COHERENT INTERACTION OF ELECTRONS OF A SEMICONDUCTOR WITH A STRONG ELECTROMAGNETIC WAVE

V. F. ELESIN

Moscow Engineering-Physics Institute

Submitted March 12, 1970

Zh. Eksp. Teor. Fiz. 59, 602–614 (August, 1970)

We consider the influence of relaxation processes on the coherent interaction between electrons of a semiconductor and a strong electromagnetic wave. The coherence parameter is assumed to be the gap in the energy spectrum of the electron excitation. As shown in [1], this gap is due to the external field that causes transitions between the valence band and the conduction band. The Keldysh diagram technique for non-equilibrium processes[6] is used to obtain a system of equations for the Green's functions and the particle distribution functions, with allowance for recombination, electron-phonon interaction, and $T \neq 0$. The Green's functions and the kinetic equations for the quasiparticles are obtained in the strong-field approximation. The damping and the spectrum of the excitations are calculated. It is shown that in a definite temperature interval, the electron-phonon interaction has practically no influence on the gap, and the damping is small. Consequently, in order for a gap to exist in a pure semiconductor, it suffices to fulfill the regularly-satisfied condition that the frequency of the transition of the electrons between the bands exceed the recombination probability. The problem of absorption of a strong external field (the saturation effect) in semiconductors is solved.

IT is known that the character of the interaction between a strong electromagnetic field and two-level systems depends on the ratio of the time required to destroy the coherence by the relaxation processes to the time of transition of the electrons between the levels under the influence of the field. An estimate of this ratio in semiconductors entails great difficulties, in view of the complexity of the interaction between electrons and crystals. At the same time, this problem must be solved for a better understanding of the operation of semiconductor quantum generators, amplifiers, and converters at high intensities of the electromagnetic field.

The parameter of the coherence of the interaction with the field in the semiconductor is customarily chosen to be the gap in the spectra of the electronic excitations; this gap, as shown in [1], is a result of the action of the external field producing transitions between the valence band and the conduction band. Then the problem of the disruption of the coherent interaction with the field reduces to a calculation of the gap and of the damping of the electron excitations, and can be solved with the aid of the effective methods developed in superconductivity theory.[2-4] It was shown in this manner in [5] that at $T = 0$ and in the absence of recombination the electron-phonon and the electron-electron interactions practically leave the gap unchanged, and the damping of the excitations is small. The latter is connected with the fact that the electrons taking part in the interband transitions are near the Fermi quasilevel.

The purpose of the present paper is to find the spectrum and the damping of the excitations at $T \neq 0$ and with allowance for recombination, which plays the principal role. The main difficulty of the problem lies in the fact that the system is essentially not in equilibrium. The electron distribution function differs from the equilibrium Gibbs distribution because of the action of the strong external field. By strong is meant a field in which the electron-hole pairs are produced more rapidly than they recombine. To describe such a state, it is necessary to solve a simultaneous system of equations for the Green's functions and for the distribution functions; we obtain this system with the aid of the diagram technique developed in the well known paper of Keldysh.[6] The Keldysh method is very convenient for the problem. It makes it possible not only to find the spectrum and the damping of the excitations, but also to determine the saturation of the absorption in the semiconductors. The main result consists in the following. In a definite temperature interval, for a gap to exist in a pure semiconductor it suffices that the field be strong in the sense indicated above, i.e., that the frequency of the transitions of the electrons between the bands exceed the recombination probability.

We consider in the paper the model of two symmetrical bands with direct optical interband transitions and with a quadratic dispersion, and use the system of units in which $\hbar = c = m = 1$.

1. FORMULATION OF PROBLEM. HAMILTONIAN OF THE SYSTEM

We consider a semiconductor in the field of a strong electromagnetic wave

$$\mathbf{E}(t) = \mathbf{E}_0 \sin(\Omega t - \mathbf{kr}), \quad \mathbf{kE}_0 = 0,$$

with frequency Ω exceeding the width of the forbidden band $\mathscr{E}$. The field is assumed to be turned on at $t \to -\infty$. Following the Keldysh method,[6] we determine the density matrix of the system $\rho(t)$ from the equation

$$i\frac{\partial \rho}{\partial t} = [H_i(t)\rho(t)]_- \equiv H_i(t)\rho(t) - \rho(t)H_i(t) \qquad (1)$$

with boundary conditions

$$\rho(t=-\infty)=\rho_0=\exp\{\Psi_0-H_0(-\infty)/T\}, \quad (2)$$

where Ψ_0 is the total free energy, $H(t)=H_0+H_i(t)$ is the Hamiltonian of the system, and

$$H_0=\sum_p\{\xi_p(a_p^+(t)a_p(t)+b_p^+(t)b_p(t))$$
$$+\lambda_p a_p^+(t)b_p^+(t)+\lambda_p^* b_p(t)a_p(t)\}+H_T. \quad (3)$$

In (1)–(3) we have performed the unitary transformation

$$U(t)=\exp\left\{-i\frac{\Omega t}{2}\sum_p(a_p^+a_p+b_p^+b_p)\right\}, \quad (4)$$

which transfers the explicit time dependence from H_0 to the interaction energy $H_i(t)$. The first term in H_0 is the energy of the noninteracting electrons and holes, the second describes the resonant interaction of the electrons with the electromegnetic field, and the third the Hamiltonian of the thermostat with which the system in question is in contact. Here

$$\xi_p=p^2/2-p_0^2/2, \quad p_0^2=\Omega-\mathscr{E} \quad (5a)$$

is the dispersion law and

$$\lambda_p=\frac{eE_0v_{cv}}{2\Omega} \quad v_{cv}=\int u_{cp}\hat{p}u_{vp}\,d^3r \quad (5b)$$

is the matrix element of the transition between the valence band (v) and the conduction band (c). The quantity λ_p is henceforth assumed constant. This is valid near the edge of the band, inasmuch as for allowed transitions the matrix element v_{cv} depends little on the quasimomentum p. In (3), we neglect the wave vector k compared with p.

The operators for the creation of electrons $a_p^+(t)$ and holes $b_p^+(t)$ satisfy the free equations of motion

$$i\frac{\partial a_p^+(t)}{\partial t}=[a_p^+(t)H_0]_-, \quad i\frac{\partial b_p^+(t)}{\partial t}=[b_p^+(t)H_0]_-, \quad (6)$$

i.e., they are defined in the interaction representation.

The interaction energy operator

$$H_i(t)=H_{ef}+H_{er}(t)+H_{ee}$$

includes the interaction of the electrons with the phonons:

$$H_{ef}=\sum_{pq}g(q)(a_p^+(t)a_{p+q}(t)+b_p(t)b_{p+q}^+(t))\varphi_q(t),$$
$$\varphi_q(t)=\varphi_{-q}^+(t)=c_q^+(t)+c_{-q}(t), \quad (7)$$

The interaction with thermal photons, producing radiative recombination:

$$H_{er}(t)=\sum_{pk}M_k a_p^+(t)b_{p+k}^+(t)d_k(t)e^{i\Omega t}+\text{h.c.} \quad (8)$$

and the interaction of the electrons with one another and with the holes H_{ee}. Here $c_q^+(t)$ and $d_k^+(t)$ are the phonon and photon creation operators,

$$g^2(q)=\zeta_0\frac{\pi^2 s}{Vp_0}q, \quad M_k=e\sqrt{\frac{2\pi}{\omega_k}}(ev_{cv}), \quad (9)$$

ζ_0 is the Frohlich parameter, s the speed of sound, V = 1 the normalization volume, and ω_k the energy of a photon with polarization e.

The Hamiltonian H_0 can be diagonalized by a canonical transformation:[1]

$$a_p=u_p\alpha_p+v_p\beta_p^+, \quad b_p=u_p\beta_p-v_p\alpha_p^+,$$
$$u_p^2, v_p^2=\tfrac{1}{2}(1\pm\xi_p/\varepsilon_p), \quad \varepsilon_p=\sqrt{\xi_p^2+|\lambda|^2}, \quad (10)$$
$$H_0=\sum_p\varepsilon_p(\alpha_p^+\alpha_p+\beta_p^+\beta_p),$$

where the quasiparticles α and β are superpositions of an electron and a hole.

We are interested in the state assumed by the system within a time larger than the relaxation time, when the system "forgets" the initial state. As shown in [5], the system is described by four Green's functions: electron G_a, hole $G_b=G_a=G$, and the Gor'kov function F_+ and F, which characterize the creation and annihilation of the electron-hole pair. In the presence of recombination, knowledge of only these functions is insufficient, since we need equations describing the distribution of the particles in the nonequilibrium state. To obtain the complete system of equations, we use the Keldysh method[6] in which, besides the causal function, there appear also three Green's functions defined in the following manner:

$$G^c(pt,\ p't')=-i\,\mathrm{Sp}\{\rho_0T_C[a_p(t_+)a_{p'}^+(t_+')S_C]\}, \quad (11)$$
$$\tilde{G}^c(pt,\ p't')=-i\,\mathrm{Sp}\{\rho_0T_C[a_p(t_-)a_{p'}^+(t_-')S_C]\}, \quad (12)$$
$$G^{\pm}(pt,\ p't')=-i\,\mathrm{Sp}\{\rho_0T_C[a_p(t_\pm)a_{p'}^+(t_\mp')S_C]\}. \quad (13)$$

Here S_C is the scattering matrix defined along the contour C passing over the entire time axis from $-\infty$ to $+\infty$ (t_+), and then back to $+\infty$ to $-\infty$ (t_-); T_C denotes ordering along the contour C. The functions $G^{\pm}$ represent the particle distribution function written in different forms. We define the functions F_+ and F in the form

$$F_+^c(pt,\ p't')=\mathrm{Sp}\{\rho_0T_C[b_p^+(t_+)a_{p'}^+(t_+')S_C]\}, \quad (14)$$
$$F_+^{\pm}(pt,\ p't')=\mathrm{Sp}\{\rho_0T_C[b_p^+(t_\pm)a_{p'}^+(t_\mp')S_C]\}, \quad (15)$$
$$F^c(pt,\ p't')=\mathrm{Sp}\{\rho_0T_C[a_p(t_+)b_{p'}(t_+')S_C]\}, \quad (16)$$
$$F^{\pm}(pt,\ p't')=\mathrm{Sp}\{\rho_0T_C[a_p(t_\pm)b_{p'}(t_\mp')S_C]\} \quad (17)$$

and the remaining ones correspondingly. In similar fashion we define also the Green's function for the Bose particles, particularly the causal phonon Green's function

$$D_f^c(qt,\ q't')=-ig^2(q)\,\mathrm{Sp}\{\rho_0T_C[\varphi_q(t_+)\varphi_{q'}(t_+')S_C]\} \quad (18)$$

and the causal photon Green's function

$$D_r^c(kt,\ k't')=-iM_k^2\,\mathrm{Sp}\{\rho_0T_C[d_k(t_+)d_{k'}^+(t_+')S_C]\}. \quad (19)$$

2. SYSTEM OF EQUATIONS FOR THE GREEN'S FUNCTIONS AND THE DISTRIBUTION FUNCTIONS

Equations of the Dyson type were obtained for the saturation state in [5] without allowance for recombination, and at T = 0, with the aid of a diagram technique used in the superconductivity theory.[2,3] In this section we present equations with allowance for recombination and $T\neq 0$, derived with the aid of the Keldysh diagram technique for non-equilibrium processes. We note that the applicability of the Wick theorem in the expansion of the T-products of the electron operators in the interaction representation can be demonstrated directly by changing over to the quasiparticle operators α and β.[5,1] By replacing the vertex function with a simple vertex, which is valid for the electron-phonon interaction, by virtue of the smallness of the parameters s/p_0 and the

condition $\lambda \ll p_0^2$ (see [5]), and for the electron-photon interaction by virtue of the weakness of this interaction, we obtain the following system of equations:

$$(\omega - \xi_p - \Sigma^c(\omega))G^c(p\omega) = 1 - iF_+^c(p\omega)(\lambda + \bar{F}^c) + iF_+^-\bar{F}^+ - G^-(p\omega)\Sigma^+(\omega), \quad (20)$$

$$(\omega + \xi_p + \Sigma^c(-\omega))F_+^c(p\omega) = iG^c(p\omega)(\lambda^* + \bar{F}_+^c) + F_+^-\Sigma^-(-\omega) - iG^-(p\omega)\bar{F}_+^-, \quad (21)$$

$$(\omega - \xi_p + \tilde{\Sigma}^c(\omega))G^-(p\omega) = -iF_+^-(p\omega)(\lambda - \tilde{\bar{F}}^c) - iF_+^c\bar{F}^- + G^c(p\omega)\Sigma^-(\omega), \quad (22)$$

$$(\omega - \xi_p - \Sigma^c(\omega))G^+(p\omega) = -iF_+^+(p\omega)(\lambda + \bar{F}^c) + i\bar{F}_+^c\bar{F}^+ - \bar{G}^c(p\omega)\Sigma^+(\omega), \quad (23)$$

$$(\omega + \xi_p - \tilde{\Sigma}^c(-\omega))F_+^-(p\omega) = iG^-(p\omega)(\lambda^* - \bar{\bar{F}}_+^c) - F_+^c\Sigma^+(-\omega) + iG^c(p, \omega)\bar{F}_+^-, \quad (24a)$$

$$(\omega - \xi_p + \tilde{\Sigma}^c(\omega))F^-(p\omega) = -iG^+(p, -\omega)(\lambda - \tilde{\bar{F}}^c) + F^c\Sigma^-(\omega) - iG^c(p, -\omega)\bar{F}^-, \quad (24b)$$

where

$$\Sigma^c(\omega) = \Sigma_f^c(\omega) - \Sigma_r^c(-\omega), \quad \Sigma^-(\omega) = \Sigma_f^-(\omega) - \Sigma_r^+(-\omega), \quad (25)$$

$$\Sigma^+(\omega) = \Sigma_f^+(\omega) - \Sigma_r^-(-\omega), \quad \Sigma^{c*}(\omega) = -\tilde{\Sigma}^c(\omega);$$

$$\Sigma_f^{c,\pm}(\omega) = \frac{i}{(2\pi)^4}\int d^3q_1\, d\omega_1\, G^{c,\pm}(p - q_1, \omega - \omega_1)D_f^{c,\pm}(q_1, \omega_1);$$

$$\Sigma_r^{c,\pm}(\omega) = \frac{i}{(2\pi)^4}\int d^3k_1\, d\omega_1\, G^{c,\pm}(p - k_1, \omega - \Omega + \omega_1)D_r^{c,\mp}(k_1\omega_1), \quad (26)$$

$$\overline{F_+^{c,\pm}}(\omega) = \frac{1}{(2\pi)^4}\int d^3q_1\, d\omega_1\, F_+^{c,\pm}(p - q_1, \omega - \omega_1)D_f^{c,\pm}(q_1\omega_1).$$

From (20) and (21) we see that they differ from the equations in [5] in the fact that to each integral term there is added, with the negative sign, a similar term made up of the functions $G^{\pm}$ and $F^{\pm}$. It can be shown (see below) that these terms vanish when T = 0 and $1/\tau_r = 0$. Thus, the approach used in [5] is justified.

We derive first equations for the distribution functions. Adding Eq. (22) with its Hermitian conjugate, and taking into account the relations

$$\tilde{\Sigma}^c + \Sigma^c = \Sigma^+ + \Sigma^-, \quad \bar{G}^c + G^c = G^+ + G^-,$$

we obtain

$$G^-(p\omega)\Sigma^+(\omega) - G^+(p\omega)\Sigma^-(\omega) + i(F_+^c\bar{F}^- + \bar{F}^cF_+^-) - i(\bar{F}^-\bar{F}_+^c + F_+^-\bar{F}^c) = i(\lambda^*F^- - \lambda F_+^-). \quad (27)$$

It is necessary to add to this equation the expressions for F_+^- and F^-:

$$(2\xi_p - \tilde{\Sigma}^c(-\omega) + \Sigma^c(\omega))F_+^-(p\omega) = i\lambda^*(G^-(\omega) + G^+(-\omega)) - iG^-(\omega)\bar{F}_+^c + iG^+(-\omega)\bar{F}_+^c - F_+^c\Sigma^+(-\omega) + \bar{F}_+^c\Sigma^-(\omega) + i\bar{F}_+^-(G^c(\omega) + \bar{G}^c(-\omega)), \quad (28)$$

$$(2\xi_p + \Sigma^c(-\omega) - \tilde{\Sigma}^c(\omega))F^-(p\omega) = i\lambda(G^-(\omega) + G^+(-\omega)) + iG^-(\omega)\bar{F}^c - iG^+(-\omega)\bar{F}^c + F^c\Sigma^+(-\omega) - F^c\Sigma^-(\omega) - i\bar{F}^-(\bar{G}^c(\omega) + G^c(-\omega)). \quad (29)$$

Equations (27)–(29) make it possible to obtain the kinetic equations under the assumptions that they exist, i.e., under the condition that the interaction be small.

In accordance with the assumption that the interaction is small, we should calculate the functions Σ and $\bar{F}$ in first order in the interaction, i.e., substitute in (27)–(29) Green's functions satisfying the free equations but normalized to the still-unknown electron distribution function f_p by the following conditions:[6]

$$\int_{-\infty}^{\infty}(G^+(p\omega) - G^-(p\omega))\,d\omega = 2\pi i, \quad f_p = \frac{1}{2\pi i}\int_{-\infty}^{\infty}G^+(p\omega)\,d\omega. \quad (30)$$

We write the zeroth Green's functions in the form (see [7])

$$G_0^-(p\omega) = -2\pi i[(1 - n_p)u_p^2\delta(\omega - \varepsilon_p) + n_pv_p^2\delta(\omega + \varepsilon_p)], \quad (31)$$

$$G_0^+(p\omega) = 2\pi i[n_pu_p^2\delta(\omega - \varepsilon_p) + (1 - n_p)v_p^2\delta(\omega + \varepsilon_p)];$$

$$G_0^c(p\omega) = \frac{\omega + \xi_p}{(\omega - \varepsilon_p + i\delta)(\omega + \varepsilon_p - i\delta)} + 2\pi i n_p(u_p^2\delta(\omega - \varepsilon_p) - v_p^2\delta(\omega + \varepsilon_p)), \quad (32)$$

($\delta \to +0$) etc., and the functions F_0 are expressed in terms of G_0 with the aid of the equations (20)–(24). We have introduced here a function n_p, having the meaning of the distribution function of the quasiparticles α and β. It is connected with f_p by the relation

$$f_p = n_pu_p^2 + (1 - n_p)v_p^2. \quad (33)$$

To calculate Σ and $\bar{F}$, we should determine also D_{f0} and D_{r0}. Assuming that the phonons are in thermodynamic equilibrium, and that the number of photons is equal to zero, we get

$$D_{f0}^{\pm}(q\omega) = -2\pi ig^2(q)\{(1 + N_q)\delta(\omega \pm \omega_q) + N_q\delta(\omega \mp \omega_q)\},$$

$$D_{r0}^-(k\omega) = -2\pi iM_k^2\delta(\omega - \omega_k), \quad D_{r0}^+(k\omega) = 0, \quad (34)$$

where $N_q = [\exp(\omega_q/kT) - 1]^{-1}$ is the Planck distribution function.

Let us substitute in (27) the difference $\lambda^*F^- - \lambda F_+^-$, determined from (28) and (29); we then obtain after certain calculations the kinetic equation for the quasiparticles:

$$\left(\frac{\partial n}{\partial t}\right)_f + \left(\frac{\partial n}{\partial t}\right)_r = 0,$$

$$\left(\frac{\partial n}{\partial t}\right)_f = 2\pi\sum_{q;\ i,\ j=1,\ 2}g^2(q)n_{ip}n_{jp+q}[(1 + N_q)\delta(\varepsilon_{ip} + \varepsilon_{jp+q} - \omega_q) + N_q\delta(\varepsilon_{ip} + \varepsilon_{jp+q} + \omega_q)]\Phi(ip, jp + q), \quad (35)$$

$$\left(\frac{\partial n}{\partial t}\right) = \pi\sum_{k;\ i,\ j=1,\ 2}M_k^2(-1)^i u_{ip}^2u_{jp+k}^2n_{ip}n_{jp+k}\delta(\omega_k - \Omega - \varepsilon_{ip} - \varepsilon_{jp+k}),$$

where

$$n_{1p} = n_p, \quad n_{2p} = 1 - n_p, \quad \varepsilon_{1,\,2p} = \pm\varepsilon_p, \quad u_{1p}^2 = u_p^2, \quad u_{2p}^2 = v_p^2,$$

$$\Phi(1p, 1p') = -\Phi(2p', 2p) = (v_pu_{p'} - v_{p'}u_p)^2,$$

$$\Phi(1p, 2p') = -\Phi(2p', 1p) = (u_pu_{p'} + v_pv_{p'})^2,$$

This equation coincides with that derived in [8] by the Bogolyubov method.

The Green's functions (32) in conjunction with the kinetic equation (35) yield the solution of the problem for weak interactions. We note that the recombination integral can be simplified:

$$\left(\frac{\partial n}{\partial t}\right)_r = \frac{1}{\tau_r}[v_p^4(1 - n_p)^2 - n_p^2u_p^4]; \quad \frac{1}{\tau_r} \approx \frac{4}{3}e^2|v_{cv}|^2\Omega, \quad (35a)$$

by omitting small terms of the order of p_0k/Ω. Let us consider some limiting cases.

If there is no recombination, then we obtain from the condition that $(\partial n/\partial t)_f$ must vanish, but n_p is the Fermi function with a potential equal to zero:[1]

$$n_p = [e^{\varepsilon_p/T} + 1]^{-1}. \quad (36)$$

When T = 0, the function n_p vanishes, and the Green's function (32) coincides with the expression obtained in [5]. This proves that the Landau theory in the saturation state is valid. If recombination prevails ($\tau_r \ll \tau_f$), then, by equating the integral (35a) to zero, we get[8]

$$n_p = v_p^2 \quad (37)$$

In this case the Green's function (32) and the distribution function are respectively equal to

$$G_0^c(p\omega)=\frac{u_p^2}{\omega-\varepsilon_p+i\delta}+\frac{v_p^2}{\omega+\varepsilon_p-i\delta}$$
$$+2\pi i v_p^2(u_p^2\,\delta(\omega-\varepsilon_p)-v_p^2\,\delta(\omega+\varepsilon_p)),$$
$$f_p=\lambda^2/2(\xi_p^2+\lambda^2).$$

Let us consider now a situation in which recombination and quasiparticle interaction with the phonons take place simultaneously. We assume that the probability of the electron-phonon scattering exceeds the recombination probability (this corresponds, as a rule, to the real situation). In this case, a quasiequilibrium is rapidly established for the produced quasiparticles, so that the distribution function takes the form

$$n_p=[\exp\{(\varepsilon_p-\mu)/T\}+1]^{-1}. \tag{38}$$

The chemical potential of the quasiparticles is obtained from the condition for the equality of the number of creative and annihilated quasiparticles:

$$\sum_{pq}2\pi\Phi(1p,1p+q)\,\delta(\varepsilon_p+\varepsilon_{p+q}-\omega_q)\,[n_p n_{p+q}-N_q(1-n_p-n_{p+q})]$$
$$=\sum_p\frac{1}{\tau_r}[v_p^4(1-n_p)^2-u_p^4 n_p^2], \tag{39}$$

which is obtained by integrating (35) with respect to the momentum. At $T\ll\lambda$, the number of quasiparticles is small and

$$n_p\approx\exp\left(-\frac{\varepsilon_p-|\lambda|}{T}\right)\alpha,\quad \alpha=\frac{p_0^2}{|\lambda|}\left(\frac{s^2}{12\pi^2\zeta_0T^2\tau_r}\right)^{1/2}\ll 1. \tag{40}$$

In the case when $T\gg\lambda$, quasiparticles are produced mainly as a result of the temperature ($\mu\to 0$), and n_p is given by formula (36).

3. ABSORPTION OF STRONG ELECTROMAGNETIC WAVES. SATURATION EFFECT

The system of equations in Sec. 2 makes it possible to solve the problem of the absorption of a strong external electromagnetic field, a problem considered earlier in [8-10]. We are interested in the absorption of energy in interband transitions. We define this absorption as the time-average of the scalar product of the field by the current, and express it in terms of the functions F_+^- and F^-:

$$Q=\frac{1}{2}\overline{\langle j\rangle E(t)}=\frac{i}{2}\Omega\sum_{p\omega}[\lambda F_+^-(p\omega)-\lambda^* F^-(p\omega)]. \tag{41}$$

We integrate Eq. (27) with respect to the frequency and the momenta. We change the order of integration with respect to momenta, and using the relation

$$D_f^+(-\omega q)=D_f^-(\omega q),$$

we obtain the equality

$$\int(\lambda^* F^-(p\omega)-\lambda F_+^-(p\omega))\,d^3p\,d\omega=i\int(G^-(p\omega)\Sigma_r^-(-\omega)$$
$$-G^+(p\omega)\Sigma_r^+(-\omega))\,d^3p\,d\omega, \tag{42}$$

which expresses the law of conservation of the number of electrons. Substituting (42) in (41), we obtain

$$Q=-\Omega\int\frac{d^3p\,d\omega}{(2\pi)^4}(G^+(p\omega)\Sigma_r^+(-\omega)-G^-(p\omega)\Sigma_r^-(-\omega)). \tag{43}$$

Expression (43) has a clear cut physical meaning. The integral to the right is the number of electrons recombining per unit time and multiplied by the energy lost in one recombination act. The first term describes the loss of particles as a result of recombination, and the second the increase in the number of particles as a result of absorption of thermal phonons. It follows from (43) that absorption is possible only in the presence of recombination. Expression (43) is convenient for the calculation of absorption of both weak and strong fields. In the strong-field case of interest to us, when the recombination probability $1/\tau_r$ is small compared with λ, it is necessary to choose as the functions $G^\pm$ the zeroth-approximation functions (31). Substituting them as well as the Green's functions of the photons with $N_k = 0$, we obtain

$$Q=\frac{\Omega}{(2\pi)^5}\int d^3p\,d^3k M_k^2\,[n_p n_{p-k}u_p^2u_{p-k}^2\,\delta(\omega_k-\Omega-\varepsilon_p-\varepsilon_{p-k})$$
$$+2n_p(1-n_{p-k})v_p^2u_{p-k}^2\,\delta(\omega_k-\Omega-\varepsilon_{p-k}+\varepsilon_p)$$
$$+(1-n_p)(1-n_{p-k})v_p^2v_{p-k}^2\,\delta(\omega_k-\Omega+\varepsilon_p+\varepsilon_{p-k})], \tag{44}$$

where n_p is determined by the kinetic equation (35). Formula (44) makes it possible to obtain the dependence of Q on the field, temperature, frequency, and other parameters. Let us consider some limiting cases.

1. If $\tau_r\ll\tau_f$, then n_p is given by (37) and the absorption is equal to

$$Q\approx\frac{\Omega}{\tau_r}\sum_p 4(u_pv_p)^4\approx\frac{\Omega}{\tau_r}\frac{p_0|\lambda|}{8\pi},\quad Q\sim E_0, \tag{45}$$

if we omit small terms of order p_0^2/Ω. A relation of this type was obtained earlier in [8,10].

2. In the customarily realized case $\tau_r\gg\tau_f$, the function n_p is determined by expression (38) and we have for Q

$$Q\approx\frac{\Omega}{\tau_r}\sum_p v_p^4=\frac{\Omega}{\tau_r}n_0,\quad Q=\text{const}, \tag{46}$$

omitting small terms of the order of p_0^2/Ω, λ, T/p_0^2. Consequently, the absorbed power does not depend on the field intensity. This result, known as the saturation effect, was effect by Krokhin[9] with the aid of perturbation theory with respect to the field, which is not valid in our case. However, inasmuch as electron recombination goes from a phase volume p_0^3 that exceeds the region of interaction with external field, $p_0\lambda$, then the results coincide accurate to λ/p_0^2. The situation is different in a quantum generator. In the generation regime, the principal role is played by the region near p_0, so that the singularities of the spectrum can greatly influence the operation of the quantum generator, particularly the gain, the occurrence of mode interaction, etc.[11]

4. DAMPING AND ENERGY SPECTRUM OF ELECTRON EXCITATIONS

By following Keldysh, we change over from (20)–(25) to a system of retarded Green's functions G^R, F_+^R, advanced Green's functions G^A and F_+^A, and correlation functions ρ and ρ_+, which are connected with the previously introduced functions by the following relations

$$G^{R,A}(p\omega)=G^c(p\omega)-G^\pm(p\omega),\quad F_+^{R,A}=F_+^c-F_+^\pm, \tag{47}$$
$$\rho(p\omega)=G^+(p\omega)+G^-(p\omega),\quad \rho_+(p\omega)=\rho_-^*(p\omega)=F_+^+(p\omega)+F_+^-(p\omega).$$

As a result we get

$$(\omega - \xi_p - \Sigma^R(\omega))G^R(p\omega) = 1 - iF_+^R(p\omega)(\lambda + \bar{F}^R), \quad (48)$$

$$(\omega + \xi_p + \Sigma^{R*}(-\omega))F_+^R(p\omega) = iG^R(p\omega)(\lambda^* + \bar{F}_+^R), \quad (49)$$

$$(\omega - \xi_p - \Sigma^R(\omega))\rho(p\omega) = -i\rho_+(\omega)(\lambda + \bar{F}^R) + G^A(p\omega)\Sigma(\omega) - F_+^A\bar{F}, \quad (50)$$

$$(\omega - \xi_p - \Sigma^R(\omega))\rho_+(p\omega) = -i\rho(p, -\omega)(\lambda + \bar{F}^R) - G^A(-\omega)\bar{F} + F^A\Sigma(\omega). \quad (51)$$

The self-energy parts $\bar{\Sigma}^R$ and $\bar{F}_+^R$ are expressed in terms of the functions G^R, F_+^R, ρ, and ρ_+ with the aid of relations (47) and (52):

$$\Sigma^R(\omega) = \Sigma^-(\omega) - \tilde{\Sigma}^e(\omega), \qquad \bar{F}_+^R = \bar{F}_+^- - \bar{F}_+^e,$$
$$\Sigma = \Sigma^+ + \Sigma^-, \qquad \bar{F} = \bar{F}^+ + \bar{F}^-. \quad (52)$$

It is convenient to rewrite (48) and (49) in the form

$$G^R(p\omega) = \frac{\omega + \xi_p + \Sigma^{R*}(-\omega)}{\Omega(p\omega)}, \qquad F_+^R(p\omega) = \frac{i\lambda^* + \overline{F_+^R}(\omega)}{\Omega(p\omega)}, \quad (53)$$

where

$$\Omega(p\omega) = (\omega - \xi_p - \Sigma^R(\omega))(\omega + \xi_p + \Sigma^{R*}(-\omega)) - (\lambda + \bar{F}^R)(\lambda^* + \bar{F}_+^R).$$

To find the damping and the spectrum of the electron excitations in the saturation state, we can simplify greatly the solution of the system by taking into account the following circumstances. The strong-field condition denotes that the frequency transitions between the bands λ exceeds the recombination time τ_r^{-}, and therefore the electron-photon interaction is small compared with λ and consequently, compared with the average particle energy $\bar{\epsilon} \sim p_0^2$ (inasmuch as in our case $\lambda \ll p_0^2$):

$$\tau_r^{-1} \ll \lambda \ll \varepsilon \sim p_0^2.$$

As to the electron-phonon interaction, in the stationary state it is characterized only by the parameter $(\bar{\epsilon}\tau_f)^{-1} \sim \xi_0$,[5] and the difference between the results of the exact solution with $\xi_0 \approx 1$ and the approximate one reduces to an inessential renormalization ξ_0. (We note that for most semiconductors $\xi_0 \ll 1$.) Consequently, in first approximation we can limit ourselves to perturbation theory in the interaction, calculating Σ^R and $\bar{F}_+^R$ with zero Green's functions (31) and (32), which contain the quasiparticle distribution functions satisfying the kinetic equation (35). Substituting (31) and (32) in (52), we obtain after certain calculations

$$\Sigma_{r0}^R(\omega) = \int \frac{d^3k}{(2\pi)^3} \frac{v_{p+k}^2 M_k^2}{\Omega - \omega_k - \varepsilon_{p+k} + \omega + i\delta}, \quad (54)$$

$$\Sigma_{f0}^R(\omega) = \int \frac{d^3q}{(2\pi)^3} [v_{p+q}^2 \Pi(\omega_q, \varepsilon_{p+q}) + u_{p+q}^2 \Pi(-\omega_q, -\varepsilon_{p+q})] g^2(q), \quad (55)$$

$$\overline{F_{+0}^R}(\omega) = -\lambda \int \frac{d^3q}{2(2\pi)^3 \varepsilon_{p+q}} [\Pi(\omega_q, \varepsilon_{p+q}) - \Pi(-\omega_q, -\varepsilon_{p+q})] g^2(q), \quad (56)$$

$$\Pi(\omega_q, \varepsilon) = \frac{1}{\omega + \varepsilon + \omega_q + i\delta} + n_p\left(\frac{1}{\omega + \varepsilon - \omega_q + i\delta} - \frac{1}{\omega + \varepsilon + \omega_q + i\delta}\right) + N_q\left(\frac{1}{\omega + \varepsilon + \omega_q + i\delta} + \frac{1}{\omega + \varepsilon - \omega_q + i\delta}\right). \quad (57)$$

It is seen from (54)–(57) that ω enters in the denominator in the combination $\omega + i\delta$. Thus, all the expressions turn out to be analytic in the upper half plane of the complex variable ω, as they should.

In the isotropic case considered by us, Σ^R and $\bar{F}_+^R$ are practically independent of p when $p \approx p_0$. Let us calculate first the recombination self-energy part Σ_{r0}^R. The real part of Σ_{r0}^R does not depend on ω and is a correction to the chemical potential

$$\mathrm{Re}\,\Sigma_{r0}^R \approx -{}^1\!/_2 e^2 p_0 |v_{ev}|^2. \quad (58)$$

The imaginary part coincides with the recombination probability

$$\mathrm{Im}\,\Sigma_{r0}^R \approx -1/2\tau_r. \quad (59)$$

The function $\Sigma_{f0}^R(\omega)$ breaks up into an odd part $f_0(\omega)$ and an even part $\mu_f + if_1(\omega)$. μ_f contains a component in which the essential region of integration is far from the Fermi quasilevel. This yields the correction to the chemical potential $\mu_0 = p_0^2/2$, which is practically independent of the temperature and the recombination. The contribution to the functions $f_0(\omega)$ and $f_1(\omega)$ comes from the region near p_0. We can therefore change over from integration with respect to q to integration with respect to $q = |\mathbf{q}|$, $\epsilon = \sqrt{\xi_p^2 + \lambda^2}$ and with respect to the angle φ, where $\xi_p \approx p_0(p_1 - p_0)$, $p_1 = |\mathbf{p} + \mathbf{q}|$, $\lambda \equiv |\lambda|$. As a result we get

$$f(\omega) = f_0(\omega) + if_1(\omega) = \frac{1}{8\pi^2 p_0}\int_\lambda^\infty \frac{\varepsilon\, d\varepsilon}{\sqrt{\varepsilon^2 - \lambda^2}} \int_0^{2p_0} dq \cdot qg^2(q)\left\{\left[(1 - 2n(\varepsilon)) + \mathrm{cth}\frac{\omega_q}{2T}\right]\left[\frac{1}{\omega + \varepsilon + \omega_q + i\delta} + \frac{1}{\omega - \varepsilon - \omega_q + i\delta}\right] + \left[\mathrm{cth}\frac{\omega_q}{2T} - (1 - 2n(\varepsilon))\right]\left[\frac{1}{\omega + \varepsilon - \omega_q + i\delta} + \frac{1}{\omega - \varepsilon + \omega_q + i\delta}\right]\right\}. \quad (60)$$

If $n(\epsilon)$ is given by (36), then the function $f(\omega)$ coincides with the expression obtained by Éliashberg[4] with the aid of the temperature Green's functions in a superconductor. Thus, allowance for the nonequilibrium nature of the system reduces to the substitution $\tanh(\epsilon/2T) \to 1 - 2n(\epsilon)$, with $n(\epsilon)$ determined by the solution of the kinetic equation.

The real part $f_0(\omega)$ of this equation hardly differs from its value at $T = 0$ and $1/\tau_r = 0$. Let us examine in greater detail the imaginary part $f_1(\omega)$:

$$f_1(\omega) = -\frac{1}{8\pi p_0}\int_\lambda^\infty \frac{d\varepsilon \cdot \varepsilon}{\sqrt{\varepsilon^2 - \lambda^2}} \int_0^{2p_0} dq \cdot qg^2(q)\left\{\left[(1 - 2n(\varepsilon)) + \mathrm{cth}\frac{\omega_q}{2T}\right][\delta(\omega + \varepsilon + \omega_q) + \delta(\omega - \varepsilon - \omega_q)] + \left[\mathrm{cth}\frac{\omega_q}{2T} - (1 - 2n(\varepsilon))\right][\delta(\omega + \varepsilon - \omega_q) + \delta(\omega - \varepsilon + \omega_q)]\right\}. \quad (61)$$

The first term vanishes when $\omega \le \lambda$. The second term differs from zero at all ω, but vanishes when $T = 0$ and $1/\tau_r = 0$. We are most interested in the second term $f^{(2)}(\omega)$, since this is precisely the term determining the damping when $\omega \le \lambda$. We note that in very strong fields at $\lambda - \omega > \omega_0 = 2p_0 s$, the term $f_1^{(2)}$ also vanishes. In the opposite limiting case, $\lambda \ll \omega_0$, the contribution to (61) is made by $\omega_q \sim T \ll \omega_0$, and we can write

$$f_1^{(2)}(\omega) \approx -\frac{2\pi\xi_0}{\omega_0^2}\int_\lambda^\infty \frac{d\varepsilon \cdot \varepsilon(\omega^2 + \varepsilon^2)}{\sqrt{\varepsilon^2 - \lambda^2}} n(\varepsilon), \quad (62)$$

Omitting the terms with N_q, which were calculated in [4].

Let us consider the low-temperature case, $T \ll \lambda$, when $n(\epsilon)$ is given by formula (40). We then obtain from (62)

$$f_1^{(2)}(\omega) \approx -2\pi^{3/2}\xi_0\left(\frac{T}{2\lambda}\right)^{1/2}\frac{\lambda}{\omega_0^2}(\omega^2 + \lambda^2)\alpha. \quad (63)$$

We call attention to the fact that the exponential de-

pendence of the damping on the temperature $e^{-\lambda/T}$ is replaced by $\alpha(T)$. The physical reason for the abrupt increase of the damping lies in the fact that the quasiparticles are produced as a result of the redistribution of the particles under the influence of the external field and as a result of recombination, and not temperature.

If $T \gg \lambda$, then $n(\epsilon)$ is determined by formula (36) and the damping $f_1^{(2)}$ is given by the expression obtained in [4]:

$$f_1^{(2)}(\omega) \approx -8.2\pi\zeta_0(T/\omega_0)^2 T, \quad \omega \lesssim \lambda \tag{64}$$

Let us calculate now $\overline{F}^R_{+0}$. The imaginary part of $\overline{F}^R_{+0}$ has a structure similar to $f_1(\omega)$. An investigation shows that the damping connected with $\overline{F}^R_{+0}$ is smaller than $f_1(\omega)$. The real part of $\overline{F}^R_{+0}$ gives the correction to the gap Δ:

$$\Delta = \lambda + \mathrm{Re}\, \overline{F}_{+0}^R(\omega). \tag{65}$$

The expression for Δ can be written, accurate to $(T/\omega_0)^2$, in the form

$$\Delta = \lambda - \frac{\lambda}{4\pi^2 p_0}\int_\lambda^\infty \frac{d\varepsilon}{\sqrt{\varepsilon^2-\lambda^2}}(1-2n(\varepsilon))\int_0^{2p_0} dq\cdot q g^2(q)\left(\frac{1}{\varepsilon+\omega_q+\omega}+\frac{1}{\varepsilon+\omega_q-\omega}\right). \tag{66}$$

This result coincides at $n(\epsilon) = 0$ with the equation for the gap in [5]. It is seen from (66) that the increase of $n(\epsilon)$ as a result of temperature recombination leads to a growth of the gap, which tends to the value λ. Thus, the gap exists in a wide temperature interval. But it must be remembered that the very concept of the gap becomes meaningless if the damping of the excitations becomes comparable with Δ. However, if the damping is small compared with Δ, then we can determine approximately the gap that depends on the temperature and recombination. From a comparison of the damping (59), (63), and (64) with the gap (65) it follows that for the gap to exist at low temperatures ($T < \lambda$) it is necessary to have

$$\lambda \gg \tau_r^{-1}, \tag{67}$$

i.e., to satisfy the strong-field condition.[5] At high temperatures, $T \gg \lambda$, the gap exists if

$$\lambda > 8\pi\zeta_0(T/\omega_e)^2 T. \tag{68}$$

The analysis presented in the paper for the model of the electron-phonon interaction can be repeated in similar fashion for the electron-electron and electron-hole interactions. In this case there is obtained an additional damping analogous to the electron-phonon damping, with the substitution

$$\zeta_0\left(\frac{\lambda, T}{\omega_0}\right)^2 \to \sqrt{\frac{e^2}{p_0}}\left(\frac{T,\lambda}{p_0^2}\right)$$

(see also [12]).

The electron-hole interaction has a somewhat different influence on the gap. The equation for the gap at $T = 0$, obtained in the approximation $e^2/p_0 \ll 1$, is (see [13])

$$\Delta = \frac{\lambda}{1-e^{*2}\ln(2\omega_e/\Delta)}, \quad e^{*2} = \frac{e^2}{4\pi p_0}\ln\frac{4p_0^2}{\varkappa_e^2}, \tag{69}$$

where $\omega_e = 2p_0\kappa_e$, $\kappa_e^2 = 4p_0e^2/\pi$, e^2 is the electron charge divided by the dielectric constant. At $\lambda = 0$, Eq. (69) goes over into the Keldysh-Kopaev equation for the gap of an exciton insulator. It is seen from (69) that the gap increases greatly as a result of the electron-hole attraction.

In conclusion we present typical values of parameters of semiconductor lasers based on GaAs:

$m \approx 0.1m_0$, $n_0 \approx 10^{18}$ cm^{-3}, $\omega_0 \approx 3\cdot 10^{12}$ sec^{-1}, $\mu_0 \approx 0.04$ eV, $\tau_r \approx 10^{-9}$ sec $\lambda \approx (10^{11} - 10^{13})$ sec^{-1} at $E_0 = (10^3 - 10^5)$ V/cm

The author is grateful to Yu. A. Bykovskiĭ, V. M. Galitskiĭ for a discussion of the work.

[1] V. M. Galitskiĭ, S. P. Goreslavskii, and V. F. Elesin, Zh. Eksp. Teor. Fiz. **57**, 207 (1969) [Sov. Phys.-JETP **30**, 117 (1970)].

[2] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965)].

[3] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **38**, 996 (1960) [Sov. Phys.-JETP **11**, 716 (1960)].

[4] G. M. Éliashberg, ibid **39**, 1437 (1960) [**12**, 1000 (1961)].

[5] V. F. Elesin, Fiz. Tverd. Tela **11**, 820 (1969) [Sov. Phys.-Solid State **11**, 672 (1970)].

[6] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys.-JETP **20**, 1018 (1965)].

[7] M. P. Kemoklidze and L. P. Pitaevskiĭ, ibid **52**, 1556 (1967) [**25**, 1036 (1967)].

[8] S. P. Goreslavskiĭ and V. F. Elesin, Voprosy teorii atomnykh stolknoveniĭ (Topics in the Theory of Atomic Collisions), Atomizdat, 1970, p. 158.

[9] O. N. Krokhin, Fiz. Tverd. Tela **7**, 2612 (1965) [Sov. Phys.-Solid State **7**, 2114 (1966)].

[10] Yu. L. Klimontovich and E. V. Pogorelova, Zh. Eksp. Teor. Fiz. **51**, 1722 (1966) [Sov. Phys.-JETP **24**, 1165 (1967)].

[11] S. P. Goreslavskiĭ and V. F. Elesin, ZhETF Pis. Red. **10**, 491 (1969) [JETP letters **10**, 000 (1969)].

[12] V. S. Elesin, Fiz. Tekh. Poluprov. **4**, 1522 (1970) [Sov. Phys.-Semiconductors **4**, 000 (1970)].

[13] V. Yu. Nikitin and I. A. Poluektov, ibid **3**, 851 (1969) [**3**, 000 (1969)].

[13] L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela **6**, 2789 (1964) [Sov. Phys.-Solid State **6**, 2255 (1965)].

Translated by J. G. Adashko
70