RAYLEIGH AND ELECTRON WAVES IN METALS IN A WEAK MAGNETIC FIELD

A. M. GRISHIN, É. A. KANER, O. I. LYUBIMOV, and N. M. MAKAROV

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

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It is shown that surface electron waves should exist in pure metals and weak magnetic fields near the frequencies of the electronic transitions between magnetic surface levels. Such waves can interact with Rayleigh sound vibrations. Spectra, damping, and the mutual transformation of electronic and Rayleigh waves are investigated.

1. INTRODUCTION

IN 1885, Lord Rayleigh predicted the existence of surface sound waves in solids. These vibrations are localized near the surface of the sample and represent the superposition of transverse and longitudinal waves existing in the bulk of the material. The spectra and damping of the sound in metals, especially at low temperatures, are determined to a considerable degree by the electronic conductivity. We shall be interested in the effect of a weak magnetic field on the surface sound wave, which is realized through the medium of the electron conductivity. It is natural to expect that the surface sound vibrations will interact principally with electrons moving near the surface of the metal.

The states of such electrons in the magnetic field H differ from the well-known Landau levels which exist in the bulk of the conductor. If the vector H is parallel to the boundary of separation and the scattering of the electrons is specular, then, thanks to multiple reflection, the electrons drift along the surface in a direction perpendicular to the magnetic field (Fig. 1). Their motion along the normal to the boundary is finite and periodic and, according to the correspondence principle, should be quantized. Such quantum states are called magnetic surface levels.[1,2] Among surface electrons, a special place is occupied by "skipping" electrons, for which the center of the classical orbit lies outside the metal at a distance somewhat less than the orbit of revolution in the magnetic field. The period of vibrational motion of the skipping electrons is much less than the cyclotron period, since their trajectories consist of small arcs of the Larmor circle. Therefore the transition frequency ω_{ab} between the magnetic surface levels a and b fall off in the microwave frequency range even for weak fields H ~ 1–10 Oe. The resonance absorption of bulk sound waves at frequencies ω_{ab} was considered by Fischbeck and Mertsching.[3] As was to be expected, the effect was weak because of the surface character of the absorption. It must be emphasized that the interaction of the skipping electrons with the Rayleigh waves will be significantly greater than with the bulk sound.

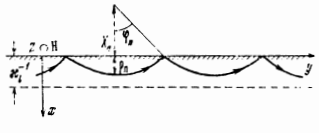

FIG. 1. Trajectory of skipping electrons in a magnetic field parallel to the surface of the metal.

In addition to the fact that the Rayleigh waves and the states of the skipping electrons are localized near the surface of separation, coupling between them actually has a resonance character. The fact is that, near the transition frequencies ω_{ab}, surface waves should exist which represent a characteristic electron sound.[4] This electron sound and the Rayleigh vibrations interact with one another. The present research is devoted to the theoretical investigation of their coupling and mutual transformation. A preliminary communication on this work has been published previously.[5]

2. COMPLETE SET OF EQUATIONS AND STATEMENT OF THE PROBLEM

1. Let a metallic half-space be located in a constant and homogeneous magnetic field H parallel to its surface. We choose the z axis along the vector H and direct the x axis along the interior normal to the boundary of separation x = 0 (Fig. 1). We consider a metallic model whose Fermi surface has axial symmetry with the axis p_z, i.e., the dispersion relation for electrons is

$$E(\mathbf{p}) = \varepsilon(p_z) + \mathbf{p}_\perp^2/2m, \tag{2.1}$$

where p is the momentum and m the transverse effective mass. The quantum states of the electron in the magnetic field H are characterized by the complete set $|a\rangle = |n, X, p_z\rangle$. Here n is the magnetic quantum number, $X = -cp_y/eH$ the projection of the center of revolution on the x axis, e the absolute value of the electronic charge, and c the velocity of light. We shall not take into account the spin distribution of quantum levels. For the vector potential A, we use the Landau gauge: $A_y = xH$, $A_x = A_z = 0$. The wave function of the electron in this gauge has the form[2]

$$|a\rangle = \frac{1}{2\pi\hbar}\exp\left[\frac{i}{\hbar}(p_y y + p_z z)\right] D_{\eta-1/2}\left(\frac{x-X}{\mu}\right), \tag{2.2}$$

where $\mu = (\hbar c/2eH)^{1/2}$ is the magnetic length, $D_\nu(\xi)$ is the parabolic cylinder function, normalized to unity. This function vanishes for $\xi = \infty$ and satisfies the equation

$$D_\nu''(\xi) + (\nu + 1/2 - 1/4\xi^2)D_\nu(\xi) = 0.$$

The quantum number η represents the energy of transverse motion of the electron in units of $\hbar\Omega$ (Ω is the cyclotron frequency). For specular reflection of the

electrons from the metallic surface, the quantum rule $\eta = \eta(n, X)$ is determined by the boundary condition

$$D_{\eta-1/2}(-X/\mu) = 0. \tag{2.3}$$

In the presence of a boundary, degeneracy along the X coordinate is removed and the energy levels can be represented in the form

$$E_a = \eta(n, X)\hbar\Omega + \varepsilon(p_z). \tag{2.4}$$

2. We shall assume that the metal is acoustically isotropic. Then the equations of elasticity theory, which describe the propagation of sound waves, have the form

$$\ddot{\mathbf{u}} = s_t^2\Delta\mathbf{u} + (s_l^2 - s_t^2)\nabla \operatorname{div}\mathbf{u} + \mathbf{f}/\rho. \tag{2.5}$$

Here $\mathbf{u}(\mathbf{r}, t)$ is the displacement vector in the sound wave, s_l and s_t are the velocities of longitudinal and transverse sound, ρ the density of the metal, $\mathbf{f}$ the volume force density exerted by the electrons on the lattice. Obtaining the quantum mechanical expression for the force $\mathbf{f}$ is most important.

We shall begin with the variational principle

$$\delta\langle\hat{\mathcal{H}}\rangle = -\int dV \mathbf{f}\delta\mathbf{u}. \tag{2.6}$$

This formula expresses the change in the total energy of the electron in a virtual displacement $\delta\mathbf{u}$. The brackets $\langle\ \rangle$ denote quantum-mechanical averaging. The operator $\hat{\mathcal{H}} = \hat{E} + \hat{U}$ is the sum of the unperturbed single-electron Hamiltonian $\hat{E}$ and the perturbation potential $\hat{U}$, which describes the interaction of the electron with sound. It is known that, in weak magnetic fields, we can limit ourselves to the direct deformation interaction of electrons with sound, and take the operator $\hat{U}$ in the form

$$\hat{U} = {}^1/_2[\Lambda_{\alpha\beta}(\hat{\mathbf{p}}),\ u_{\alpha\beta}]_+, \tag{2.7}$$

where $u_{\alpha\beta} = {}^1/_2(\partial u_\alpha/\partial x_\beta + \partial u_\beta/\partial x_\alpha)$ is the elastic deformation tensor, $\Lambda_{\alpha\beta}$ the symmetric tensor of the deformation potential, which vanishes in averaging over the Fermi surface, $[\hat{A}, \hat{B}]_+ = \hat{A}\hat{B} + \hat{B}\hat{A}$ is the anticommutator, and sumation over the repeated indices α and β is implied. Thanks to the smooth dependence of the deformation potential $\Lambda_{\alpha\beta}$ on $\mathbf{p}$ in the quasiclassical approximation, one can neglect the noncommutativity of the operators $\hat{\mathbf{p}}$ and $\hat{\mathbf{r}}$. In other words, one can assume $\Lambda_{\alpha\beta}(\hat{\mathbf{p}})$ to be a c number, and the perturbation $\hat{U}$ to be a multiplication operator.

The average value of the total energy of the electron is given by the usual formula

$$\langle\hat{\mathcal{H}}\rangle = \int dV\psi^*\hat{\mathcal{H}}\psi = \int dV\psi^*(\hat{E} + \Lambda_{\alpha\beta}u_{\alpha\beta})\psi, \tag{2.8}$$

where ψ is the exact wave function of the electron with Hamiltonian $\hat{\mathcal{H}}$. In the calculation of the variation of the mean energy $\delta\langle\mathcal{H}\rangle$ one cannot vary the wave functions ψ and ψ^* because of the equation of continuity for the probability density (the analog of Liouville's theorem). Consequently,

$$\delta\langle\hat{\mathcal{H}}\rangle = \int dV\psi^*(\Lambda_{\alpha\beta}\delta u_{\alpha\beta})\psi. \tag{2.9}$$

Integrating (2.9) by parts and equating the result to (2.6), we obtain

$$f_\alpha = \frac{\partial}{\partial x_\beta}\Lambda_{\alpha\beta}\psi^*\psi. \tag{2.10}$$

We now determine the density matrix with the help of the relation

$$\mathcal{F}(\mathbf{r}', \mathbf{r}) = \psi^*(\mathbf{r}')\psi(\mathbf{r}). \tag{2.11}$$

Then the electron force operator is equal to

$$\hat{f}_\alpha = \frac{\partial}{\partial x_\beta}\{\Lambda_{\alpha\beta}(\hat{\mathbf{p}})\hat{\mathcal{F}}(\mathbf{r}', \mathbf{r})\}_{\mathbf{r}'=\mathbf{r}}, \tag{2.12}$$

and the mean value of the force is

$$f_\alpha(\mathbf{r}) = 2\frac{\partial}{\partial x_\beta}\operatorname{Sp}\Lambda_{\alpha\beta}(\hat{\mathbf{p}})\hat{\mathcal{F}}\delta(\hat{\mathbf{r}} - \mathbf{r}'). \tag{2.13}$$

The coefficient 2 comes from the summation over the spins. One can show that the final expressions (2.12) and (2.13) are actually valid in the general case and not only in the quasiclassical approximation.

3. We now write down the equation of motion for the density matrix $\hat{\mathcal{F}}$. In the absence of scattering of electrons, this equation has the form

$$\frac{\partial\hat{\mathcal{F}}}{\partial t} + \frac{i}{\hbar}(\hat{\mathcal{H}}\hat{\mathcal{F}} - \hat{\mathcal{F}}\hat{\mathcal{H}}) = 0. \tag{2.14}$$

We linearize the density matrix, setting $\hat{\mathcal{F}} = \mathcal{F}^0(\hat{\mathcal{H}}) + \hat{\mathcal{F}}'$. The nonequilibrium addition to the density matrix $\hat{\mathcal{F}}'$ satisfies the equation

$$\frac{\partial\hat{\mathcal{F}}'}{\partial t} + \frac{i}{\hbar}(\hat{E}\hat{\mathcal{F}}' - \hat{\mathcal{F}}'\hat{E}) = -\frac{\partial\hat{\mathcal{F}}^0(\hat{\mathcal{H}})}{\partial t}. \tag{2.15}$$

The matrix element $\partial\mathcal{F}^0(\hat{\mathcal{H}})/\partial t$ in the representation of the unperturbed Hamiltonian is easily found with the help of a relation which is valid in the linear approximation in the perturbation $\hat{U}$:

$$(\hat{E}\mathcal{F}^0(\hat{\mathcal{H}}) - \mathcal{F}^0(\hat{\mathcal{H}})\hat{E})_{ab} = (E_a - E_b)\mathcal{F}_{ab}^0 \simeq (\mathcal{F}_a^0 - \mathcal{F}_b^0)U_{ab}, \tag{2.16}$$

where $\mathcal{F}^0_{ab} = \langle a|\mathcal{F}^0(\hat{\mathcal{H}})|b\rangle$, $\mathcal{F}^0_a = \mathcal{F}^0(E_a)$. With the help of (2.16), the equation for the matrix elements $\mathcal{F}'_{ab}$ can be represented in the form

$$\frac{\partial\mathcal{F}_{ab}'}{\partial t} + \frac{i}{\hbar}(E_a - E_b)\mathcal{F}_{ab}' = \frac{\mathcal{F}_a^0 - \mathcal{F}_b^0}{E_b - E_a}\dot{U}_{ab}, \tag{2.17}$$

where the dot indicates the partial time derivative. For the monochromatic perturbation $\hat{U}(t) \sim e^{-i\omega t}$, we write out the solution of Eq. (2.17) in the form

$$\mathcal{F}_{ab}' = \hbar\omega\frac{\mathcal{F}_a^0 - \mathcal{F}_b^0}{E_b - E_a}\frac{\langle a|\Lambda_{\alpha\beta}u_{\alpha\beta}|b\rangle}{E_b - E_a + \hbar\omega + i\gamma}. \tag{2.18}$$

In the account of collisions of electrons with scatterers, the adiabatic parameter $\gamma \to +0$ can be replaced under certain conditions[6] by the relaxation time: $\gamma = \hbar/\tau$. In the case of elastic scattering of the electrons, a similar substitution in the dissipative part of the density matrix can be made only after the use of the exact law of conservation of energy $E_a = E_b + \hbar\omega$ in the factor $(\mathcal{F}^0_a - \mathcal{F}^0_b)/(E_b - E_a)$. Keeping this remark in mind, we write down the final expression for the mean electron force

$$f_\alpha(\mathbf{r}) = 2\hbar\omega\sum_{a,b}\frac{\mathcal{F}_a^0 - \mathcal{F}_b^0}{E_b - E_a}\frac{\langle a|\Lambda_{ik}u_{ik}|b\rangle}{E_b - E_a + \hbar\omega + i\hbar\nu}\Lambda_{\alpha\beta}\frac{\partial}{\partial x_\beta}\langle b|a\rangle. \tag{2.19}$$

Here $\nu = 1/\tau$ is the collision frequency of electrons with scatterers. It should be remarked that there is a contribution to the electron force because of the dependence of the quasi-equilibrium part of the density matrix $\mathcal{F}^0(\hat{\mathcal{H}})$ on the perturbation potential $\hat{U}$. It leads to renormalization of the elastic moduli and sound velocities, which do not depend on the magnetic field in the region of weak magnetic fields considered. This contribution can be taken into account if we assume the

sound velocities s_l and s_t in (2.5) to be renormalized quantities.

4. In order to simplify further calculations, we make the assumption that the deformation potential tensor is diagonalized in the indices α and β, i.e., $\Lambda_{\alpha\beta}(p) = \Lambda(p)\delta_{\alpha\beta}$. This assumption corresponds to the Fröhlich model of electron-phonon interaction, in which the effect of electrons on the propagation of transverse sound is not taken into account. Although such a model is inapplicable to real anisotropic metals, nevertheless, it can be assumed that the results obtained by means of this model give the correct qualitative description of the effect, even in the general case.

Following Landau and Lifshitz,[7] we represent the displacement vector u in the form of the sum of a potential and a vortical part: $u = u^l + u^t$, while div $u^t = 0$, curl $u^l = 0$. Because of the diagonality of the tensor $\Lambda_{\alpha\beta}$, the equations for u^l and u^t are separated:

$$\omega^2 u^t + s_t^2 \Delta u^t = 0, \qquad \omega^2 u^l + s_l^2 \Delta u^l + f/\rho = 0, \tag{2.20}$$

and only the potential part of the displacement u^l enters into the expression for the force f. Such a division of the equations for u^l and u^t materially simplifies the calculations.

The equation (2.20) should be supplemented with boundary conditions. It is seen from (2.19) that the electron force f on the surface of separation vanishes with the wave function of the electron (2.3). For this reason, the boundary conditions on the free surface x = 0, the conditions from which the spectra of the surface waves are determined,

$$u_{xy} = u_{xz} = 0; \qquad u_{xx} + u_{yy} + \frac{s_l^2 - 2s_t^2}{s_l^2} u_{zz} = 0, \tag{2.21}$$

turn out to be the same as for $f \equiv 0$.

The solution of the set of equations (2.20) is sought in the form of a plane surface wave $u(r) = u(x)e^{ikz}$ propagating along the z axis. Here the component $u_y \equiv 0$, as in the absence of interaction of the electrons with the sound. The solution of the first of Eqs. (2.20) is a free wave of "transverse" sound

$$u_x^t(r) = iAk \exp(-\varkappa_t x + ikz),$$
$$u_z^t(r) = A\varkappa_t \exp(-\varkappa_t x + ikz), \tag{2.22}$$

where $\kappa_{t,l}^2 = k^2 - \omega^2/s_{t,l}^2$; A is some constant. We write down Eqs. (2.20) for "longitudinal" sound, expanding $u^l(x)$ in the Fourier integral

$$u^l(x) = (1/\pi)\int_0^\infty dq\, u^l(q)\cos(qx)$$

$$2\frac{\partial u_\alpha^l(0)}{\partial x} + (\varkappa_l^2 + q^2)u_\alpha^l(q) = \frac{1}{\pi}\int_0^\infty dq' Q_{\alpha\beta}(q, q')u_\beta^l(q'),$$
$$\alpha, \beta = x, z. \tag{2.23}$$

The right hand side of (2.23) represents the Fourier component of the quantity $f_\alpha(r)/\rho s_l^2$. The solution of Eqs. (2.23) is expressed in terms of two constant $\partial u_\alpha^l(0)/\partial x$. The dispersion equation for surface waves is obtained by equating to zero the determinant of Eqs. (2.21), which contain these constants and the quantity A.

2. CALCULATION OF THE FOURIER COMPONENT OF THE ELECTRON FORCE

The Fourier transform of the electron force is determined by means of the formula

$$f_\alpha(k, q) = 2e^{-ikz}\int_0^\infty dx f_\alpha(x, z)\cos(qx). \tag{3.1}$$

Using (2.19) and the diagonality of the deformation potential tensor, we write out this expression in the form

$$f_\alpha(k, q) = 4\hbar\omega e^{-ikz}\sum_{a,b}\frac{\mathscr{F}_a^0 - \mathscr{F}_b^0}{E_b - E_a}\frac{\Lambda^2}{E_b - E_a + \hbar\omega + i\hbar\nu}$$

$$\cdot \langle a|\operatorname{div} u^l(r)|b\rangle\int_0^\infty dx\cos(qx)\frac{\partial}{\partial x_\alpha}\langle b|a\rangle. \tag{3.2}$$

After identity transformations, this expression takes the form

$$\frac{f_\alpha(k, q)}{\rho s_l^2} = \frac{1}{\pi}\int_0^\infty dq' k^2 \sum_{a,b}\Phi_{ab}\Psi_{ab,\alpha}(q)\Psi_{ab,\beta}^*(q')u_\beta(q'). \tag{3.3}$$

Here we have introduced the following notation

$$\Phi_{ab} = \frac{\hbar\omega}{\rho s_l^2(\pi\hbar)^2}\frac{\mathscr{F}_a^0 - \mathscr{F}_b^0}{E_b - E_a}\frac{\Lambda^2\delta(p_{ya} - p_{yb})\delta(p_{za} - p_{zb} - \hbar k)}{E_a - E_b - \hbar\omega - i\hbar\nu}, \tag{3.4}$$

$$\Psi_{ab,x}(q) = \frac{q}{k}(a|\sin(qx)|b), \qquad \Psi_{ab,z}(q) = i(a|\cos(qx)|b),$$

$$(a|e^{iqx}|b) = \int_0^\infty dx e^{iqx} D_{\eta_a - 1/2}\left(\frac{x - X}{\mu}\right)D_{\eta_b - 1/2}\left(\frac{x - X}{\mu}\right), \tag{3.5}$$

$\delta(p)$ is the Dirac delta function. Equating Eq. (3.3) to the right hand side of (2.23), we can easily write down the expression for the kernel of the integral transformation $Q_{\alpha\beta}(q, q')$ in the form

$$Q_{\alpha\beta}(q, q') = k^2\sum_{a,b}\Phi_{ab}\Psi_{ab,\alpha}(q)\Psi_{ab,\beta}^*(q').$$

The electron states in a metal divide naturally into volume and surface states. The interior (volume) electrons do not collide with the surface of the sample and have the Landau spectrum $\eta(n, X) = n + 1/2$, which is degenerate in the X coordinate of the center of revolution. For these electrons, $X \geq R_\perp = cp_\perp/eH$, where $R_\perp$ is the classical radius of cyclotron rotation of the electron. The surface electrons with $|X| \leq R_\perp$ undergo reflection from the interface. Inasmuch as we are interested in the region of weak magnetic fields, not exceeding several tens of Oersteds, the contribution to the electron force from the interior electrons can be assumed to be independent of the magnetic field. The interaction of these electron with sound, as is well known, leads to a relatively small value of damping and change in the velocity of the interior oscillations, and is characterized by the dimensionless parameter

$$\zeta\frac{\omega}{kv_F} \approx \zeta\frac{s_l}{v_F}. \tag{3.6}$$

The parameter of electron-phonon interaction $\zeta = (\Lambda/\epsilon_F)^2$ is of the order of unity.

Among the surface electrons, the most important for the effect being studied are the skipping electrons. Their interaction with the sound wave in weak magnetic fields has a resonance character. Therefore it is necessary above all to find the electron force due to the skipping electrons.

For this purpose, we compute the matrix element (3.5) in the quasiclassical approximation, when

$$n \gg |n - n'|. \tag{3.7}$$

The quasiclassical asymptote of the parabolic cylinder functions has the form

$$D_{\eta-1/2}\left(\frac{x-X}{\mu}\right)=\frac{n^{-1/4}[1-(x-X)^2/4\eta\mu^2]^{-1/4}}{\sqrt{\mu}\arccos(-X/2\mu\sqrt{\eta})}C_\eta\left(\frac{x-X}{\mu}\right), \quad (3.8)$$

where

$$C_\eta(t)=\cos\left\{\eta\left[\arccos\left(\frac{t}{2\sqrt{\eta}}\right)-\frac{t}{2\sqrt{\eta}}\left(1-\frac{t^2}{4\eta}\right)^{1/2}\right]-\frac{\pi}{4}\right\}. \quad (3.8')$$

Integration in (3.5) with the wave functions (3.8) should be carried out over the classical region $0 \le x \le R_\perp + X$. Transforming to the new variable $t = (x - X)/\mu$, we represent the matrix element (3.5) in the form

$$(a|e^{iqx}|b)=\eta^{-1/4}[\arccos(-X/2\mu\sqrt{\eta})]^{-1}$$
$$\times\int_{-X/\mu}^{2\sqrt{\eta}}dt\frac{C_\eta(t)C_{\eta'}(t)\exp[iq(\mu t+X)]}{(1-t^2/4\eta)^{1/4}}. \quad (3.9)$$

In place of the X coordinate of the center, we introduce the angle variable

$$\varphi=\arccos(-X/2\mu\sqrt{\eta}), \quad (3.10)$$

which is the angle of skipping of the electron at the moment of its collision with the surface (Fig. 1). We transform in the integral (3.9) from the variable t to the running angle variable λ by the formula $t = 2\sqrt{\eta}\cos\lambda$. Then

$$(a|e^{iqx}|b)= \quad (3.11)$$
$$=\frac{2}{\varphi}\int_0^\varphi d\lambda C_\eta(2\sqrt{\eta}\cos\lambda)C_{\eta'}(2\sqrt{\eta}\cos\lambda)\exp[2iq\mu\sqrt{\eta}(\cos\lambda-\cos\varphi)].$$

In the quasiclassical approximation, the relation

$$2C_\eta(2\sqrt{\eta}\cos\lambda)C_{\eta'}(2\sqrt{\eta}\cos\lambda)\approx\cos[\lambda(\eta'-\eta)]$$

is valid, in the derivation of which the function $C_{\eta'}(2\sqrt{\eta}\cos\lambda)$ is expanded in the small parameter $\lambda|\eta'-\eta|/\eta(\lambda-\cos\lambda\sin\lambda)\sim|n-n'|/n$. Thus, the matrix element (3.5) is equal to

$$(a|e^{iqx}|b)=\varphi^{-1}\int_0^\varphi d\lambda\cos[\lambda(\eta'-\eta)]\exp[-2iq\mu\sqrt{\eta}(\cos\varphi-\cos\lambda)] \quad (3.12)$$

The expression for the tensor $Q_{\alpha\beta}(q, q')$ contains a summation over the quantum numbers n and n′ and integration over p_{za}, p_{zb}; p_{ya}, p_{yb}. Integration over p_{yb} and p_{zb} is performed by means of the δ function contained in Φ_{ab}. In place of integration over p_{ya}, we transform to an integral over the angle variable φ in correspondence with Eq. (3.10). Here we must keep in mind in the substitution of variables that the quantity η is a function of n and φ. The quasiclassical condition of quantization for $\eta(n, \varphi)$ is easily obtained from (2.3) and (3.8′) in the form

$$\eta(n,\varphi)=\frac{\pi(n-1/4)}{\varphi-\sin\varphi\cos\varphi}, \qquad n=1,2,3\ldots \quad (3.13)$$

The difference $\eta - \eta' = \eta(n, \varphi) - \eta(n', \varphi')$ is expressed in terms of the transition frequency ω_{ns} between the levels $n' = n + s$ and n by the following formula:

$$\eta'-\eta\equiv\omega_{ns}/\Omega\approx\pi s/\varphi. \quad (3.14)$$

As a result, the expression for the kernel $Q^{(s)}_{\alpha\beta}(q, q')$, due to magnetic surface levels, is written in the form

$$Q^{(s)}_{\alpha\beta}(q,q')=\frac{\hbar k^2}{\mu}\sum_{n=1}^{\infty}\sum_{s=-\infty}^{\infty}\int dp_z\int_0^\pi\frac{\varphi\sin\varphi\sqrt{\eta(n,\varphi)}\,d\varphi}{\varphi-\sin\varphi\cos\varphi}\Phi_{ns}(\varphi,p_z)$$
$$\times\Psi_{ns,\alpha}(q)\Psi^*_{ns,\beta}(q'), \quad (3.15)$$

where Ψ_{ns}, (q) is determined by the formulas (3.5) and (3.12) with account of (3.14), and the quantity $\Phi_{ns}(\varphi, p_z)$ is equal to the coefficient of the δ functions in (3.4).

Equation (3.15) admits a further simplification if we take it into account that the ratio $(\mathcal{F}^0_\alpha - \mathcal{F}^0_b)/(E_b - E_a)$ can be replaced by $\delta[E_n(\varphi, p_z) - \epsilon_F]$, where ϵ_F is the Fermi energy. In order to get criteria for the possibility of a similar substitution, we must take into account the presence of the a resonance factor in (3.4), which is connected with the law of conservation of energy in the absorption of the quantum $\hbar\omega$. The difference of the Fermi functions $\mathcal{F}^0_a - \mathcal{F}^0_b$ differs from zero in the circle $E = \epsilon_F$ and changes over the characteristic energy interval $T + \hbar\omega$ (because of the finite temperature T and the quantum energy $\hbar\omega$). The relative rate of change of the difference in the Fermi functions has the order $(T + \hbar\omega)/\epsilon_F$, and for the resonance denominator, $-\nu/\omega$. Therefore replacement of $(\mathcal{F}^0_a - \mathcal{F}^0_b)/(E_b - E_a)$ by a delta function is valid under the condition[4]

$$\frac{T+\hbar\omega}{\varepsilon_F}\ll\frac{\nu}{\omega}.$$

By means of $\delta[E_n(\varphi, p_z) - \epsilon_F]$ it is easy to carry out integration over φ. In the interval $0 \le \varphi \le \pi$, there is no more than one root of the equation $E_n(\varphi, p_z) = \epsilon_F$, the solution of which represents the condition of quantization of the skipping angle $\varphi = \varphi_n$.

In order to obtain the final expression for the kernel $Q^{(s)}_{\alpha\beta}$, it must be taken into account that the principal role is played in weak magnetic fields by the skipping electrons with small values of the angle φ. The quantization condition for these electrons has the form

$$\varphi_n=\left[\frac{3\pi}{2}\frac{\hbar\Omega}{\varepsilon_F-\varepsilon(p_z)}\left(n-\frac{1}{4}\right)\right]^{1/3}. \quad (3.16)$$

We have left in the kernel $Q^{(s)}_{\alpha\beta}(q, q')$ only one resonance component, in which the frequency ω is close to one of the transition frequencies ω_{ns}

$$Q^{res}_{\alpha\beta}(q,q') \quad (3.17)$$
$$=-\frac{\omega k^2}{2\pi^2\hbar^2\rho\mu s l^2\Omega}\int dp_z\frac{\Lambda^2(p_z)[\hbar\Omega/(\varepsilon_F-\varepsilon(p_z))]^{1/3}}{\omega-\omega_{ns}-kv_z+i\nu}\Psi^*_{ns,\alpha}(q)\Psi_{ns,\beta}(q').$$

Here the components of the vector $\Psi_{ns,\alpha}(q)$ for the skipping electrons have the form

$$\Psi_{ns,x}(q)=(q/k)\int_0^1 dx\cos(\pi sx)\sin[q\rho_n(1-x^2)],$$

$$\Psi_{ns,z}(q)=i\int_0^1 dx\cos(\pi sx)\cos[q\rho_n(1-x^2)]. \quad (3.18)$$

The parameter $\rho_n = R_\perp\varphi_n^2/2$ is the maximum distance of the n-th quantized trajectory from the surface of the metal (Fig. 1); $v_z = \partial\epsilon(p_z)/\partial p_z$ is the velocity of the electron along the magnetic field; the value of the deformation potential $\Lambda(p_z)$ is taken for $E = \epsilon_F$ and $\varphi_n = 0$.

Let us compare the resonance and nonresonance

parts of the kernel of the integral transform $Q_{\alpha\beta}(q, q')$. Its nonresonance part is determined principally by the interior electrons and, in accord with (3.6), $Q^{V}_{\alpha\beta} \sim \zeta\omega/v_F$. The maximum value of $Q^{res}_{\alpha\beta}$ for $|\Psi_{ns,\alpha}| \sim 1$ is of the order of $\hbar k^2/\omega\zeta/mv_F\nu$. The relative values of these two components are determined by the parameter $\hbar k^2/m\nu$, which is the ratio of the energy of the electron with momentum $\hbar k$ to the collision width of the level $\hbar\nu$. Thus, upon satisfaction of the condition

$$\hbar k^2/m\nu \gg 1 \tag{3.19}$$

the resonance component in $Q_{\alpha\beta}$ from the skipping electrons gives the principal contribution to the electron force.

4. SOLUTION OF THE EQUATIONS

The solution of Eqs. (2.23) and the analysis of the resulting dispersion equation for surface waves are much simpler if we use the model of a cylindrical Fermi surface with axis p_z. In other words, we assume $\epsilon(p_z) = 0$. Then the quantization condition (3.16) for the angle φ does not contain $\epsilon(p_z)$ and in (3.17) integration over p_z reduces to the multiplication by the dimension of the cell in the p_z direction. The expression for Q^{r}_{α} can be rewritten in the form

$$Q^{res}_{\alpha\beta}(q, q') = -2\zeta\frac{\hbar k^2}{p_F}\frac{\omega}{\omega - \omega_{ns} + i\nu}\Psi_{ns,\alpha}(q)\Psi^{*}_{ns,\beta}(q'). \tag{4.1}$$

Thanks to the fact that the kernel (4.1) is degenerate, Eqs. (2.23) reduce to an inhomogeneous set of linear algebraic equations for the quantities

$$\frac{1}{\pi}\int_0^{\infty} dq'\Psi^{*}_{ns,r}(q')u_r^{l}(q'), \qquad r = x, z.$$

The solution of this set is expressed in terms of the constants $\partial u^{l}_{\alpha}(0)/\partial x$. The boundary conditions (2.21) connect these constants with A. Setting the determinant of Eqs. (2.21) equal to zero, we obtain the dispersion equation for the surface waves. We shall not attempt to give here all these essentially simple but rather cumbersome calculations, and immediately write down the result:

$$\varkappa_t k\frac{4\varkappa_t\varkappa_l k^2 - (\varkappa_t^2 + k^2)^2}{(\varkappa_t^2 + k^2)^2(\varkappa_l^2 - k^2)} = \Xi\frac{\omega}{\omega - \omega_{ns}(1 - \Delta) + i\nu}. \tag{4.2}$$

Here

$$\Xi = \zeta\frac{\hbar k}{p_F}\alpha_{ns}^2 \tag{4.3}$$

plays the role of a dimensionless coupling constant

$$\Delta = \zeta\frac{\hbar\omega^2}{\varkappa_l p_F s_l^2}\left(\beta_{ns} + \frac{\alpha_{ns}^2}{2}\right) \tag{4.4}$$

is the small relative shift in the resonance frequency. The dimensionless parameters α_{ns} and β_{ns} are expressed by the following formulas:

$$\alpha_{ns} = \int_0^1 dx\cos(\pi sx)\exp[-\varkappa_l\rho_n(1 - x^2)],$$

$$\beta_{ns} = \int_0^1 dx\cos(\pi sx)\int_0^x dy\cos(\pi sy)\exp[-\varkappa_l\rho_n(x^2 - y^2)]. \tag{4.5}$$

For the cylindrical model of the Fermi surface considered,

$$\omega_{ns} = \frac{\pi s\Omega}{\varphi_n}, \quad \rho_n = \frac{cp_F}{2eH}\varphi_n^2, \quad \varphi_n = \left[\frac{3\pi}{2}\frac{\hbar\Omega}{\varepsilon_F}\left(n - \frac{1}{4}\right)\right]^{2/3}. \tag{4.6}$$

The dispersion relation (4.2) describes the coupling between the two types of waves. One of them corresponds to the vanishing left hand side of (4.2) and gives the spectrum $\omega = s_t k\xi$ ($\xi = \xi(s_t/s_l)$ is determined by the solution of some transcendental equation[7]) of the unperturbed Rayleigh wave. The other solution

$$\omega = \omega_{ns}(1 - \Delta) - i\nu \tag{4.7}$$

corresponds to the vanishing of the denominator of the right hand side of (4.2) and represents the spectrum of surface electron sound in the case considered. The parameter Ξ characterizes the coupling between the surface sound and the electron surface waves. For $\Xi \to 0$, both branches are independent and intersect at $\omega_{ns} \approx s_t k\xi$. Near the point of intersection, if $\Xi \neq 0$, there is resonance interaction and mutual transformation of both types of oscillations.

As is well known,[8] one should distinguish between the cases of weak and strong coupling in the resonance coupling of waves, depending on the ratio of the parameter Ξ and the square of the total relative damping of both waves $\gamma \approx \nu/\omega + s_l/\nu_F$. In the case of strong coupling ($\Xi \gg \gamma^2$) the "repulsion" of the resonance curves $\delta\omega$ near the point of intersection is symmetric and $\delta\omega/\omega \approx \pm\,\Xi^{1/2}$ (Fig. 2). To find the corrections to the spectrum and the damping for weak coupling ($\Xi \ll \gamma^2$), it is necessary to make use of the perturbation theory with respect to the coupling constant Ξ, substituting the unperturbed spectrum in the correction.[8] We shall not attempt to write out these well-known formulas, which describe the transformation of the waves.

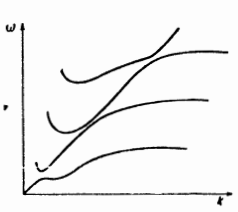

FIG. 2. Schematic picture of the mutual transformation of Rayleigh and surface electron waves.

5. SPECTRUM OF SURFACE ELECTRON SOUND

The possibility of the appearance of the resonance effect described above is due to the existence of surface electron excitations, the spectrum and damping of which are determined by Eq. (4.7). We shall give the expressions for the spectrum of surface electron waves in two limiting cases.

1. The trajectory of the skipping electrons is located in the "skin-layer of the Rayleigh wave, $\kappa_l\rho_n \ll 1$. In this case, the asymptotic expressions for α_{ns} and β_{ns} have the form

$$|\alpha_{ns}| = 2\varkappa_l\rho_n/(\pi s)^2, \quad \beta_{ns} = \varkappa_l\rho_n/2(\pi s)^2. \tag{5.1}$$

Substituting these expressions in (4.4) and (4.7), we get

$$\omega(k) = \omega_{ns}\left\{1 - \zeta\frac{\hbar\Omega}{4ms_l^2}\left[1 + \left(\frac{2}{\pi s}\right)^2\varkappa_l\rho_n\right]\right\}, \tag{5.2}$$

where $\kappa_l = (k^2 - \omega_{ns}^2/s_l^2)^{1/2}$. It is seen from this formula that the spectrum begins with $k = \omega_{ns}/s_l$, and

the group velocity of the electron wave at this point goes to negative infinity. The dispersion of the wave directly after the threshold has an anomalous character, i.e., the signs of the phases and group velocity are different. There is a difference here in the spectrum of the surface electron waves in their interaction with the acoustic oscillations from the case of interaction with the electromagnetic field, for which the dispersion has a normal character.[4] The difference shown is due to the fact that in the electromagnetic field the unperturbed surface wave is absent.

2. In the opposite limiting case $\kappa_l \rho_n \approx \kappa \rho_n \gg 1$, the asymptotic expressions for α_{ns} and β_{ns} have the form

$$|\alpha_{ns}| = (2k\rho_n)^{-1}, \quad \beta_{ns} = (4k\rho_n)^{-1} \ln (k\rho_n), \quad k \gg \omega_{ns}/s_l. \tag{5.3}$$

The spectrum of the electron wave in this case has the form

$$\omega(k) = \omega_{ns}\left[1 - \zeta \frac{(\pi s)^2}{8} \frac{\hbar\Omega}{ms_l^2} \frac{\ln(k\rho_n)}{(k\rho_n)^2}\right]. \tag{5.4}$$

In this region of wave numbers, the frequency dispersion is normal. For large k, the limiting frequency in the spectrum is identical with ω_{ns}. It should be noted that the behavior of the dispersion curve in the limiting case of large k has the same character as for surface electromagnetic waves.[4] This fact confirms our interpretation of the excitations with the spectrum (4.7) as the proper surface oscillations in the electron Fermi gas.

From a comparison of Eqs. (5.2) with (5.4), it can be concluded that there is a gap $|\Delta\omega| = \zeta\hbar\Omega\omega_{ns}/4ms_l^2$ between the threshold and the limiting frequency. Figure 3 shows schematically the path of the dispersion of the characteristic frequency as a function of k. The dashes indicate that part of the curve in which it is difficult to obtain an analytic expression $\omega = \omega(k)$.

The analysis of the spectrum given for the cylindrical Fermi surface is essentially based on the fact that the resonance frequency ω_{ns} is independent of p_z. In the general case, thanks to the dependence of ω_{ns} on p_z, resonance takes place close to the extremal values of $\omega_{ns}(p_z) = \omega_{ns}^{ext}$. The analysis given in[4] for the study of surface electromagnetic waves showed that the dispersion equation, in the case of a spherical Fermi surface, does not have a solution for surface electron excitations. This result is connected with the fact that the kernel $Q_{\alpha\beta}^{res}$ (3.17) has no singularity for $\nu \to 0$, when the extremum of the resonance curve corresponds to the maximum of $\omega_{ns}(p_z)$. The given conclusion is valid also for the local maxima of $\omega_{ns}(p_z)$ and for an arbitrary electron dispersion law. The surface electron excitations in this case exist only near the local minima of the functions $\omega_{ns}(p_z)$ and their spectrum has the form

$$\omega(k) = \omega_{ns}^{min} - \alpha k^2, \quad \alpha > 0. \tag{5.5}$$

The investigation, which is easily carried out by the same method as in[4] leads to analogous conclusions. That is, the spectrum of the surface electron sound, in the interaction with Rayleigh waves, exists in the vicinity of the local minima of $\omega_{ns}(p)$ and is determined by Eq. (5.5) for small k.

The resonance effects considered above were observed in experiments with surface hypersonic waves (frequency $\omega \sim 10^{10}$–10^{11} sec^{-1}) in pure monocrystalline metals at low temperatures. Here the characteristic value of the coupling constant is $\Xi \sim 10^{-2}$–10^{-3} and the relative resonance changes in the spectrum and the damping of the surface waves can reach values on the order of 0.01–0.1.

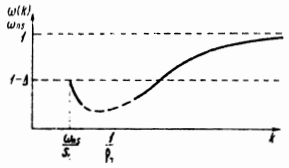

FIG. 3. Spectral curve of a surface electron wave (the part of the curve where it is difficult to obtain an explicit analytic dependence $\omega = \omega(k)$ is shown by the dashed curve).

[1] T. W. Nee and R. E. Prange, Phys. Lett. 25A, 582 (1967).
[2] É. A. Kaner, N. M. Makarov and I. M. Fuks, Zh. Eksp. Teor. Fiz. 55, 931 (1968) [Soviet Phys.-JETP 28, 483 (1969)].
[3] H. J. Fischbeck and J. Mertsching, Phys. Stat. Sol. 31, 107 (1969).
[4] É. A. Kaner and N. M. Makarov, ZhETF Pis. Red. 10, 253 (1969) [JETP Lett. 10, 160 (1969)]; Zh. Eksp. Teor. Fiz. 58, 1972 (1970) [Soviet Phys.-JETP 31, 1063 (1970)].
[5] A. M. Grishin, É. A. Kaner, O. I. L'ubimov and N. M. Makarov, Solid State Commun. 8, 581 (1970).
[6] É. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. 53, 375 (1967) [Soviet Phys.-JETP 26, 281 (1968)].
[7] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) (Nauka Press, 1965) Sec. 24.
[8] V. G. Skobov and É. A. Kaner, Zh. Eksp. Teor. Fiz. 46, 273 (1964) [Soviet Phys.-JETP 19, 189 (1964)].

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