THERMAL RADIATION IN A RANDOMLY INHOMOGENEOUS MEDIUM

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Expressions are obtained for the correlation functions of an electromagnetic fluctuation field in a randomly inhomogeneous non-absorbing medium (dielectric) with small-scale dielectric-constant fluctuations. The calculations are carried out in an approximation in which only the first term in the mass-operator series is taken into account (Bourret approximation). It is shown that spatial dispersion and scattering processes due to macroscopic inhomogeneities of the medium play an important role in the formation of a thermal electromagnetic field. The case is also considered in which the appearance of longitudinal field oscillations in an inhomogeneous medium becomes possible (cold inhomogeneous plasma with a frequency close to the Langmuir frequency). The physical meaning of the quantities $\operatorname{Im} \epsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})$ in this case is analyzed in detail for a one-dimensional model of a randomly inhomogeneous plasma. Scattering ceases to play an important role, and the whole picture is determined by the plasma oscillations at points at which $\epsilon(\omega, \mathbf{r}) = 0$.

1. INTRODUCTION

In the theory of propagation and emission of waves in a medium with random inhomogeneities, many of the results obtained in recent years go beyond the limits of the perturbation method, which is limited by the condition of weak inhomogeneity of the medium. In particular, equations of the Dyson and of the Bethe-Salpeter type were derived for the first and second moments of random fields with the aid of a diagram technique[1-6].

There exists a class of problems related to the problem of wave propagation in media with refractive-index fluctuations. These include the statistical problems of radio engineering (for example, the current in a circuit with variable parameters), the problem of the behavior of particles in random fields[7], and many others.

In the electrodynamics of a randomly inhomogeneous medium, the most convenient form of describing the average field is to introduce the tensor of the effective dielectric constant of the medium. With the aid of $\epsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})$, the problem of the average field reduces to the corresponding problem in a homogeneous absorbing medium. The significance of such a characteristic of the medium as $\epsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})$ is not limited, however, to this fact. It was shown in[8] that a number of quantities quadratic in the field can be calculated in terms of $\epsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})$. The loss to radiation by given currents (for example, the radiation of a charged particle moving with constant velocity in an inhomogeneous medium[9]), and the radiation resistance and the intensity of the thermal fluctuation fields in randomly inhomogeneous media, and so on, can be obtained in terms of $e_{ij}^{\text{eff}}(\omega, \mathbf{k})$.

In this paper, which is a continuation of[8], we calculate the correlation function of a thermal electromagnetic field in a strongly inhomogeneous non-absorbing dielectric, and investigate the influence of spatial dispersion (connected with the inhomogeneity of the medium) on the formation of the fluctuation field. In spite of the absence of absorption, the thermal field contains a longitudinal component that is "tied-in" with the inhomogeneities of the medium.

In the case when the random function $\epsilon(\omega, \mathbf{r})$ can vanish (in random fashion) at individual points of the volume of the medium (we consider a cold plasma at a frequency close to the Langmuir frequency), the picture on the whole changes significantly. At these points there occur plasma oscillations which eventually determine the character of the damping of the average field. The inhomogeneity of the medium leads in this case only to a change in the resonant frequency of the oscillations and to the appearance of a damping decrement connected with the transformation of the energy of the average field into energy of longitudinal oscillations[5].

2. CORRELATION FUNCTIONS OF THERMAL FIELD IN AN INHOMOGENEOUS DIELECTRIC

The correlation functions of fluctuating electric and magnetic fields in an unbounded medium with random inhomogeneities are expressed in terms of the effective dielectric constant of the medium[8]:

$$\begin{gathered}
\langle \mathbf{E}(\mathbf{r}_1)\mathbf{E}(\mathbf{r}_2)\rangle_\omega = \frac{\omega\Theta}{2\pi^3 c^2}\operatorname{Im}\int\left[\frac{2}{k^2 - k_0^2\varepsilon_{\text{eff}}^{tr}(\omega, k)} + \frac{1}{k_0^2\varepsilon_{\text{eff}}^{l}(\omega, k)}\right]e^{i\mathbf{k}\mathbf{R}}d\mathbf{k},\\
\langle \mathbf{H}(\mathbf{r}_1)\mathbf{H}(\mathbf{r}_2)\rangle_\omega = \frac{\omega\Theta}{\pi^3 c^2}\operatorname{Im}\int\frac{\varepsilon_{\text{eff}}^{tr}(\omega, k)}{k^2 - k_0^2\varepsilon_{\text{eff}}^{tr}(\omega, k)}e^{i\mathbf{k}\mathbf{R}}d\mathbf{k},\\
\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2, \quad \Theta = \varkappa T, \quad k_0 = \omega / c.
\end{gathered} \tag{1}$$

We investigate here the case when the effective dielectric constant of the medium is determined by the first term of the series of the mass operator[5] (the Bourret approximation). This approximation is expressed by the formulas

$$\begin{gathered}
\varepsilon_{\text{eff}}^{l}(\omega, k) = \varepsilon_0(\omega)\left[1 + \xi_{\text{eff}}^{l}(\omega, k)\right],\\
\varepsilon_{\text{eff}}^{tr}(\omega, k) = \varepsilon_0(\omega)\left[1 + \xi_{\text{eff}}^{tr}(\omega, k)\right],\\
\xi_{\text{eff}}^{l}(\omega, k) = -2\langle\xi^2\rangle\, q(p, p_0),
\end{gathered}$$

$$\xi_{\mathrm{eff}}^{tr}(\omega, k)=\frac{p_0^2\langle\xi^2\rangle}{p}\int_0^\infty \Gamma_\xi(x)e^{ip_0x}\sin px\,dx+\langle\xi^2\rangle q(p,p_0), \quad (2)$$

$$q(p,p_0)=\sqrt{\frac{\pi}{2}}\int_0^\infty \Gamma_\xi(x)\frac{J_{5/2}(px)}{\sqrt{px}}\frac{dx}{x}+$$

$$+\frac{p_0^2}{2p}\int_0^\infty \Gamma_\xi(x)\left[\frac{1}{px}\left(\frac{\sin px}{px}-\cos px\right)-\sin px\right]dx$$

$$-i\frac{p_0^3}{3p}\int_0^\infty \Gamma_\xi(x)x\sin px\,dx.$$

Here $\xi_{\mathrm{eff}}^{l}(\omega, k)$ and $\xi_{\mathrm{eff}}^{\mathrm{tr}}(\omega, k)$ are the effective longitudinal and transverse polarizabilities of the medium, $p = kl$, $p_0 = k_0\sqrt{\epsilon_0(\omega)}\,l$, and $\Gamma_\xi(x)$ is the normalized correlation function of the random polarizability of the medium $\xi(\omega, \mathbf{r})$ ($B_\xi(\mathbf{r}) = \langle\xi^2\rangle\Gamma_\xi(\mathbf{r}/l)$). Formulas (2) are valid when $|\xi_{\mathrm{ij}}^{\mathrm{eff}}(\omega, k)| \ll 1$. The random polarizability of the medium $\xi(\omega, \mathbf{r})$ plays an important role in the electrodynamics of randomly inhomogeneous forces with strong fluctuations of the dielectric constant $\epsilon(\omega, \mathbf{r})$. It is expressed in terms of $\epsilon(\omega, \mathbf{r})$ by the formula

$$\xi(\omega,\mathbf{r})=3\frac{\varepsilon(\omega,\mathbf{r})-\varepsilon_0(\omega)}{\varepsilon(\omega,\mathbf{r})+2\varepsilon_0(\omega)}. \quad (3)$$

The quantity $\epsilon_0(\omega)$ is the limiting value of $\epsilon_{\mathrm{eff}}^{l,\mathrm{tr}}(\omega, k)$ as $k_0, k \to 0$

$$\varepsilon_0(\omega)=\lim_{k_0,\,k\to 0}\varepsilon_{\mathrm{eff}}^{l,\,tr}(\omega, k). \quad (4)$$

The dielectric constant $\epsilon_0(\omega)$ describes the long-wave components of the average quasistationary field. Its value is determined by the equation

$$\langle\xi(\omega,\mathbf{r})\rangle=0. \quad (5)$$

The averaging is over the realizations of the process $\epsilon(\omega, \mathbf{r})$. The dependence of $\epsilon_{\mathrm{ij}}^{\mathrm{eff}}(\omega, k)$ on $p = kl$, connected with the spatial dispersion of the inhomogeneous medium, is naturally characterized by the parameter l/λ. This dependence is important, since it determines the character of the convergence of the integrals (1).

We begin our analysis with a dielectric, assuming that $\epsilon(\omega, \mathbf{r}) > 0$. For a dielectric, $\epsilon_0(\omega)$ and $\langle\xi^2\rangle$ are real positive quantities, and therefore

$$\mathrm{Im}\,\varepsilon_{\mathrm{eff}}^{l}(\omega, k)=\mathrm{Im}\,\varepsilon_{\mathrm{eff}}^{tr}(\omega, k)=\frac{2}{3}\varepsilon_0(\omega)\langle\xi^2\rangle p_0^3\int_0^\infty \Gamma_\xi(x)x\frac{\sin px}{p}dx. \quad (6)$$

The real parts of $\epsilon_{\mathrm{eff}}^{l}(\omega, k)$ and $\epsilon_{\mathrm{eff}}^{\mathrm{tr}}(\omega, k)$ differ from each other. We write $\epsilon_{\mathrm{eff}}^{l}(\omega, k)$ and $\epsilon_{\mathrm{eff}}^{\mathrm{tr}}(\omega, k)$ in the form

$$\begin{gathered}\varepsilon_{\mathrm{eff}}^{l}(\omega, k)=\varepsilon_0(\omega)[u_1{}^l+iu_2],\\ \varepsilon_{\mathrm{eff}}^{tr}(\omega, k)=\varepsilon_0(\omega)[u_1{}^{tr}+iu_2],\\ u_2\ll u_1{}^l,\quad u_2\ll u_1{}^{tr}.\end{gathered} \quad (7)$$

We have assumed that the inhomogeneities of the medium have a small scale, i.e., $p_0 = k_0\sqrt{\epsilon_0}\,l \ll 1$. When $p_0 \ll 1$, we can neglect in u_1^l and u_1^{tr} the terms proportional to p_0^2 at arbitrary p:

$$\begin{gathered}u_1{}^l=1-2\langle\xi^2\rangle\sqrt{\frac{\pi}{2}}\int_0^\infty \Gamma_\xi(x)\frac{J_{5/2}(px)}{\sqrt{px}}\frac{dx}{x},\\ u_1{}^{tr}=1+\langle\xi^2\rangle\sqrt{\frac{\pi}{2}}\int_0^\infty \Gamma_\xi(x)\frac{J_{5/2}(px)}{\sqrt{px}}\frac{dx}{x}.\end{gathered} \quad (8)$$

It is easy to estimate the value of the integral in (8), which depends on $p = kl$. Let us denote by $f(p)$. The function $f(p)$ increases from zero ($p = 0$) to a value $f(\infty) = 1/3$. The last value is reached asymptotically[8]. Near $p = 0$, the function $f(p)$ increases slowly and $f(p) \ll 1$ when $p \approx 1$. The function $u_2(p)$ decreases to zero quite rapidly as $p \to \infty$ (no slower than $1/p^4$). For example, at $\Gamma_\xi(x) = e^{-x}$ we have $u_2 \sim 2p/(1+p^2)^2$; when $\Gamma_\xi(x) = e^{-x^2}$, we have $u_2 \sim \exp(-k^2/4)$.

Let us estimate now the intensity of the random polarizability $\langle\xi^2\rangle$. The bilinear function (3) maps the complex ϵ plane conformally into the ξ plane. Let us consider the function $W = 3(z-1)/(z+2)$, $z = \epsilon/\epsilon_0$. We easily see that the right half-plane of the variable $z = x + iy$, $x \geq 0$ is mapped in the interior of a circle with center at the point $W = 3/4$ and radius $R = 9/4$. The upper half-plane of z is then transformed in the upper half-plane of W, and the imaginary axis $z = iy$ is transformed into the boundary of the indicated circle. It follows therefore that for a dielectric ($z > 0$), regardless of the magnitude of the fluctuation of $\epsilon(\omega, \mathbf{r})$, the random polarizability $\xi(\omega, \mathbf{r})$ satisfies the inequality $-3/2 \leq 3$. Actual calculations for different media show that $\langle\xi^2\rangle$ is of the order of unity. Recognizing that when $\langle\Delta\epsilon^2\rangle/\langle\epsilon\rangle^2 \ll 1$ we have $\langle\xi^2\rangle \approx \langle\Delta\epsilon^2\rangle/\langle\epsilon\rangle^2$, we can state that the transition from the weakly inhomogeneous medium into a medium with strong fluctuations is accompanied by a change of $\langle\xi\rangle^2$ from zero to a value on the order of unity.

Let us return to formulas (1). We consider the correlation function of the longitudinal fluctuating electric field

$$\langle E^l(\mathbf{r}_1)\,E^l(\mathbf{r}_2)\rangle_\omega=\frac{4\Theta}{(2\pi)^3\omega}\int_0^\infty\frac{\mathrm{Im}\,\varepsilon_{\mathrm{eff}}^{l}(\omega, k)}{|\varepsilon_{\mathrm{eff}}^{l}(\omega, k)|^2}e^{i\mathbf{k}\mathbf{R}}d\mathbf{k}. \quad (9)$$

Recognizing that $\mathrm{Im}\,\epsilon_{\mathrm{eff}}(\omega, q)$ becomes small when $k \sim 1/l$ ($p \approx 1$), for which $f \ll 1$, we neglect the dependence of $\epsilon_{\mathrm{eff}}^{l}(\omega, k)$ in the denominator of (9), i.e., we put $\epsilon_{\mathrm{eff}}^{l}(\omega, k) \approx \epsilon_0(\omega)$. Substituting (6) in (9) we obtain[1)]

$$\langle E^l(\mathbf{r}_1)E^l(\mathbf{r}_2)\rangle_\omega=\frac{2\omega^2\Theta\varepsilon_0{}^{1/2}(\omega)\langle\xi^2\rangle}{3\pi c^3}\Gamma_\xi\left(\frac{R}{l}\right). \quad (10)$$

In a physically transparent medium, the longitudinal field is connected with the inhomogeneity of the medium. Formula (10) shows that the correlation function of a longitudinal thermal field is expressed in terms of the correlation function of the polarizability of the medium and the scale of its correlation coincides with the scale of $\xi(\omega, \mathbf{r})$.

In the approximation of the perturbation method ($\langle\Delta\epsilon^2\rangle/\langle\epsilon\rangle^2 \ll 1$) we have

$$\begin{gathered}\varepsilon_0(\omega)=\langle\varepsilon\rangle-\frac{1}{3}\frac{\langle\Delta\varepsilon^2\rangle}{\langle\varepsilon\rangle}+\ldots,\\ \langle\xi^2\rangle\approx\frac{\langle\Delta\varepsilon^2\rangle}{\langle\varepsilon^2\rangle}+\ldots,\quad \Gamma_\xi(x)=\Gamma_\varepsilon(x),\end{gathered} \quad (11)$$

where $\Gamma_\epsilon(x)$ is the normalized correlation function of the fluctuations $\epsilon(\omega, \mathbf{r})$. Substituting (11) in (10), we obtain an expression for the fluctuations of the longi-

1) In calculating (10) it must be taken into account that

$$\int_{-\infty}^{+\infty}\frac{\sin px}{p}e^{ip\mathbf{R}}\,dp=\frac{2\pi^2}{R}[\delta(x-R)-\delta(x+R)].$$

tudinal field in a weakly inhomogeneous medium:

$$\langle \mathbf{E}^l(\mathbf{r}_1)\mathbf{E}^l(\mathbf{r}_2)\rangle_\omega = \frac{2\omega^2\Theta\langle\Delta\varepsilon^2\rangle}{3\pi c^3\langle\varepsilon\rangle^{3/2}}\Gamma_\varepsilon\left(\frac{R}{l}\right). \quad (12)$$

Formula (12) can be obtained in another way. In a physically transparent homogeneous medium, the fluctuation field is transverse, and its intensity is equal to

$$\langle \mathbf{E}^2\rangle_\omega = \frac{2\omega^2\Theta\sqrt{\varepsilon(\omega)}}{\pi c^3}. \quad (13)$$

The appearance of random inhomogeneities $\Delta\epsilon(\omega, \mathbf{r})$ in the medium will be accompanied by scattering of the field (13). Let us calculate the intensity of the longitudinal component of the scattered field. In the approximation of the perturbation method we have

$$\Delta e_i - \frac{\partial^2 e_j}{\partial x_i\,\partial x_j} + k_0^2\langle\varepsilon\rangle e_i = -k_0^2\Delta\varepsilon(\omega,\mathbf{r})E_i^0(\mathbf{r}). \quad (14)$$

Here $\Delta\epsilon(\omega, \mathbf{r}) = \epsilon(\omega, \mathbf{r}) - \langle\epsilon(\omega, \mathbf{r})\rangle$, $e_i = E_i - \langle E_i\rangle$, and E_i^0 is the transverse thermal field with intensity given by formula (13).

The solution for the longitudinal component of the field is

$$E_i^l(\mathbf{r}) = -k_0^2\int G_{ik}^l(\mathbf{r}-\mathbf{r}')\Delta\varepsilon(\omega,\mathbf{r}')E_k^0(\mathbf{r}')\,d\mathbf{r}',$$
$$G_{ik}^l = \int G_{ik}^l(\mathbf{p})e^{i\mathbf{pr}}\,d\mathbf{p}, \quad (15)$$
$$G_{ik}^l(\mathbf{p}) = \frac{1}{(2\pi)^3}\frac{p_ip_k}{p^2}\frac{1}{k_0^2\langle\varepsilon(\omega)\rangle}.$$

The intensity of the longitudinal field is

$$\langle(\mathbf{E}_\omega^l)^2\rangle = \frac{\langle E_i^0E_j^0\rangle}{3\langle\varepsilon^2\rangle}\int\Phi_\varepsilon(\mathbf{k})\frac{k_ik_j}{k^2}\,d\mathbf{k}, \quad (16)$$

where

$$\Phi_\varepsilon(\mathbf{k}) = \frac{1}{(2\pi)^3}\int B_\varepsilon(\mathbf{r})e^{-i\mathbf{kr}}\,d\mathbf{r}.$$

From (16) it follows that ($\int\Phi_\epsilon(\mathbf{k})\,(k_ik_j/k^2)d\mathbf{k} = \delta_{ij}\langle\Delta\epsilon^2\rangle$)

$$\langle(E_\omega^l)^2\rangle = \frac{2\omega^2\Theta\langle\Delta\varepsilon^2\rangle}{3\pi c^3\langle\varepsilon\rangle^{3/2}}. \quad (17)$$

We now consider the transverse fluctuating field. We have from (1)

$$\langle\mathbf{E}^{tr}(\mathbf{r}_1)\mathbf{E}^{tr}(\mathbf{r}_2)\rangle_\omega = \frac{8k_0^4\Theta}{(2\pi)^2R\omega}\,\mathrm{Im}\int_{-\infty}^{+\infty}\frac{\varepsilon_{\mathrm{eff}}^{tr}(\omega,k)}{k^2-k_0^2\varepsilon_{\mathrm{eff}}^{tr}(\omega,k)}\frac{\sin kR}{k}\,dk. \quad (18)$$

Substituting here (7), we reduce the problem to a calculation of the integrals

$$I_1 = \int_{-\infty}^{+\infty}\frac{u_1^{tr}(k)}{k^2-k_0^2\varepsilon_{\mathrm{eff}}^{tr}(\omega,k)}\frac{\sin kR}{k}\,dk,$$
$$I_2 = \int_{-\infty}^{+\infty}\frac{u_2(\omega,k)}{k^2-k_0^2\varepsilon_{\mathrm{eff}}^{tr}(\omega,k)}\frac{\sin kR}{k}\,dk. \quad (19)$$

By virtue of the conditions $u_2 \ll u_1^{tr}$ and $f(k_0\sqrt{\epsilon_0(\omega)}\,l) \ll 1$, it is clear that in determining the poles of the integrands in (19) it is possible to use the perturbation method, putting at the zeroth approximation $k = k_0\sqrt{\epsilon_0(\omega)}$. The next approximation is of the form

$$k = \pm k_1 = \pm k_0\left[\sqrt{\varepsilon_0(\omega)} + i\frac{\varepsilon_2(\omega, k_0\sqrt{\varepsilon_0(\omega)})}{2\sqrt{\varepsilon_0(\omega)}}\right]. \quad (20)$$

The quantity $u_1^{tr}(k)$ in the numerator of the integrand in I_1 can be replaced by unity, since $|f(k_1)| \ll 1$. The integral can be then readily calculated ($I_1 = (\pi/k_1^2)[\exp(ik_1R) - 1]$). Calculation of I_2 shows that $|I_2|\,I_1| \approx \langle\xi^2\rangle p_0^2 \ll 1$ and consequently

$$\langle\mathbf{E}^{tr}(\mathbf{r}_1)\mathbf{E}^{tr}(\mathbf{r}_2)\rangle_\omega = \frac{2\omega\Theta}{\pi c^2}e^{-\alpha R}\frac{\sin\beta R}{R},$$
$$\alpha = k_0\frac{\varepsilon_2(\omega, k_0\sqrt{\varepsilon_0(\omega)})}{2\sqrt{\varepsilon_0(\omega)}} \approx k_0\frac{\varepsilon_2(\omega,0)}{2\sqrt{\varepsilon_0(\omega)}}, \quad (21)$$
$$\beta = k_0\sqrt{\varepsilon_0(\omega)}, \quad \varepsilon_2 = \mathrm{Im}\,\varepsilon_{\mathrm{eff}}^{tr}.$$

It follows from (21) that in calculating the correlation functions of the transverse electric field it is possible to disregard the spatial dispersion of the inhomogeneous medium.

A similar calculation for the magnetic field leads to the expression

$$\langle\mathbf{H}(\mathbf{r}_1)\mathbf{H}(\mathbf{r}_2)\rangle_\omega = \frac{2\omega\Theta\varepsilon_0(\omega)}{\pi c^2}e^{-\alpha R}\frac{\sin\beta R}{R}. \quad (22)$$

Let us discuss the result. Expressions (10), (21), and (22) determine the correlation functions of the electromagnetic field in a randomly inhomogeneous dielectric. They are valid at the frequencies ω for which the inequality $p_0 = \omega c^{-1}\sqrt{\epsilon_0(\omega)}\,l \ll 1$ is satisfied. It is clear that this condition guarantees satisfaction of the fundamental inequality $|\xi_{ij}^{\mathrm{eff}}| \ll 1$, if the statements made above concerning $\langle\xi^2\rangle$ are taken into account. The foregoing calculations demonstrate the role of the spatial dispersion connected with the inhomogeneity of the medium in the formation of the electromagnetic thermal radiation. In particular, in calculating $\langle\mathbf{E}^l(\mathbf{r}_1)\mathbf{E}^l(\mathbf{r}_2)\rangle_\omega$, the character of the decrease of the function $\mathrm{Im}\,\epsilon_{\mathrm{eff}}^l(\omega, k)$ as $k \to \infty$ becomes important. To find $\langle\mathbf{H}(\mathbf{r}_1)\mathbf{H}(\mathbf{r}_2)\rangle$, it is also necessary to take into account the spatial dispersion. Let us illustrate this by comparing (22) with the formula for the correlation function of the magnetic field in a homogeneous absorbing medium[10]:

$$\langle\mathbf{H}(\mathbf{r}_1)\mathbf{H}(\mathbf{r}_2)\rangle_\omega = \frac{2\omega\Theta}{\pi c^2}\left\{\varepsilon_{tr}'\frac{\sin k'R}{R} + \varepsilon_{tr}''\frac{\cos k'R}{R}\right\}e^{-k''R} \quad (23)$$

When $R = 0$, the second term of (23) has a singularity. In a randomly inhomogeneous non-absorbing medium, the analogous term has no singularity. This singularity is eliminated by allowance for the spatial dispersion. Moreover, this term is small (it is proportional to p_0^2 and lies beyond the limits of accuracy of the calculation), and has been omitted in writing down (22).

3. FLUCTUATION FIELD IN AN INHOMOGENEOUS PLASMA

Let us assume that a random function $\epsilon(\omega, \mathbf{r})$ vanishes at individual points of the volume occupied by the inhomogeneous medium. The possibility of intersecting the zero level (the distribution function $W(x)$ for the quantity ϵ differs from zero at $x = 0$) causes the quantity $\epsilon_0(\omega)$ to become a complex function of the frequency, as given in[8]. For example, in an inhomogeneous plasma with electron-density fluctuations at a frequency close to the Langmuir frequency[5] we have

$$\varepsilon_0(\omega) = 0.56\langle\varepsilon(\omega)\rangle + 0.52i\sigma_N/\langle N\rangle, \quad \langle\xi^2\rangle \approx -1.7;$$
$$\sigma_N^2 = \langle(N-\langle N\rangle)^2\rangle, \quad \varepsilon(\omega,r) = 1 - 4\pi e^2N/m\omega^2, \quad (24)$$
$$\langle\varepsilon\rangle \ll \sigma_N/\langle N\rangle, \quad \sigma_N/\langle N\rangle \ll 1.$$

Now the quantity $\mathrm{Im}\,\epsilon^{\mathrm{eff}}(\omega, k)$ is no longer determined by expression (6). The attenuation of the field due to the scattering (a quantity proportional to p_0^3) can in general be neglected because of the appearance of the much stronger effect connected with the plasma oscillations at the points of the medium where $\epsilon = 0$. It is easy to see that in this case

$$\varepsilon^{l}_{\text{eff}}(\omega, k) \approx \varepsilon_0(\omega) u_1{}^l(\omega, k), \quad \varepsilon^{tr}_{\text{eff}}(\omega, k) \approx \varepsilon_0(\omega) u_1{}^{tr}(\omega, k), \tag{25}$$

where the imaginary part of $\epsilon_0(\omega)$ does not depend on k. Since u_1^l and u_1^{tr} do not decrease when $k \to \infty$, it is clear, in particular, that the integral (9), which determines the correlation function of the longitudinal field, diverges. To clarify the meaning of this result, let us determine the cause of the imaginary part of $\epsilon_0(\omega)$ more rigorously than in[5,8]. Using as an example a one-dimensional randomly inhomogeneous plasma placed in a quasistatic electric field E^l, the direction of which coincides with the direction of the variation of $\epsilon(\omega, x)$, we shall show that $\operatorname{Im} \epsilon_0(\omega)$ determines the damping of the long-wave quasistatic field due to the losses connected with transformation of energy of the latter into the energy of the plasma oscillations at the points where $\epsilon = 0$.

Let the field outside the plasma layer (at infinity) be equal to E_0^l. The solution of the quasistatic equations in this case is

$$E^l(x) = E_0{}^l / \varepsilon(\omega, x). \tag{26}$$

The average field $\langle E^l \rangle$ is determined by the relation

$$\langle E^l(x) \rangle = E_0{}^l \int_{-\infty}^{+\infty} \frac{W(\varepsilon)}{\varepsilon} d\varepsilon, \tag{27}$$

where $W(\epsilon)$ is the distribution function of the random function $\epsilon(\omega, x)$. To determine the rule for going around the pole in (27) (we recognize that $\epsilon(\omega, x)$: can vanish, i.e., $W(0) \neq 0$), we assume the presence of a small imaginary part in $\epsilon(\omega, x)$:

$$\langle E^l \rangle = E_0{}^l \,\mathrm{P} \int_{-\infty}^{+\infty} \frac{W(\varepsilon)}{\varepsilon} d\varepsilon - i\pi W(0) E_0{}^l \tag{28}$$

(the symbol P indicates integration in the sense of the principal value).

We assume that ϵ has a normal distribution with a variance $\sigma^2_{\Delta\epsilon} = \langle \Delta\epsilon^2 \rangle$ and a mean value $\langle \epsilon \rangle = 0$ (at the resonant frequency $\omega_p^2 = 4\pi e^2 \langle N \rangle / m$). Formula (28) yields

$$\langle E^l \rangle = -i \sqrt{\frac{\pi}{2}} \frac{E_0{}^l}{\sigma_{\Delta\varepsilon}} \tag{29}$$

(the principal value at $\omega = \omega_p$ is equal to zero). We note that from (28) follows a formula for $\epsilon_0(\omega)$ (see[11]):

$$\varepsilon_0{}^{-1} = \mathrm{P} \int_{-\infty}^{+\infty} \frac{W(\varepsilon)}{\varepsilon} d\varepsilon - i\pi W(0). \tag{30}$$

Let us calculate the effective losses per unit volume, determined by the quantity $\operatorname{Im} \epsilon^{\text{eff}}(\omega, k = 0) = \operatorname{Im} \epsilon_0(\omega)$:

$$Q^{\text{eff}} = \frac{\omega}{2\pi} \operatorname{Im} \varepsilon^{\text{eff}} |\langle E^l \rangle|^2. \tag{31}$$

Substituting (29), (30), in (31), we have

$$Q^{\text{eff}} = \frac{\omega}{2} W(0) |E_0{}^l|^2 = \frac{\omega |E_0{}^l|^2}{2\sigma_{\Delta\varepsilon}\sqrt{2\pi}}; \tag{32}$$

The same quantity can be calculated using considerations that indicate directly the connection between the losses (32) and the plasma oscillation²⁾.

²⁾The Joule losses in the vicinity of the point $\epsilon = 0$ were considered in [12,13].

The Joule loss in the inhomogeneous medium is

$$Q = \frac{\omega}{2\pi} \int \varepsilon''(\omega, x) |E^l(x)| dx. \tag{33}$$

Integration is carried out along a segment of unit length. Substituting (26) and recognizing that $\epsilon''/(\epsilon^2 + \epsilon''^2) = \pi\delta(\epsilon)$ as $\epsilon'' \to 0$, we find the value of the loss, which does not depend on the introduced quantity $\epsilon''(\omega, x)$:

$$Q = \frac{\omega}{2} |E_0{}^l|^2 \sum_{s=1}^{N} \frac{1}{|\partial\varepsilon/\partial x|_{\varepsilon=0}}, \tag{34}$$

where the summation is carried out over the points of the segment at which $\epsilon(\omega, x) = 0$. The average number of such points on the segment is $N = 1/l\sqrt{2\pi^2}$ (for the normal process), where l is the correlation scale of the function $\epsilon(\omega, x)$. Let us average (34), using the distribution function of the derivatives $y = \partial\epsilon/\partial x$ at the points $\epsilon = 0$ in the normal process[14]:

$$W(y) = \frac{|y|}{\sigma_{\Delta\varepsilon}{}^2 R_2} \exp\left\{\frac{-y^2}{2\sigma_{\Delta\varepsilon}{}^2 R_2}\right\}, \qquad R_2 = -\left[\frac{d^2\Gamma_\varepsilon}{dx^2}\right]_{x=0}, \quad \langle y \rangle = 0, \quad R_2 > 0 \tag{35}$$

(at $\Gamma_S(x) = \exp(-x^2/l^2)$, $R_2 = 2/l^2$). With the aid of (35) we obtain from (34)

$$\langle Q \rangle = \frac{\omega |E_0{}^l|^2}{2\sigma_{\Delta\varepsilon}\sqrt{2\pi}} = Q^{\text{eff}}. \tag{36}$$

The foregoing arguments establish the origin of the imaginary part of $\epsilon_0(\omega)$, when the distribution $W(x)$ admits the possibility of the vanishing of $\epsilon(\omega, \mathbf{r})$. The appearance of plasma oscillations leads to a strong decrease of the role of scattering by inhomogeneities and the corresponding spatial dispersion. It is clear that the divergence of the longitudinal fluctuation field at the resonant frequency ω_p is connected with the presence of an infinite number of plasma oscillations with different k, excited at the zero-level points, and cannot be eliminated solely by the spatial dispersion determined by the macroscopic inhomogeneities. It is necessary to take into account the thermal motion of the plasma particle[10].

4. CONCLUSION

It is shown in the present paper that the processes of scattering and diffraction of the field by small-scale macroscopic random inhomogeneities play an essential role in the form of the random thermal field in a physically transparent medium. These processes are automatically taken into account with the aid of the dielectric constant $\epsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})$, which makes it possible to reduce the problem formula to a consideration of a homogeneous absorbing medium. The use of calculations of this type for inhomogeneous media such as the plasma at the resonant frequency entail difficulties in the sense that the scattering itself no longer plays the principal role in the formation of the field, and the corresponding spatial dispersion is not ensured convergence of the energy integrals of the type (9). Apparently, only the theory of a randomly inhomogeneous plasma, which takes into account thermal motion of the particles, is capable of overcoming this difficulty.

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