\vartheta}^\infty(T)$ for a direction making an angle ϑ with the [0001] axis was also obtained by recalculating the resistivities $\rho_{i\parallel}^\infty(T)$ and $\rho_{i\perp}^\infty(T)$, which were determined from measurements of the total resistance R(T) of bulky samples:

$$\rho_i^\infty(T) = \frac{R^\infty(T) - R^\infty(0)}{R^\infty(293)} \rho^\infty(293).$$

The residual resistance R(0) was obtained by extrapolating R(T) to 0°K. For the quantity $\rho^\infty(293)$ we used the following values in units of 10^{-6} ohm-cm[16]: for zinc $\rho_\parallel^\infty = 6.15$ and $\rho_\perp^\infty = 5.83$ and for cadmium $\rho_\parallel^\infty = 8.45$ and $\rho_\perp^\infty = 7.1$. Analogously we obtained

$$\rho_{i\vartheta}^d(T) = \frac{R^d(T) - R^d(0)}{R^d(293)} \rho_\vartheta^d(293).$$

At room temperature, the residual resistance due to the chemical impurities was disregarded, since $\rho^\infty(0) \approx 10^{-4} \rho(293)$. The resistance $\rho_\vartheta^d(293)$ was calculated under the assumption that total diffuse reflection takes

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[3)] We shall henceforth omit the symbol °K.

	Cd-1	Cd-2	Cd-3	Cd-4	Cd-5	Cd-5*	Zn-1	Zn-2	Zn-2*	Zn-3	Zn-4	Zn-4**
Orientation	$[11\bar{2}3]$?	$[11\bar{2}2]$	$[11\bar{2}2]$	$[11\bar{2}3]$	$[11\bar{2}1]$		$[11\bar{2}2]$	$[11\bar{2}2]$		$[11\bar{2}1]$	$[11\bar{2}1]$	
Angle ϑ	30°	40°	40°	30°	60°		40°	40°		60°	60°?	
Thickness d, μ	1,45	1,6	1,85	2,2	2,6	2,6	1,3	2,05	2,0	2,6	3,35	3.2
$\rho^d(4.2°K) \times 10^{-8}$, ohm-cm	4,8	3,6	3,0	3,5	4,4	5,0	3,5	2,2	2,5	1,8	1,6	2.8

*Samples etched once.
**Samples etched twice.

place at room temperature, using the expression $\rho_\vartheta^d(293) = \rho_\vartheta^\infty(283) + (3/4)A_\vartheta/d$.

We note here two factors of importance in the investigation of the function p = p(T). First, the size effect at room temperature cannot be neglected, i.e., it is impossible to use $\rho^\infty(293)$ in lieu of $\rho^d(293)$ to determine $\rho_i^d(T)$, since this distorts greatly the true course of the difference $\rho_i^d(T) - \rho_i^\infty(T)$ near room temperature. Second, the plots of $\rho_i^d(T)$ and $\rho_i^\infty(T)$ should correspond to the same angle ϑ. This is connected with the fact that, as has become clear during the course of the measurements, the resistance anisotropy $k = \rho_{i\parallel}^\infty/\rho_{i\perp}^\infty$ depends significantly on the temperature. Fortunately, this has made it possible to determine the whisker orientation (more accurately, the angle ϑ) within the limits of the measurement accuracy. This method will be described below.

At first the experimental points for $R^d(T)$ were used to obtain the values of $\rho_{i,av}^d(T)$, which is the first approximation to the true $\rho_i^d(T)$ dependence. To this end, the value A_{av} averaged over three angles (ϑ = 30, 40, and 60°) was introduced: $A_{av} = 1.6 \times 10^{-11}$ ohm-cm² for zinc and 1.95×10^{-11} ohm-cm³ for cadmium. Then

$$\rho_{i,\,\mathrm{av}}^{d}(T) = \frac{R^d(T) - R^d(0)}{R^d(293)}\left[\rho_\perp^\infty(293) + \frac{3}{4}\frac{A_{\mathrm{av}}}{d}\right]$$

$$= \frac{k_\vartheta(T)\rho_{i\perp}^\infty(T) + A_\vartheta\Delta f(p)/d}{k_\vartheta(293)\rho_\perp^\infty(293) + {}^3/_4A_\vartheta/d}\left[\rho_\perp^\infty(293) + \frac{3}{4}\frac{A_{\mathrm{av}}}{d}\right]$$

$$= B\frac{k_\vartheta(T)\rho_{i\perp}^\infty(T) + A_\vartheta\Delta f(p)/d}{k_\vartheta(293)},$$

where

$$k_\vartheta(T) = \rho_{i\vartheta}^\infty(T)\,/\,\rho_{i\perp}(T), \quad \Delta f(p) = f(p) - f(p_c).$$

Since the anisotropy k at 293°K is small (k_{Zn}(293 = 1.05, $k_{\mathrm{Cd}}(293) = 1.19$[16]), and the values of A_ϑ do not differ by more than 25% from A_{av}, it can be assumed with the same accuracy, 25%, that the constant B is equal to unity.

We then found the difference between $\rho_{i.av}^d$ and $\rho_{i\perp}^\infty$:

$$\Delta\rho_{i\,\mathrm{av}}^{\infty}(T) = \rho_{i\,\mathrm{av}}^{d}(T) - \rho_{i\perp}^\infty(T) \approx \rho_{i\perp}^\infty(T)\left[\frac{k_\vartheta(T)}{k_\vartheta(293)} - 1\right] + \frac{A_\vartheta\Delta f(p)}{dk_\vartheta(293)}. \tag{2}$$

If k does not depend on the temperature, then the first term in expression (2) vanishes. Then the true course of p(T) is determined with sufficient accuracy by the difference $\Delta\rho_{i.av}^d(T)$. However, the measurements have shown that the resistance anisotropy in Zn and Cd depends strongly on the temperature. To be able to take this dependence into account in expression (2), we measured the resistances $\rho_{i\parallel}^\infty(T)$ and $\rho_{i\perp}^\infty(T)$ in the temperature interval 4.2–293°K. The measurement results were reduced by means of the formula

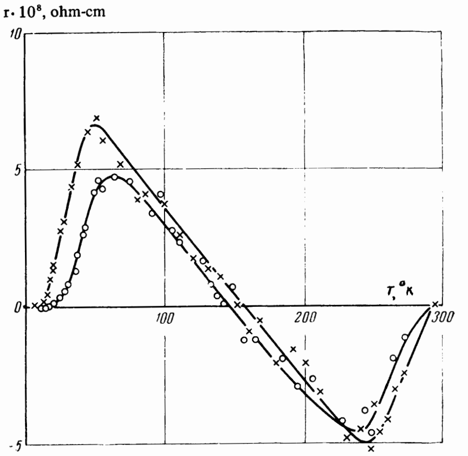

FIG. 1. Temperature dependence of the difference between $\rho_{i\parallel}^\infty(T)$ and $\rho_{i\perp}^\infty(T)$, due only to the change of the anisotropy coefficient k, as calculated from Eq. (3): X—for cadmium, O—for zince; $r = \rho_{i\perp}^\infty(T)[k(T)/k(293)-1]$.

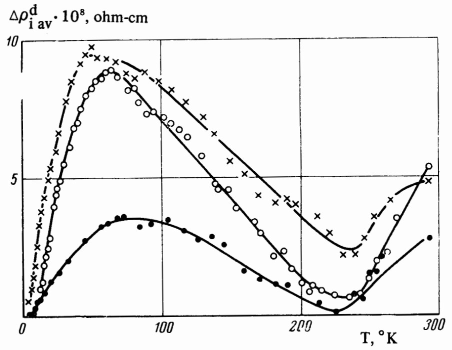

FIG. 2. Temperature dependence of the difference between the resistivity of a thin sample (for the average angle ϑ_{av}) and the resistivity of a bulky sample (for ϑ = 90°), calculated from Eq. (2): X—Cd-2, O—Zn-1, O—Zn-3.

$$\frac{\rho_{i\parallel}^\infty(T)}{\rho_\parallel^\infty(293)}\rho_\perp^\infty(293) - \rho_{i\perp}^\infty(T) = \rho_{i\perp}^\infty(T)\left[\frac{k(T)}{k(293)} - 1\right]. \tag{3}$$

This expression represents that part of the difference between the resistances $\rho_{i\parallel}^\infty(T)$ and $\rho_{i\perp}^\infty(T)$, which is connected with the temperature dependence of the anisotropy k. The results of the reduction are shown in Fig. 1. The distinctive temperature dependence of the difference (3) (the curve shows clearly a maximum and a minimum) has made it possible to propose a method of determining the angles ϑ for the whisker axis. It is based on the assumption that above 20–25°K the quantity Δf(p) is monotonic and depends little on the temperature. This assumption is justified, in particular,

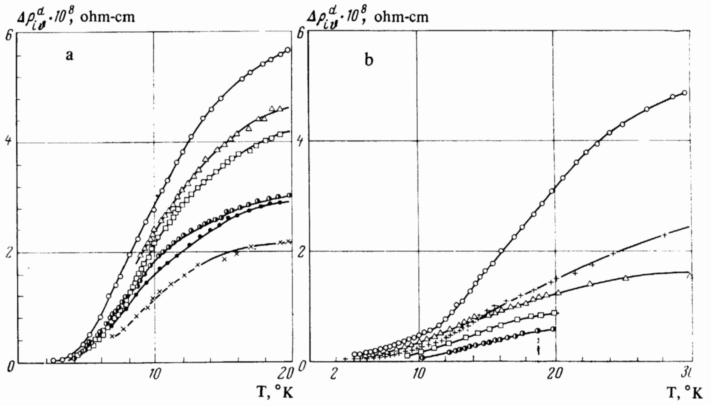

FIG. 3. Temperature dependence of the difference between the resistivities of thin and bulky samples of metal, determined from (4), for the same angle ϑ: a—cadmium, points: ○—Cd-1, △—Cd-2, □—Cd-3, ◑—Cd-4, ●—Cd-5, X—Cd-5 after the first etching; 6—zinc, points: ○—Zn-1, +—Zn-2, △—Zn-3, □—Zn-4, ◑—Zn-4 after second etching.

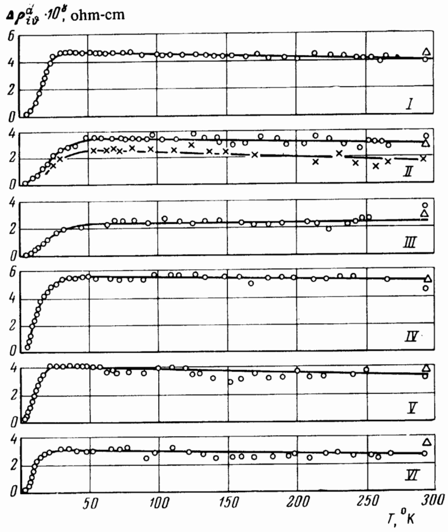

FIG. 4. Temperature dependence of the difference between the resistivities of thin and bulky samples, calculated from (4) for zinc and cadmium. Samples: I—Zn-1, II—Zn-2 (○—prior to etching, X—after the first etching), III—Zn-3, IV—Cd-2, V—Cd-3, VI—Cd-4. Points △—calculated limiting values of the difference $\rho_{i\vartheta}{}^d-\rho_{i\vartheta}{}^\infty$.

by the fact that up to 20—25°K the influence of the temperature dependence of the resistance anisotropy is small, whereas the difference $\Delta\rho^d_{\mathrm{i.av}}(T)$ of the measured whiskers reaches approximately half its maximum value.

In this case, the principal role is played in (2) by the first term, which can be obtained for the three angles $\vartheta = 30$, 40, and 60° of interest to us by recalculating the results shown in Fig. 1. In such a recalculation, naturally, the "wave-like" character of the curves of Fig. 1 remains unchanged. Therefore the difference between the maximum (at T = 50—60°K) and the minimum (at T = 230—240°K) values of the experimental curve $\Delta\rho^d_{\mathrm{i.av}}(T)$ makes it possible to estimate the angle ϑ.

Figure 2 shows measurement results reduced in accordance with expression (2). From a comparison of the curves of this figure with those of Fig. 1 we see that the temperature variation of the resistance anisotropy has little influence up to 20—25°K, but exerts a decisive role above these temperatures.

After estimating the angle ϑ from the $\Delta\rho^d_{\mathrm{i.av}}(T)$ curves, the experimental points were reduced again, yielding values for

$$\rho_{i\vartheta}{}^d(T)=\frac{R^d(T)-R^d(0)}{R^d(293)}\left[\rho_\vartheta{}^\infty(293)+\frac{3}{4}\frac{A_\vartheta}{d}\right].$$

Finally, the difference

$$\Delta\rho_{i\vartheta}{}^d(T)=\rho_{i\vartheta}{}^d(T)-\rho_{i\vartheta}{}^\infty(T)=\frac{A_\vartheta}{d}\Delta f(p), \tag{4}$$

was determined by using for $\rho^\infty_{i\vartheta}(T)$ the curves obtained by recalculating the $\rho^\infty_{i\parallel}(T)$ and $\rho^\infty_{i\perp}(T)$ curves, obtained in turn by graphically averaging the experimental points. The accuracy with which the values of $\Delta\rho^d_{i\vartheta}(T)$ were determined was approximately 10% at temperatures above 50°K and 1—5% below this temperature.

Figures 3 and 4 show the plots of $\Delta\rho^d_{i\vartheta}(T)$ obtained in this manner for zinc and cadmium.

According to the expressed point of view concerning the nature of the difference between the temperature variations of the resistance of thin and bulky samples, one should expect $\Delta\rho^d_{i\vartheta}(T)$ to decrease with decreasing coefficient of specular reflection p_0. In order to verify this, an attempt was made to decrease the coefficient p_0 of zinc and cadmium whiskers by etching the surface. The etching took place in hydrochloric acid vapor without dismounting the sample, with continuous observation of the state of the surface with a microscope. The first etching was carried out prior to the appearance of a dull deposit on the greater part of the whisker surface. No measurable thinning of the whisker was observed in this case. After a second longer etching, the average whisker thickness decreased approximately 10%.

The first etching was always accompanied by an increase of the resistance at T = 4.2°K (see the table). This, however, cannot be interpreted uniquely as favoring the decrease of p_0, since it may be connected with the appearance of additional physical defects in the sample. To the contrary, the temperature part of the resistance $\rho^d_i(T)$ is not sensitive to physical defects of the sample. The first etching caused a noticeable decrease of this part. The second and succeeding etchings, which could sometimes be performed on the same samples, no longer led to noticeable changes of the resistances. The influence of the etching can be seen in Figs. 3 and 4. The result of etching is equivalent in our case to a decrease of the specularity coefficient by an approximate factor of 2 (for the etched samples p_0 is equal to 0.35—0.20).

DISCUSSION OF RESULTS

The plots of $\Delta\rho^{d}_{i\vartheta}(T)$ shown in Figs. 3 and 4 have the same form. This indicates that the experimental data were reduced correctly. These curves are characterized by a sharp rise up to a temperature 15 to 20°K, followed by saturation. The latter should mean that the reflection of the conduction electrons from the boundaries has become completely diffuse. According to the experimental curves, it can be assumed that $T_{\mathrm{dif}} \approx 20-30°\mathrm{K}$ for cadmium and $T_{\mathrm{dif}} \approx 30-40°\mathrm{K}$ for zinc. Thus, above these temperatures, the values of $A = \rho^{\infty}\lambda^{\infty}$ for Zn and Cd, determined from the size effect, should not depend on the surface quality of the samples and have a maximum value.

The opinion expressed at the beginning of the article concerning the nature of the difference between the behavior of the $\rho^{d}_{i}(T)$ and $\rho^{\infty}_{i}(T)$ curves has two consequences that admit of experimental verification. First, at high temperatures ($T > T_{\mathrm{dif}}$), when $p = 0$, it is possible to calculate the limit to which the $\Delta\rho^{d}_{i\vartheta}(T)$ curves should tend. According to (4), it should equal $(A_{\vartheta}/d)\,[3/4 - (1 - p_0)/(1 + p_0)]$. For the investigated samples, this limiting value is shown in Fig. 4 (we used in the calculation the experimental values of A[16] and $p_0 = 0.5$). It can be noted that within the limits of the measurement accuracy, good agreement was observed between experiment and calculation. Second, the decrease of the value of p_0 should lead to a decrease of the values of $\Delta\rho^{d}_{i}(T)$ in the entire temperature range. This conclusion was also confirmed experimentally. Thus, the assumption that the coefficient of specular reflection depends on the temperature makes it possible to explain well both qualitatively and quantitatively the features of the temperature variation of the resistance of thin metal samples.

When plotting $p = p(T)$ and $q = q(T)$ from the experimental data, it is necessary to use two expressions for $f(p)$, namely $f(p) = (1 - p)/(1 + p)$ when $\lambda^{\infty} \gg d$ and $f(p) = (3/4)(1 - p)$ when $\lambda^{\infty} \ll d$. In our case such a procedure was made difficult by the fact that the transition region $\lambda^{\infty} \approx d$ occurs at temperatures close to T_{dif}. The use of the first expression in the entire temperature range from 4°K to T_{dif} has made it necessary to alter noticeably the experimental values of p_0 or A. At the same time, the use of the second relation gave very good quantitative agreement in the region $T \approx T_{\mathrm{dif}}$. At a temperature $T = (1/2)T_{\mathrm{dif}}$, both expressions led to identical values of p. The first expression was therefore used to reduce the results for $f(p)$ up to temperatures $T \approx (1/2)T_{\mathrm{dif}}$, and the second expression was used in the region $T > (1/2)T_{\mathrm{dif}}$.

In the region $T < (1/2)T_{\mathrm{dif}}$, where the results for $\Delta\rho^{d}_{i}(T)$ do not depend on the change of the anisotropy of the resistance with changing temperature within a measurement accuracy 1–5%, we determined the law governing the variation of the diffuseness coefficient with temperature. It turned out that $q \propto T^{3.7 \pm 7.3}$ for all the zinc and cadmium samples. This relation holds true for cadmium up to 7°K and for zinc up to 12°K.

This result can be explained in the following manner: the physical nature of the temperature dependence of the coefficient q is the same as that of the resistance. Its value therefore depends both on the number of phonons and on the efficiency of the electron-phonon interaction. We assume for simplicity that $p_0 = 1$. The diffuse scattering is characterized by large scattering angles. At temperatures much lower than the Debye temperature, $T \ll \Theta_D$, a large scattering angle is produced only after $(\Theta_D/T)^2$ collisions with the phonons. On the other hand, the number of surface phonons is proportional to T^2. Thus, the coefficient q should be proportional to T^4. The experimental results favor such a variation.

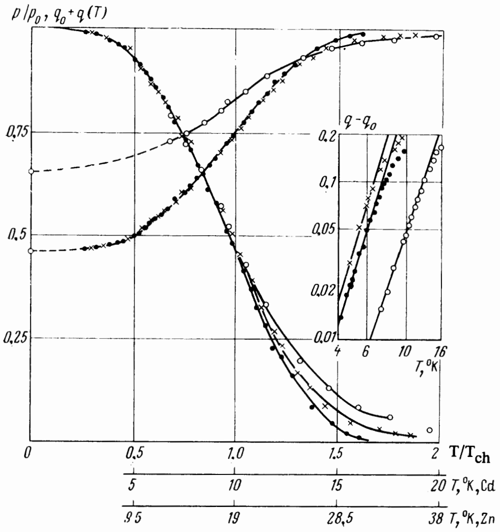

FIG. 5. Temperature dependence of the diffuseness and specularity coefficients: ×–Cd-1, ●–Zn-1, ○–Cd-5 (after first etching). On the right side is shown, in a logarithmic scale, the dependence of the diffuseness coefficient on the temperature: ×–Cd-1, ●–Cd-4, ○–Zn-1; the straight lines correspond to the relation $q \propto T^{3.7}$.

If we write in general form $q(T) = (1 - q_0)(T/T_{\mathrm{ch}})^{3.7}$, then according to the measurements the characteristic temperature T_{ch} is equal, on the average, to 10°K for cadmium and 19°K for zinc.

The results of the method described above for extracting the q(T) and p(T) dependences for the thinnest samples of Zn and Cd are shown in Fig. 5. The ordinates represent here the total diffuseness coefficient $q = q_0 + q(T)$ and the normalized specularity coefficient p/p_0 as functions of both the absolute and of the reduced temperature T/T_{ch}. (In the region $T > (1/2)T_{\mathrm{dif}}$ the curves are only qualitatively correct, since T_{dif} and the formula for f(p) were not determined accurately in this region.) In terms of the coordinates T/T_{ch}, the p/p_0 curves are the same for zinc and cadmium, thus indicating that the temperature T_{ch} is universal. Apparently, T_{ch} plays the same role in the surface scattering of electrons as does the Debye temperature in volume scattering, and in this sense it has a more definite character than T_{dif}. It is not quite clear, however, why T_{ch} is much smaller than the Debye temperature ($T_{\mathrm{ch}} \approx 0.1\,\Theta_D$). The obtained experimental data do not suffice for a serious discussion of this question. In particular, the question of the dependence of T_{dif} and T_{ch} on the sample thickness and on the value of q_0 remains unclear.

The question of the temperature variation of the resistance of very thin wires ($d \ll \lambda^{\infty}$) was considered theoretically by Azbel' and Gurzhi[17]. The calculation was based on the mechanism proposed by Olsen[7] for scattering from a surface, namely, at low temperature $T \ll \Theta_D$, but such that $T/\Theta_D > d/\lambda^\infty$, each act of electron-phonon scattering leads to a collision of the electron with the surface, where it is diffusely scattered. As a result of the Bloch-Gruneisen law, the factor $(T/\Theta_D)^2$ should drop out and $\rho_i^d \propto (T)/\Theta_D)^3$.

The qualitative reasoning and the exact calculation indicate convincingly that in the case of thin samples the decrease of the resistance with temperature should be much slower than for bulky samples. We emphasize that an essential factor in Olsen's mechanism is the assumption that even at acute incidence angles the electron is scattered diffusely when it reaches the surface.

Experiments performed on whiskers for the purpose of verifying the main conclusions of[17] are described in[5,6]. No agreement was observed in these experiments between the experimental data and the theoretical calculations: the power-law growth of the resistance with increasing temperature for very thin and for bulky samples turned out to be approximately the same; nor was the expected appreciable difference between the resistivities of thin and bulky samples observed. This disparity between theory and experiment must be attributed to the fact that in the case of whiskers the assumption that the electrons traveling at acute angles to the surface are diffusely reflected is not valid, since at low temperatures even the specular-reflection coefficient $p_{4.2}$, averaged over all the angles of incidence, is large. On the other hand at high temperatures, when the reflection becomes completely diffuse, the Olsen mechanism is no longer applicable.

The disparity between the conclusions of the theoretical papers and the experimental results is noted also in a paper by Holwech and Jeppesen[9] devoted, in particular, to the temperature dependence of the resistance of thin aluminum foils. It was observed that the difference $\Delta\rho^d(T)$ increases with temperature, reaches a maximum, and then decreases. The experimentally determined product $\rho^\infty\lambda^\infty$ likewise increases with temperature, from 0.8×10^{-11} ohm-cm^2 at T = 4.2°K to a maximum value 1.17×10^{-11} ohm-cm^2 at T = 25°K. A comparison of the results with the theory of Azbel' and Gurzhi[17] did not lead to any agreement. Holwech and Jeppesen[9] explained their results qualitatively as being due to a peculiar deviation from the Matthiessen rule in the scattering of electrons by the surface, in a manner similar to that occurring in the case of dilute alloys[18]. From our point of view, these results admit of a different explanation, if it is assumed that the coefficient of specular reflection p_0 of the investigated foils was different from zero. Assuming that at $T \approx 25$°K the reflection is practically diffuse, and using the formulas for films for the case $\lambda^\infty \gg d$[19], we obtain

$$\frac{(\rho^\infty\lambda^\infty)_{4.2}}{(\rho^\infty\lambda^\infty)_{25}} = \frac{1-p_{4.2}}{1+p_{4.2}} = 0.7.$$

Hence $p_{4.2} \approx 0.18$. Such a relatively low value of $p_{4.2}$ is sufficient for a good quantitative explanation of the observed effects. It is more difficult to explain the clearly pronounced maximum on the $\Delta\rho^d(T)$ curves, since the method of reducing the experimental points is not described in[9]. On the other hand, if it is assumed that values of $\rho^d(T)$ were determined from the formula $\rho^d(T) = [R(T)/R(0)]\rho^\infty(293)$, i.e., without taking into account the size effect at room temperature, then the appearance of the maximum is inevitable.

In accord with the results obtained in the present paper, the "surface" resistance in the case of a static size effect for wires can be represented in the two limiting cases $\lambda^\infty \gg d$ and $\lambda^\infty \ll d$ in the form of the sum

$$\rho^{\text{sur}} = \rho_0^{\text{sur}} + \rho_i^{\text{sur}}. \tag{5}$$

Just as in the case of the Matthiessen rule for volume resistance, ρ_0^{sur} is a constant quantity that depends on the coefficient p_0, while ρ_i^{sur} is the temperature part of the resistance, connected with the scattering of the electrons by the surface phonons. However, unlike the Matthiessen rules, the quantity ρ_i^{sur} depends strongly on ρ_i^{sur} and can be represented in turn by

$$\rho_i^{\text{sur}} = (\rho_{\text{dif}}^{\text{sur}} - \rho_0^{\text{sur}})F(T), \tag{6}$$

where $\rho_{\text{dif}}^{\text{sur}}$ is the "surface" resistance in the case when the coefficient p_0 vanishes, and the function F(T) changes from zero to unity with increasing temperature. Using the concrete values of these quantities at $\lambda^\infty \gg d$, we get

$$\rho_{\text{dif}}^{\text{sur}} = \frac{\rho^\infty\lambda^\infty}{d}, \quad \rho_0^{\text{sur}} = \frac{\rho^\infty\lambda^\infty}{d}\left(\frac{1-p_0}{1+p_0}\right);$$

$$F(T) = \frac{p_0 - p}{(1+p)p_0}, \quad p = p(T).$$

Expressions (5) and (6), describe completely, in a lucid general form, the static size effect in the resistance. Nonetheless, at the present time there are not enough experimental data to refute the scattering mechanism proposed by Olsen. The problem consists here, on the one hand, of developing an experimental method for the determination of the dependence of the specular-reflection coefficient on the incidence angle, and on the other hand, of obtaining surfaces on which the scattering is diffuse at any incidence angle.

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