INVESTIGATION OF QUANTUM OSCILLATIONS OF THE PHASE VELOCITY OF HELICONS IN ALUMINUM

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Submitted February 6, 1970

Zh. Eksp. Teor. Fiz. 59, 96–102 (July, 1970)

An investigation was made of the anisotropy of the quantum oscillations of the phase velocity of helicons with respect to the direction of the wave vector in an aluminum crystal. Measurements were made of the absolute amplitude $\partial M/\partial B$ of the oscillations which were due to certain parts of the Fermi surface of aluminum in the third Brillouin zone. The dependence of this amplitude on the magnetic field was measured for three samples and the Dingle temperatures were calculated for these samples.

It is shown in[1] that the quantum oscillations observed by Grimes[2] in an investigation of helicons in aluminum are the oscillations of the phase velocity of helicon waves and not of the attenuation coefficient, as suggested in[2,3]. We attributed these oscillations[1] to the de Haas-van Alphen effect.

The present paper describes further experiments on aluminum and gives a derivation of the dispersion relationship in the local limit ($kl \ll 1$), which predicts quantum oscillations of the phase velocity of helicons and is in full agreement with our experiments.

DISPERSION RELATIONSHIP

We shall consider an uncompensated metal with a closed Fermi surface. We shall assume that this metal is subjected to a magnetic field which is sufficiently strong to satisfy the condition $\omega_c\tau \gg 1$, where ω_c is the cyclotron frequency and τ is the time taken by electrons to traverse a distance equal to one mean free path. Weakly damped plasma waves—helicons—can propagate in the metal under these conditions.

The wave equation for a helicon wave in a metal has the following form in the local case, i.e., when $kl \ll 1$ (k is the wave vector of the helicon and l is the mean free path of the electrons):

$$[\mathbf{k}\hat{\rho}\,[\mathbf{kh}]] = \frac{4\pi i\omega}{c^2}\mathbf{b}. \quad (1)$$

Here, $\hat{\rho}$ is the magnetoresistance tensor; $\mathbf{h}$ and $\mathbf{b}$ are, respectively, the intensity and the induction of the magnetic field of the helicon wave.

The most important aspect of our derivation is an allowance for the nonlinear oscillatory dependence of $\mathbf{h}$ on $\mathbf{B}$, where $\mathbf{B}$ is the sum of the static field to which the metal is subjected and the wave field: $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$.

We shall now write the expression for $\mathbf{h}$:

$$\mathbf{h} = \mathbf{b} - 4\pi\mathbf{M}(\mathbf{B}), \quad (2)$$

where $\mathbf{M}(\mathbf{B})$ is the magnetic moment per unit volume. We shall now expand $\mathbf{M}(\mathbf{B})$ as a series in $\mathbf{b}$. Retaining terms of the first order with respect to the components of $\mathbf{b}$, we obtain

$$h_x = b_x - 4\pi\left(\frac{\partial M_x}{\partial b_x}b_x + \frac{\partial M_x}{\partial b_y}b_y + \frac{\partial M_x}{\partial b_z}b_z\right). \quad (3)$$

The expressions for h_y and h_z are similar.

Let us assume that the magnetic field $\mathbf{B}_0$ makes an angle θ with the normal to the flat surface of the metal. We shall use the following rectangular Cartesian system of coordinates: the z axis is normal to the surface and directed into the metal; the x axis is parallel to the projection of $\mathbf{B}_0$ onto the surface; the y axis is also directed along the surface of the metal but is perpendicular to the z and x axes.

In this system of coordinates, the vectors and the tensor $\hat{\rho}$ in Eq. (1) have the following components:

$$\mathbf{k}(0, 0, k), \quad \mathbf{b}(b_x, b_y, 0), \quad \mathbf{B}_0(B_0\sin\theta, 0, B_0\cos\theta),$$

$$\hat{\rho} = \begin{vmatrix} 0 & RB\cos\theta & 0 \\ -RB\cos\theta & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (4)$$

(R is the Hall coefficient). We have retained only the Hall components of $\hat{\rho}$ since the other components are smaller than the Hall terms when $\omega_c\tau \gg 1$. The non-Hall components govern the weak damping of helicons and, consequently, they make no contribution to the oscillations of the phase velocity observed in our experiments. Substituting Eqs. (2) and (4) into Eq. (1), we obtain a homogeneous system of two equations for b_x and b_y. When the determinant of this system is equated to zero, we obtain

$$[1 - 4\pi(M_{xx} + M_{yy}) + 16\pi^2(M_{xx}M_{yy} - M_{xy}M_{yx})]\,k^4R^2B^2\cos^2\theta + 16\pi^2(M_{xy} - M_{yx})k^2\frac{i\omega}{c^2}RB\cos\theta - \frac{16\pi^2\omega^2}{c^4} = 0, \quad (5)$$

where M_{xx}, M_{xy}, etc. are the derivatives of the thermodynamic potential. Thus, the second term in Eq. (5) vanishes. Finally, we obtain the following dispersion relationship for helicons:

$$k^2 = 4\pi\omega / c^2RB\cos\theta\sqrt{q}, \quad (6)$$

where

$$q = 1 - 4\pi(M_{xx} + M_{yy}) + 16\pi^2(M_{xx}M_{yy} - M_{xy}^2). \quad (7)$$

We thus find that the oscillations of the phase velocity of helicons are governed by the derivatives of the magnetic moment with respect to the field. The diamagnetic moment of a metal is small but it oscillates

*[kh] ≡ k × h.

rapidly with the magnetic field. Consequently, the derivatives of the moment with respect to the field may give rise to a considerable oscillatory effect in Eq. (6). It is known[4,5] that the quantity $4\pi\partial M/\partial B$ may reach values of the order of unity. We can then expect particularly large effects,[6] as predicted by Eqs. (6) and (7).

Since our intention is to interpret the experimental data on aluminum, we shall consider the special case of a Fermi surface in the form of a cylindrical tube of arbitrary cross section. The diamagnetic moment of electrons on such a Fermi surface is directed along the tube for any direction of the magnetic field. The oscillating component of the magnetic moment can be written in the form:

$$\mathbf{M}=\mathbf{n}(\cos\alpha,\cos\beta,\cos\gamma)M_0\cos\frac{2\pi F_0}{B_x\cos\alpha+B_y\cos\beta+B_z\cos\gamma}. \quad (8)$$

Here, F_0 is the frequency of the oscillations in a magnetic field parallel to the tube; $n(\cos\alpha, \cos\beta, \cos\gamma)$ is a unit vector along the direction of the tube; M_0 is the amplitude of the oscillations, which depends monotonically on the magnetic field (this field dependence is ignored in differentiation).

Differentiating with respect to b_x and b_y, we obtain the following expressions for the amplitudes of the quantities which occur in Eq. (7):

$$M_{xx}=\frac{-2\pi F_0M_0\cos^2\alpha}{B_0^2(\sin\theta\cos\alpha+\cos\theta\cos\gamma)^2}$$

$$M_{xy}=\frac{-2\pi F_0M_0\cos\alpha\cos\beta}{B_0^2(\sin\theta\cos\alpha+\cos\theta\cos\gamma)^2}, \quad (9)$$

$$M_{yy}=\frac{-2\pi M_0F_0\cos^2\beta}{B_0^2(\sin\theta\cos\alpha+\cos\theta\cos\gamma)^2}.$$

If the Fermi tube lies in the xz plane and the magnetic field is directed along the tube, it follows that $\cos\alpha = 0$, $\cos\beta = \sin\theta$, and $\cos\gamma = \cos\theta$. Then, Eq. (7) for the oscillating factor reduces to

$$q=1-4\pi\frac{\partial M}{\partial B}\sin^2\theta. \quad (10)$$

This dependence of the investigated effect on the angle θ between the tube axis and the wave vector of a helicon was checked experimentally.

SAMPLES. EXPERIMENTAL CONDITIONS

Our measurements were carried out on three disk-shaped single crystals of aluminum of 10 mm diameter and 0.5 mm thick. These samples were prepared by crystallization from the melt in a demountable graphite mold. The resistivity ratios $\rho(300°K)/\rho(4.2°K)$ of all three samples are listed in Table II.

The normal to the flat surface of one of the samples

Table I

H, kOe	$a = \Delta\omega/\omega$ *		$\frac{a(73°)}{a(17°)}$	$\frac{\sin^2 73°}{\sin^2 17°}$
	a(73°)	a(17°)		
14.0	0.074	0.0069	10.7±0.4	11
15.0	0.079	0.0071	11.1±0.4	11
16.0	0.081	0.0072	11.3±0.4	11
17.0	0.082	0.0073	11.2±0.4	11

*The error in measurement of α was 2%.

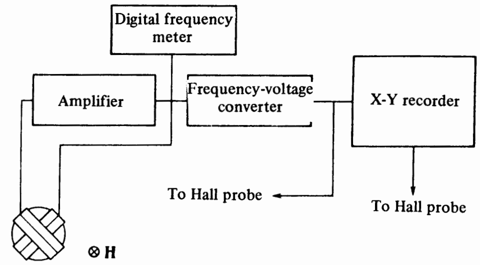

FIG. 1. Block diagram of the experimental set-up.

(Al-1) coincided with the [110] axis. The orientations of the other two samples (Al-2 and Al-3) were the same but they differed from the orientation of the first sample by rotation through an angle of 28° about the [001] axis, parallel to the surface of the disk. These samples were placed in a cryostat in such a way that the normal to the flat surface was in the horizontal plane. The direction of the magnetic field could be varied in this plane by rotating the electromagnet. The measurements were carried out in magnetic fields of 10–20 kG at 1.3°K, in the frequency range 0.1–1.0 kHz.

EXPERIMENTAL SETUP

Quantum oscillations of the phase velocity of helicons were observed, as in our earlier investigation,[1] using a helicon generator.[7] The resonance element of this generator was a metal slab placed inside two crossed coils. The excitation coil, consisting of 10–20 turns, was connected to the output of a special amplifier and the detector coil with ~500 turns was connected to the input of this amplifier. When a magnetic field was applied, helicons were generated at a frequency such that the helicon half-wavelength was equal to the thickness of the sample. Thus, the helicon generator frequency was governed by the dispersion relationship (6), in which the value $k = \pi/d$, where d is the thickness of the sample, should be substituted:

$$\omega=\frac{\pi c^2}{4d^2}RB_0\cos\theta\cdot\sqrt{q}. \quad (11)$$

According to this relationship, an increase in the magnetic field results in a linear increase in the generator frequency which is modulated by the oscillations governed by q of Eq. (10).

The linear rise in the frequency with increasing magnetic field was compensated by a Hall probe and the oscillations were plotted, by an X-Y automatic recorder, as a function of the magnetic field. The amplifier in the helicon generator included an integrating circuit, which rotated the phase of the signal by 90° throughout the investigated range of frequencies, and a phase-inversion stage with a switch, which enabled us to switch the phase by 180° so as to match the direction of rotation of the polarization of helicon waves. A block diagram of the experimental set-up is given in Fig. 1.

ANISOTROPY OF THE WAVE VECTOR DIRECTION

When the magnetic field was directed along the cylindrical part of the Fermi surface, the relative amplitude of the oscillations produced by our generator should be—according to Eqs. (10) and (11)—proportional to $\sin^2\theta$, where θ is the angle between the wave vector of helicons (this vector was aligned along the normal to the Fermi surface) and a generator of the Fermi cylinder. An experimental check was made on the oscillations corresponding to certain parts of the Fermi surface of aluminum in the third Brillouin zone;[8] these parts were tubes which were almost cylindrical in their central cross sections (see[9], γ-type orbits) and were parallel to the [110] axes. Such parts of the Fermi surface made the greatest contribution to the oscillations observed in fields from 10 to 20 kG. The aluminum samples were oriented and held in such a way that, by rotating the electromagnet, we could direct the magnetic field along one of two or three tubes making different angles with the normal to the Fermi surface.

Sample Al-1 was oriented so that the [111] axis was vertical and the magnetic field could be directed along any of the three [110] tubes, one of which was parallel to the normal and the other two, which were symmetrical, formed angles of 60° with the normal. The oscillations recorded in both cases are shown in Fig. 2. The tube parallel to the normal made no contribution to the oscillations, in full agreement with Eqs. (10) and (11), because in this case we had $\sin^2\theta = 0$.

Samples Al-2 and Al-3 were oriented so that the [001] axis was vertical and the horizontal plane contained two tubes making angles of, respectively, 17° and 73° with the normal. The ratio of the relative amplitudes of the frequency oscillations for both tubes was determined for several values of the magnetic field applied to sample Al-2. The results are given in Table I. The deviation from the value of the ratio $\sin^2\theta_1/\sin^2\theta_2$ did not exceed the experimental error.

MEASUREMENT OF THE AMPLITUDE OF THE DE HAAS-VAN ALPHEN OSCILLATIONS AND DETERMINATION OF THE DINGLE TEMPERATURE

Measurements of the dependence of the amplitude $\partial M/\partial B$ on the temperature and on magnetic field made it possible to determine the effective mass of carriers and the Dingle temperature for those parts of the Fermi surface which were responsible for the observed oscillations. Shoenberg presented a paper at a conference in Zurich in 1968[10] and discussed in detail the possibility of investigating the relaxation mechanisms of conduction electrons in metals by means of such measurements.

The observation of the quantum oscillations of the phase velocity of helicons by means of a helicon generator provided us with a convenient and accurate method for measuring the amplitudes of the de Haas-van Alphen oscillations. Substituting q from Eq. (10) into Eq. (11) and using the observation that the value of $4\pi\partial M/\partial B$ of aluminum was small (under our experimental conditions) compared with unity, we obtained the following expression for the relative amplitude of oscillations of the generator frequency:

$$\frac{\Delta\omega}{\omega} = 2\pi\frac{\partial M}{\partial B}\sin^2\theta. \quad (12)$$

Thus, we were able to determine $\partial M/\partial B$ by measuring $\Delta\omega/\omega$. An important advantage was that there were no instrumental factors in Eq. (12) and that the frequency could be measured very accurately without any special difficulties.

Out first experiments indicated that the amplitude $\Delta\omega/\omega$ of the oscillations depended on the amplitude of the generated waves. Obviously, the magnetic field of a helicon wave within a sample became greater than the field corresponding to one period of the oscillations of $\partial M/\partial B$. This field of a helicon wave was $Q \sim 10$ times stronger than the field generated by the excitation coil (here, Q is the Q factor of the helicon resonance). In subsequent experiments, we used a variable resistor, connected in series with the excitation coil at the amplifier output, to reduce the excitation field to a value which no longer affected the amplitude of $\Delta\omega/\omega$. We measured $\partial M/\partial B$ for γ-type orbits[9] on tubes in the third zone of the Fermi surface of aluminum. These measurements were based on experimentally obtained curves, recorded for each of the samples in a magnetic field. The field was directed along that tube which made an angle of 60° with the normal to the surface for sample Al-1, and an angle of 73° for samples Al-2 and Al-3. The results of these measurements for sample Al-1 are presented in Fig. 3.

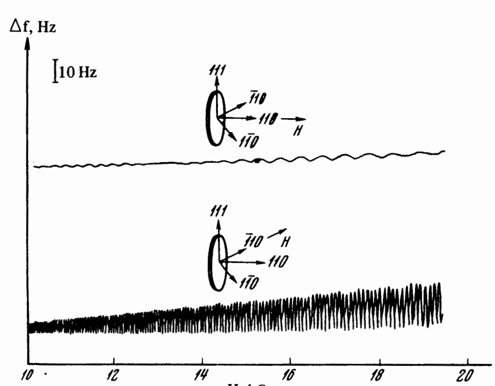

FIG. 2. Oscillations of the frequency of a helicon generator based on sample Al-1. The geometry of the various directions is shown in the figure. The monotonic component of the dependence of the frequency on the magnetic field, compensated by a signal provided by a Hall probe, increased from 300 to 520 Hz in the upper curve and from 150 to 260 Hz in the lower curve. Both curves show short-period γ-type oscillations and long-period α-type oscillations (see [9]).

Table II

Sample	$\rho(300°K)/\rho(4.2°K)$	T^*, °K
Al-1	5220±5%	0.68±0,1
Al-2	4550±5%	0.88±0,1
Al-3	7200±5%	1.14±0,1

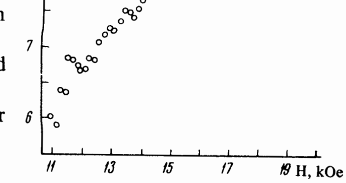

FIG. 3. Dependence of $\partial M/\partial B$ on the magnetic field for γ-type orbits on a tube in the third zone of the Fermi surface of aluminum with the magnetic field directed along the tube. This dependence is derived from the lower curve in Fig. 2. This curve is reproducible and the scatter of the points is not due to experimental errors.

Using the expression for the amplitude of the de Haas-van Alphen effect derived by Lifshitz and Kosevich[11] and the effective mass for orbits of interest to us (obtained in cyclotron resonance experiments[12]), we calculated—for each sample—the Dingle temperature T^* of the electrons on those parts of the Fermi surface which were referred to earlier. The results of these calculations are presented in Table II together with the values of the resistivity ratio $\rho(300°K)/\rho(4.2°K)$. The ratio $\rho(300°K)/\rho(4.2°K)$ was determined by a contactless method[13] for each of the samples. It was interesting to note that the Dingle temperatures estimated from the resistivity ratios were almost an order of magnitude smaller than those listed in Table II. A similar discrepancy was observed by other workers and it was discussed by Shoenberg.[10]

The results presented in this paper simply illustrate the possibilities of the method. A systematic investigation of the dependence of the Dingle temperature on various factors (impurities, heat treatment, etc.) is a subject for a separate paper.

The authors are deeply grateful to S. F. Kosterev and S. N. Nikonov for their technical help in setting up the apparatus.

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Translated by A. Tybulewicz
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