THEORY OF EXCITATION AND HIGH-FREQUENCY STABILIZATION OF THE KADOMTSEV-NEDOSPASOV HELICAL INSTABILITY IN AN ELECTRON-HOLE PLASMA IN A STRONG MAGNETIC FIELD

V. V. VLADIMIROV

Physics Institute, Ukrainian Academy of Sciences

Submitted December 29, 1969

Zh. Eksp. Teor. Fiz. 59, 162–167 (July, 1970)

It is shown that the spatial structure of helical waves[2] is modified in strong magnetic fields ($\omega_c\tau > 1$). As a consequence the excitation threshold for the helical instability depends upon the length of the sample, the threshold becoming higher in short samples and strong magnetic fields. The efficiency of high-frequency-stabilization for this instability increases under these conditions (the modulation coefficient required for stabilization is reduced and the stabilization region is expanded).

1. THE theory of the helical instability in the plasma in the positive column was formulated by Kadomtsev and Nedospasov (KN).[1] Using this theory, Glicksman[2] was able to explain the results of some well-known experiments by Ivanov and Ryvkin as well as others[3,4] in which an instability of the "oscillator" type was investigated, wherein current oscillations are observed when sufficiently strong longitudinal electric and magnetic fields are applied to a semiconductor sample.

It follows from the KN theory that the helical instability in long discharge tubes develops at an optimum value of the wavelength (pitch of the helix), which corresponds to the minimum value of the critical magnetic field at which the instability can be excited (in the positive column the only free parameter is the magnetic field, since the electric field is determined by it). The existence of an optimum wavelength can be understood as follows. At long wavelengths of the helical perturbations the drift flux in the wave field, which leads to the instability, is small and is dominated by transverse diffusion; at short wavelengths, although the drift flux is large, a strong role is played by the longitudinal diffusion, which quenches the instability. In the KN picture the optimum wavelength (or the wave vector k) and the excitation criterion are determined from the condition that at the threshold of the instability we must have

$$\operatorname{Im}\omega(k) = F(E, H, k) = 0; \quad dF/dk = 0, \tag{1}$$

that is to say, on going through zero the curve F(k) is tangent to the wave-vector axis, since the excitation occurs at one value of k.

In the positive-column plasma the optimum values of k as determined from (1) correspond to rather long wavelengths, of the order of ten tube radii.[1] Such perturbations can develop in long tubes. The KN theory shows that in short tubes the optimum wavelength can be greater than the length of the tube, so that these perturbations cannot develop, by virtue of the boundary conditions. In this case the wavelength of the helical perturbation is bounded from above by the length of the discharge tube or by appropriate spatial harmonics. For this reason the excitation criterion will be more stringent in short tubes than in long tubes.

A similar situation obtains for the excitation of an oscillistor. It has been shown by Glicksman[2] that in weak magnetic fields ($y = \omega_c\tau \ll 1$ where ω_c is the cyclotron frequency of the carriers while τ is the relaxation time) the optimum wavelength for the helical perturbations, as determined from (1), is independent of the magnetic field, being approximately 2a in samples with "dirty" surfaces ($D_a/as \ll 1$) and approximately 8a for a "clean" surface ($D_a/as \gg 1$).[2,5] Here, D_a is the coefficient of ambipolar diffusion, s is the rate of surface recombination, and a is the radius of the sample.

When there are nodes at the ends[5] the fine structure of the wave is modulated by a long-wave envelope with a period equal to the length of the sample and appropriate harmonics. In strong magnetic fields the period of the fine structure increases with increasing magnetic field, and when $y \gg 1$ this period becomes comparable with the length of the sample L. Under these conditions the spatial structure of the wave is determined completely by the sample geometry. In samples with clean surfaces this situation arises at lower values of the magnetic field. Hence, the criterion for excitation of the oscillistor in samples of finite length will be appreciably stronger than the corresponding criterion derived for samples of infinite length;[2] this is the case because longitudinal diffusion starts to play an important role, since the wavelength is independent of the magnetic field. Thus, a strong magnetic field leads to a reduction of the "effective length" of the sample and effects associated with the boundary conditions (with longitudinal diffusion) become important. The dependence of the threshold for the oscillatory perturbations on the length of the sample in strong magnetic fields was first observed in experiments by Dubovoĭ and Shanskiĭ,[6] who investigated the oscillistor in samples of Ge. It was shown by Shanskiĭ[7] that in strong magnetic fields the dependence of the threshold electric field (E_{th}), at which the oscillistor is excited, on the magnetic field exhibits a minimum due to the increasing role of the longitudinal diffusion, which is independent of the magnetic field in finite samples.

Actually, at reasonably modest magnetic fields the basic factor in the appearance of the instability is the transverse diffusion flux $\sim H^{-2}$ and $E_{th} \sim H^{-1}$ since the drift flux in the field of the wave, which causes the instability, is $\sim E \cdot H^{-1}$, and for very strong magnetic fields the primary role is played by longitudinal diffusion and $E_{th} \sim H$. As the length of the sample is increased the position of the minimum $E_{th}(y)$ is shifted in the direction of stronger magnetic fields. It is evident that a rigorous description of these effects requires a theory that takes account of the boundary conditions of the ends.

In the present work, using the two-mode picture for the oscillistor,[5] we obtain criteria for excitation for samples of finite length and these criteria are found to be in agreement with experiment.[6] The calculation is carried out for a high level of injection of nonequilibrium carriers (for example impact ionization, optical ionization) and a "dirty" sample surface, in which case the distribution of carrier concentration over the cross-section falls off sharply towards the surface (Schottky condition). This case is the closest to the gas-discharge plasma and is relatively simple for calculation purposes since the strong magnetic field has no effect on the spatial distribution of the electron-hole plasma.

It should be noted that the conditions in the Dubovoĭ-Shanskiĭ experiments[6] actually apply to a clean surface, so that the agreement between theory and experiment is qualitative.

In this work we present a theory for the high-frequency stabilization (HFS) of the oscillations in strong magnetic fields. This effect was first observed by Dubovoĭ and Shanskiĭ[8] in weak magnetic fields and was explained by Kadomtsev and Vladimirov.[5] Subsequent experiments[8] verified the basic conclusions of the theory.[5]

It will be shown below that for a given sample length the efficiency of high-frequency stabilization increases with increasing magnetic field (the stabilization region is expanded and stabilizing coefficients of the modulation $\eta = \tilde{E}/E_c$ of the electric field, where E_c is the constant component, are reduced). We shall determine the dependence of the stabilizing modulation coefficient on the effective length of the sample and the modulation frequency. It should be noted that the efficiency of HFS of the oscillistor is much higher in samples with clean surfaces.[5] For this reason the calculations carried out below, which apply to a dirty surface, are aimed primarily at explaining the qualitative features of HFS in strong magnetic fields.

For reasons of simplicity, all the calculations are carried out for the case in which the electron and hole mobilities are equal (the helical instability is excited in this case, too, because in a fixed electric field the electron and hole density helices drift in opposite directions).

We note that in Ge the ratio of the mobilities is $b_e/b_h \approx 2, 2$ at $T = 300°K$ and $1-1.5$ at $T = 77°$, so that the case being considered is a close approximation to the real situation.[2]

2. The initial equation of the oscillistor in an alternating electric field in the presence of a longitudinal magnetic field and equal densities of electrons and holes can be obtained from the equation given by Glicksman[2] much in the same way as is done in [5]. When the electron and hole mobilities are the same ($b_e = b_h = b$, $D_e = D_h = D$) the equation for the perturbed density n', written in dimensionless form is

$$\hat{L}n' = \frac{\partial}{\partial\vartheta}\left(\frac{\partial^2 n'}{\partial x^2} - n'\right) + \mu\left(\frac{\partial^2 n'}{\partial x^2} - n'\right) - i\alpha(1 + \eta\sin\beta\vartheta)\frac{\partial n'}{\partial x} = \frac{\partial^4 n'}{\partial x^4} - n', \tag{2}$$

where all quantities are computed for a dirty surface and are defined as follows:

$$x = \frac{3{,}44}{\sqrt{1+y^2}}\frac{z}{a}, \quad \vartheta = \frac{11{,}8D}{a^2(1+y^2)}t,$$

$$\alpha = \frac{0{,}1v_{0c}a}{D}y\sqrt{1+y^2}, \quad \beta = 0{,}1\frac{\omega_0 a^2(1+y^2)}{D},$$

$v_{0c} = bE_c$ is the drift velocity in the constant electric field, ω_0 is the modulation frequency and $\mu = 2.25$.

The boundary conditions are written in the form

$$n'|_{x=0,x_L} = 0, \tag{3}$$

where $x_L = 3.44l$ and $l = (1+y^2)^{-1/2}L/a$ is the effective length of the sample.

The form of (2) is exactly the same as that of the corresponding equation for the case $y \ll 1$.[5] The principal difference lies in the fact that the effective length of the sample l depends on the magnetic field. This feature then determines the basic characteristics of excitation and HFS of the oscillistor in strong magnetic fields.

In choosing zero boundary conditions we have assumed that the rate of surface recombination at the surface of the contacts is very large. The experimental investigation of the spatial structure of the oscillistor[7,8] shows that the boundary conditions (3) are well satisfied.

The solution of Eq. (2) in the form of plane waves $\exp(ikz)$ does not satisfy the boundary conditions (3). Since the variables in (2) do not separate, we seek the solution written in the form of an expansion in the characteristic coordinate functions Φ_n of the operator $\hat{L}$, which satisfy the boundary conditions (3):[5]

$$n' = \sum_n C_n(\vartheta)\Phi_n(x), \tag{4}$$

where $\Phi_n(x) = \exp(l\rho_n x) \times \sin\kappa_n x$, $\rho_n = \sqrt{1+\kappa_n^2}$, $\kappa_n = \pi n/x_L$, $n = 1, 2, \ldots$.

This approach to the solution of Eq. (2) represents one of the perturbation-theory methods, since the right side of (2), with substitution of the solution in the form (4), is a quantity of order $1/x_L$ and is small in long samples. We note that the equation $\hat{L}n' = 0$ (zeroth approximation) does not describe the HFS effect;[5] hence, in very long samples, in which the right side of (2) is small, the HFS effect will not be observed, as has been verified experimentally.[7,8]

It follows from the form of the function $\Phi_n(x)$ that the spatial structure of the helical wave in long samples and weak magnetic fields ($l \gg 1$) will be determined by the exponential factor and that the period of the wave will be comparable with the radius of the sample; in short samples and strong magnetic fields ($l \approx 1$) the period of the wave coincides with harmonics of the sample length.

The further analysis is carried out in the two-mode approximation ($n = 1, 2$)[5,9] since the first two modes have the largest instability growth rates.[5-8] The equation for the first mode is of the form[5,9]

$$\ddot{C}_1 + 2\gamma\dot{C}_1 + \varkappa C_1 = 0. \tag{5}$$

The expressions for the coefficients γ and κ (and for all the coefficients given below) have the same form as the corresponding coefficients for the case of weak magnetic fields.[5,9] In carrying out the calculations it is necessary to make the substitution $L/a \to l$. The equation for the second mode is of the same form. The solution of (5) is

$$C_1 = \text{const}\cdot\exp\{-\textstyle\int\gamma d\vartheta\}\, u(\vartheta),$$

where $u(\vartheta)$ satisfies the equation

$$\ddot{u} + (\varkappa - \gamma^2 - \dot{\gamma})u = 0. \tag{6}$$

High-frequency stabilization is possible when, in the absence of the high-frequency field, the development of the instability is given by (6):

$$\gamma(\eta = 0) > 0, \quad \varkappa(\eta = 0) < 0. \tag{7}$$

The conditions in (7) determine the range of values of α for which HFS is possible. In Fig. 1 this region is shown as the function of the effective length of the sample l. It is evident from Fig. 1 that the HFS region is reduced as l is increased. Hence, for a given sample length the HFS region increases with increasing magnetic field.

As follows from the conditions in (7), the lower limit of the region determines the threshold for excitation of the oscillistor. In Fig. 2 we show the dependence of the threshold value of the electric field $\epsilon = v_{0c}a/D$ on the magnetic field y for various sample lengths. We note, that in contrast with the helical instability in the positive column, the oscillator phenomenon is determined by two free parameters, the electric field and the magnetic field.

In Fig. 2 the solid curve determines the oscillistor excitation boundary for samples of infinite length. This curve coincides with the corresponding curve given by the Glicksman theory.[2] The dashed curve determines the threshold for excitation in samples of finite length. As is evident from Fig. 2, in weak magnetic fields ($y \ll 1$) the solid curve and the dashed curve coincide. However, in strong magnetic fields, in which the effective length of the sample is reduced, the dashed curves do not coincide with the solid curves and the conditions

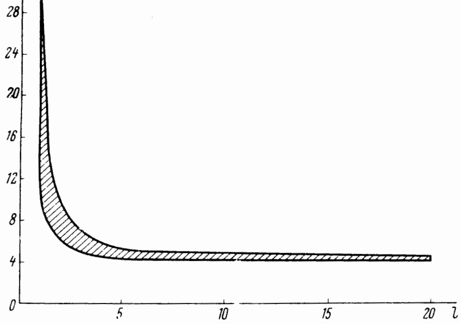

FIG. 1. Region of high-frequency stabilization.

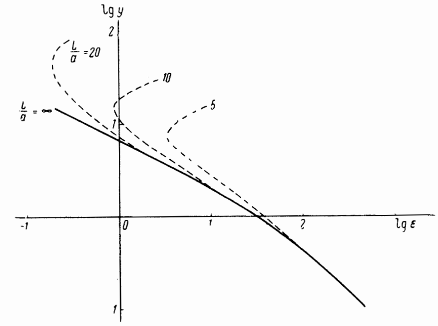

FIG. 2. The electric field $\epsilon = v_{0c}a/D$ as a function of magnetic field $y = bH/c = \omega_c\tau$ at the excitation threshold.

for excitation become more and more stringent. The dependence of the threshold electric field on the magnetic field has a minimum, in accordance with the qualitative discussion given above. As the sample length is increased the position of the minimum is displaced in the direction of stronger magnetic fields.

Equation (6) for the function $u(\vartheta)$ is in the form of the generalized Hill equation.[5,9] The criterion for stability of the solution of this equation reduces to a biquadratic equation for the stabilizing modulation coefficient of the electric field η_s;[9]

$$\eta_s^4 + p\eta_s^2 - q > 0. \tag{8}$$

In Fig. 3 we show the dependence of η_s on the effective length of the sample l for the modulation frequencies $\beta = 5$ and 10. It will be evident from Fig. 3 that η_s increases sharply with increasing l; when $l > 3$ it varies like $l^{3/2}$. As the modulation frequency is increased η_s increases; this effect is associated with the reduction of the high-frequency corrections in the Hill equation averaged over the high-frequency. A similar situation arises in the analysis of the dynamic stabilization of an inverted pendulum with an oscillating point of support.[10] As is evident from Fig. 3, HFS can be realized more effectively in samples with small effective length, that is to say, in strong magnetic fields.

In conclusion the author wishes to thank B. B. Kadomtsev for his interest in this work; useful discussions with members of the seminar given by M. A. Leontovich are also acknowledged.

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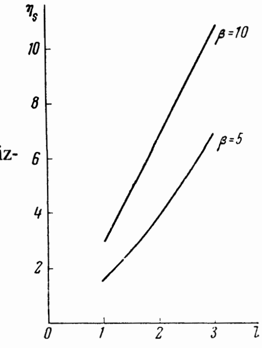

FIG. 3. The dependence of the stabilizing modulation coefficient $\eta_s = E_s/E_c$ on the effective length of the sample $l = (1 + y^2)^{-1/2} L/a$.

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Translated by H. Lashinsky
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