# IONIZING COLLISIONS AND CHARGE EXCHANGE FOR Li<sup>+</sup>, Li<sup>2+</sup> AND Li<sup>3+</sup> IONS IN

GASES (0.2-2 MeV)

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The effective cross sections for the formation of slow positive ions and free electrons on ionization of the gases  $H_2$ ,  $N_2$ , He, Ne, Ar and Kr by Li<sup>+</sup> (in the energy interval 0.2–2.0 MeV), Li<sup>2+</sup> (0.6–2.0 MeV), and Li<sup>3+</sup> (1.2–2.0 MeV) ions have been measured. The cross sections for loss and capture of electrons by the ions Li<sup>+</sup>, Li<sup>2+</sup> and Li<sup>3+</sup> have also been measured. An estimate of the cross sections for pure ionization is made on the basis of the data obtained. Starting from the Bethe formula, in the cases where this is possible, the effective charge of the lithium ions is determined.

### 1. INTRODUCTION

A large number of investigations have been made of ionization processes of gases by hydrogen and helium ions in a broad range of relative velocities of the interacting particles<sup>[1-6]</sup>. Ionization of lithium ions has practically not been studied. Moreover, ionizing collisions of fast lithium particles with gas molecules are not only of great practical interest but also of considerable scientific interest if only from the point of view of further generalizing the results of ionization of gases by fast light atomic particles.

In the present work we have measured the total cross sections for ionization of the gases He, Ne, Ar, Kr, H<sub>2</sub> and N<sub>2</sub> by differently-charged ions of the isotope <sup>7</sup>Li, with ion energies of 0.2-2 MeV for Li<sup>+</sup>, 0.6-2 MeV for Li<sup>2+</sup>, and 1.2-2 MeV for Li<sup>3+</sup>.

To estimate the cross sections for pure ionization along with the total cross sections for formation of slow positive ions  $\sigma_{+}$  and free electrons  $\sigma_{-}$ , one must have at least a knowledge of the magnitudes of the cross sections for electron loss and electron capture by the lithium ions in the gases being studied. Systematic data on the loss and capture of electrons by lithium ions were not available for the whole of the energy range investigated by us. There were data only for the gases  $H_2$ , He and  $N_2$  in the energy range  $10-475 \text{ keV}^{[7]}$ , for air at energies  $85-250 \text{ keV}^{[8]}$ , and for the gases He, N<sub>2</sub>, Ar and Kr at the fixed energy values 575 and 1155 keV<sup>[9-12]</sup>. We should note here also that these separate data are in poor agreement with each other. Therefore, in the present work we have also carried out systematic investigations of the processes of electron loss and capture by lithium ions in the energy range 0.2-2 MeV in the indicated gases.

#### 2. TECHNIQUE OF THE MEASUREMENTS

The experimental apparatus in which measurements were made of the cross sections  $\sigma_{+}$  and  $\sigma_{-}$  and of the sections for electron loss and capture by lithium ions were described earlier in the papers<sup>[6]</sup> and<sup>[13]</sup> respectively. The general scheme of the experiment in the present work is clear from Fig. 1. FIG. 1. Schematic diagram of the experimental setup. 1) Charge exchange gas chamber; 2) magnetic mass monochromator; 3) assembly for measuring the cross sections  $\sigma_4$  and  $\sigma_{-;}$  4) assembly for measuring the cross sections for loss and capture of electrons by lithium ions.

A beam of Li<sup>+</sup> ions, obtained from a thermionic source, was accelerated by means of an electrostatic accelerator and was admitted into the charge-exchange gas chamber 1; the latter was a "nozzle" target. The Li<sup>2+</sup> and Li<sup>3+</sup> ions formed in the chamber 1 were directed into the experimental assemblies 3 and 4 by means of the magnetic mass-monochromator 2. Certain modifications were made in the assembly 4 which was used to measure the cross sections for loss and capture of electrons by lithium ions. The collision chamber, the construction of which is clear from the figure, was replaced, and Faraday cylinders were installed for an additional check on the efficacy of the particle counting. The initial beam of accelerated Li<sup>+</sup> ions could be either directed immediately into the assemblies 3 and 4 or passed first through the charge-exchange chamber 1. The possibility of carrying out measurements both with Li<sup>+</sup> ions obtained from a thermionic source, i.e., ions in the ground state, and with Li<sup>+</sup> ions formed after repeated collisions with the target 1, as a result of which they can be excited, enabled us to check the effect of metastable states on the electron loss processes. Systematic errors arising in the determination of the molecular concentration in the collision chamber and in the measurement of the currents and energies amounted to about 10%.

## 3. RESULTS OF THE MEASUREMENTS AND DISCUS-SION

The results of the measurements carried out in this work of the total cross sections for formation of slow positive ions  $\sigma_{+}$  and free electrons  $\sigma_{-}$  on collision of singly, doubly and triply charged ions of lithium with molecules of the gases H<sub>2</sub>, N<sub>2</sub>, He, Ne, Ar and Kr are shown in Figs. 2–4. As has already been pointed out, data on the cross sections for ionization of gases are totally absent for lithium ions in the whole of the energy range investigated; this excludes the possibility of a



FIG. 2. Dependence of the total cross sections for formation of slow positive ions  $\sigma_+$  and electrons  $\sigma_-$  on the energy of lithium ions in H<sub>2</sub> (undashed figures) and N<sub>2</sub> (dashed figures): 1, 1'-for Li<sup>+</sup> ions; 2, 2'-for Li<sup>2+</sup> ions; 3, 3'-for Li<sup>3+</sup> ions.



FIG. 3. Dependence of the total cross sections for formation of slow positive ions  $\sigma_+$  and electrons  $\sigma_-$  on the energy of lithium ions in He (undashed figures) and Ar (dashed figures): 1, 1'-for Li<sup>+</sup> ions; 2, 2'-for Li<sup>2+</sup> ions; 3, 3'-for Li<sup>3+</sup> ions.



FIG. 4. Dependence of the total cross sections for formation of slow positive ions  $\sigma_+$  and electrons  $\sigma_-$  on the energy of lithium ions in Ne (undashed figures) and Kr (dashed figures): 1, 1'-for Li<sup>+</sup> ions; 2, 2'-for Li<sup>2+</sup> ions; 3, 3'-for Li<sup>3+</sup> ions.

direct comparison of the results obtained. It is possible, however, to make an indirect check of the accuracy of the results obtained, given the corresponding cross sections for loss and capture of electrons by lithium ions. Unfortunately, as has been noted already in the Introduction, there were insufficient data in the literature on the cross sections for loss and capture of electrons by lithium ions. The results of the measurements of the effective cross sections for loss and capture of electrons by singly, doubly and triply charged lithium ions in the gases He, Ne, Ar, Kr,  $H_2$  and  $N_2$  are shown in Figs. 5-7. Experimental data obtained by other authors are shown in the same figures for comparison. Theoretical estimates<sup>[14,15]</sup> for the cases of electron loss by singly and doubly charged lithium ions in collisions with hydrogen and helium atoms are given. From a comparison of the data on cross sections for capture and loss of electrons (Figs. 5-7) obtained in the present



FIG. 5. Dependence of the cross sections  $\sigma_{ik}$  for electron loss and capture on the energy of lithium ions in H<sub>2</sub> and N<sub>2</sub>. The dashed lines are data from [<sup>7</sup>], the underlined points are data from [<sup>9-12</sup>], and the thin continuous lines are theoretical curves from [<sup>14,15</sup>].



FIG. 6. Dependence of the cross sections  $\sigma_{ik}$  for loss and capture of electrons on the energy of lithium ions in He and Ne. The notation is the same as in Fig. 5.

work with the data obtained by Allison et al.<sup>[7]</sup> at energies up to 475 keV, large differences are seen in the magnitudes of the cross sections  $\sigma_{10}$  and  $\sigma_{12}$  in hydrogen. In nitrogen these differences are smaller. In helium, a noticeable difference is observed only for the cross section  $\sigma_{10}$ . A comparison of the data obtained with the data of Nikolaev et al.<sup>[9-10]</sup> and Dmitriev et al.<sup>[11-12]</sup> shows an appreciable difference in the magnitudes of the cross sections  $\sigma_{10}$  and  $\sigma_{12}$ . For the other cross sections there is satisfactory agreement.

The theoretically calculated cross sections  $\sigma_{12}$  in helium and  $\sigma_{23}$  in hydrogen agree, within a factor of 2, with our data. For the cases of  $\sigma_{12}$  in hydrogen and  $\sigma_{23}$ in helium the agreement with our data can only be regarded as qualitative. One possible reason for the difference in the magnitudes of the cross sections could be the effect of metastable states of the Li<sup>+</sup> ions on the extent of their electron loss. As was noted in Sec. 2, the experimental apparatus used enabled us to compare the cross sections for loss and capture of electrons by Li<sup>+</sup> ions in the ground and excited states. In Table I results are shown of such a comparison for the cross sec-



FIG. 7. Dependence of the cross sections  $\sigma_{ik}$  for loss and capture of electrons on the energy of lithium ions in Ar and Kr. The underlined points are data from papers [<sup>9-12</sup>].

Table I. Cross sections  $\sigma_{12}[10^{-17} \text{ cm}^2]$  for electron loss by Li<sup>+</sup> ions in collisions with Ar atoms with different gases in the charge-exchange target

•••••	Target								
E, MeV	No target	Charge-ex- change tar- get with- out gas	He	Ne	Ar	Kr	N2	Air	
0.4 1,0	3.3 11	3.7 10.7	5.5 13.6	5.0	5.2 13.8	4.4 13.0	5.5 14,3	5.7 12.4	

tions  $\sigma_{12}$  for electron loss by Li<sup>+</sup> ions in argon on filling the charge-exchange target with different gases, the concentrations of which maintain a stationary distribution in the beam. It is clear from the Table that the cross sections  $\sigma_{12}$  depend weakly on the type of gas in the charge-exchange target. It is also clear that the contribution to the loss cross sections due to metastable states of the Li<sup>+</sup> ions falls with increase of the ion energy. The lowest values of the cross sections  $\sigma_{12}$  correspond to passing the ions formed in the thermionic source directly (bypassing the target) into the measuring chamber and are the values of the cross sections for loss of an electron from the ground state of the Li<sup>+</sup> ion. Thus the differences in the values obtained by various authors for the cross sections for electron loss by Li+ ions may be due, to a considerable extent, to the contribution to these cross sections of metastable states of these ions. A check which we carried out on the effect of metastable states of Li<sup>+</sup> ions on the cross section  $\sigma_{13}$ for loss of two electrons showed that if there is such an effect the increase in the cross section  $\sigma_{\rm 13}$  due to it does not go beyond the limits of the errors in the measurements. However, not all the differences between the electron loss and capture cross sections obtained by various authors can be explained in this way.

We shall examine now to what extent the results of the measurement of the cross sections for formation of slow ions  $\sigma_+$  and electrons  $\sigma_-$  and of the cross sections for loss and capture of electrons by Li<sup>+</sup>, Li<sup>2+</sup> and Li<sup>3+</sup> ions are in agreement with each other. Because the positive and negative charges formed on ionization of the target molecules are equal, the difference  $\sigma_+ - \sigma_-$  in the cross sections will be caused only by processes of loss and capture of electrons by lithium ions. Starting from this fact and ignoring processes in which several electrons are captured, leading to the formation of negative ions, we can write the expressions

$$|\sigma_{+} - \sigma_{-}|_{\mathrm{Ll}^{+}} = \sigma_{10} - \sigma_{12} - 2\sigma_{13}, \qquad (1)$$

$$|\sigma_{+} - \sigma_{-}|_{\mathrm{Li}^{2^{+}}} = \sigma_{21} + 2\sigma_{20} - \sigma_{23}, \qquad (2)$$

$$|\sigma_{+} - \sigma_{-}|_{\mathrm{Li}^{3^{+}}} = \sigma_{32} + 2\sigma_{31} + 3\sigma_{30}.$$
(3)

for singly, doubly and triply charged lithium ions respectively. We have carried out a comparison, in accordance with formulas (1)-(3), of the data obtained in our work with the data for electron loss and capture cross sections from (7,9-12). For most of the cases investigated, good agreement is observed between the right and left hand sides of the expressions (1)-(3). The discrepancies do not go beyond the limits of the errors in the measurements of the ionization cross sections. It is clear from Figs. 2–4 that for the curves for the energy dependence of the cross sections  $\sigma_+$ , there is typically a broad plateau, which goes over into a monotonic decrease with increase of the ion energy. The energy dependences of the cross sections  $\sigma_-$  are curves with a maximum. For both  $\sigma_+$  and  $\sigma_-$  there is a significant increase in these cross sections with increase of the lithium ion charge. For lithium ions, as for the other fast atoms and ions studied earlier, there is a steep increase in the cross sections  $\sigma_+$  and  $\sigma_-$  with increase of the atomic number of the target atoms.

#### 4. COMPARISON WITH THEORY

Calculation of cross sections for ionization of atoms by ions in the Born approximation is possible, as is well known, for only a few very simple atomic systems. There are absolutely no corresponding calculations for cases of interaction of lithium ions with gas molecules. We shall examine therefore the applicability of a simplified Born approximation proposed by Bethe<sup>[16]</sup>, in the case under consideration. Bethe's formula was obtained by assuming that a structureless charged particle interacts with a target atom leading to pure one-electron ionization.

We shall consider the cross sections for pure ionization of molecules and atoms by singly, doubly and triply charged lithium ions. For this it is necessary to exclude from the cross sections  $\sigma_+$  and  $\sigma_-$  the contributions caused by processes of loss and capture of electrons by lithium ions. A sufficiently close estimate for the cross sections for pure ionization can be made by starting from the following expressions, written for singly, doubly and triply charged lithium ions respectively:

$$\sigma_i(\mathrm{Li}^+) \approx \sigma_-(\mathrm{Li}^+) - (2\sigma_{12} + 3\sigma_{13}), \qquad (4)$$

$$\sigma_i(\mathrm{Li}^{2+}) \approx \sigma_-(\mathrm{Li}^{2+}) - 2\sigma_{23}, \tag{5}$$

$$\sigma_i(\mathrm{Li}^{3+}) \approx \sigma_-(\mathrm{Li}^{3+}). \tag{6}$$

As was noted in<sup>[6]</sup>, a knowledge of the cross sections for electron loss is not always sufficient for an accurate determination of the corrections to the cross sections  $\sigma_{-}$ . However, taking account of the dominant role of one-electron ionization, we can assume without great error that the formulas (4)–(6) express the cross sections for pure ionization.

We note also that the cross sections for pure ionization can be estimated in an analogous way if for this we make use of the total cross sections  $\sigma_+$  for the formation of positive ions and the corresponding cross sections for loss and capture of electrons. A control check showed that the cross sections for pure ionization determined on the basis of formulas (4)–(6) and those found by using the cross sections  $\sigma_+$  are in agreement within the limits of experimental error.

The cross sections  $\sigma_i$  for pure ionization in the gases He, Ne, Ar, Kr, H<sub>2</sub> and N<sub>2</sub>, determined in accordance with the formulas (4)-(6), are shown in Figs. 8-10.

For the curves for the pure ionization cross sections  $\sigma_i(Li^*) = f(v)$ , broad flat maxima are observed in all the gases investigated; these go over to a monotonic decrease with increase of the velocity. The decreasing



FIG. 8. Dependence of the cross sections  $\sigma_i$  for pure ionization on the energy of lithium ions in H<sub>2</sub> and N<sub>2</sub>. The continuous curves correspond to Li<sup>+</sup> ions, the dot-dashed lines to Li<sup>2+</sup> ions, the dashed lines to Li<sup>3+</sup> ions, and the dotted lines are nine times the magnitude of the cross sections for ionization by protons as a function of velocity.

FIG. 9. Dependence of the cross sections  $\sigma_i$  for pure ionization on the energy of lithium ions in He and Ar. The notation is the same as in Fig. 8.

parts of these curves in the region of Li<sup>+</sup> ion energies greater than 1 MeV have an energy dependence of the form  $E^{-1}ln(BE)$  in all the cases investigated.

In Table II we show the ratios of the cross sections  $\sigma_i$  for pure ionization for singly, doubly and triply charged lithium ions to the averaged cross sections for ionization by protons of different velocities<sup>[4,17]</sup> in the energy interval 1–2 MeV:

$$\gamma_1^2 = \frac{\sigma_i(\mathrm{Li}^{+})}{\sigma_i(\mathrm{H}^{+})}, \quad \gamma_2^2 = \frac{\sigma_i(\mathrm{Li}^{2+})}{\sigma_i(\mathrm{H}^{+})}, \quad \gamma_3^2 = \frac{\sigma_i(\mathrm{Li}^{3+})}{\sigma_i(\mathrm{H}^{+})}.$$

As is clear from the table, for Li<sup>+</sup> these ratios are close to some constant value in the whole of the given energy range and for practically all the gases investigated. The differences in the various gases do not go beyond the limits of the total errors in the measurements. A different picture is observed for doubly and triply charged lithium ions. In these cases the ratios  $\gamma_2^2$  and  $\gamma_3^2$  increase systematically with increase of ion energy and nowhere is a constant value of these ratios attained. It is obvious that in the range of velocities considered it is still not possible to introduce the concept of an effective charge for these cases. In particular, for Li<sup>3+</sup> ions in the region of constancy of  $\gamma_3^2$  its value must be equal to 9, since  $z_{eff} = 3q$ , where q is the

Tabl	e II
1	1

E, MeV	Υ1 <sup>2</sup>	Y2 <sup>2</sup>	Y3 <sup>2</sup>	Υı²	Y 2 <sup>2</sup>	Υa²	۲٫²	Υ2 <sup>9</sup>	۲s
H <sub>2</sub>				Не			Ne		
1.0 1.2 1.4 1.6 1.8 2.0	1.7 1.7 1.6 1.6 1.6 1.5	3.8 4.2 4.5 4.6 4.7 4.7	8.0 8.3 8.7 9.0 9.2	1.8 1.8 2.1 2.1 2.0 2.1	3.3 3.9 4.6 4.8 4.9 4.9	5.9 7.4 7.8 7.6 7.9	$     \begin{array}{r}       1.8 \\       1.7 \\       1.7 \\       1.6 \\       1.5 \\       \end{array}   $	3.5 3,9 4.1 4.1 4,2 4,3	6.0 6.1 6.4 6.7 7.0
$N_2$				Ar			Kr		
1.0 1.2 1.4 1.6 1.8 2.0	1.7 1.9 1.8 1.8 1.7 1.7	$ \begin{array}{c c} 3.5 \\ 3.9 \\ 4.2 \\ 4.4 \\ 4.5 \\ 4.6 \\ \end{array} $	6.4 6.7 7.0 7.1 7.1	1.7 1.8 1.8 1.8 1.8 1.8 1.8	$\begin{array}{c c} 3.2 \\ 3.4 \\ 3.8 \\ 4.0 \\ 4.1 \\ 4.3 \end{array}$	$5.9 \\ 6.4 \\ 6.7 \\ 6.8 \\ 7.0$	1.6 1.6 1.7 1.7 1.7 1.7	$\begin{array}{c c} 3.1 \\ 3.2 \\ 3.6 \\ 4.0 \\ 4.0 \\ 4.1 \end{array}$	5.8 6.3 6.5 6.5



FIG. 10. Dependence of the cross sections  $\sigma_i$  for pure ionization on the energy of lithium ions in Ne and Kr. The notation is the same as in Fig. 8.

proton charge. As is clear from Table II, only in the case of hydrogen at the highest energies investigated is the value  $\gamma_3^2 \approx 9$  attained; the closest approach to this value is in helium, i.e., with particles with the smallest number of electrons. Evidently, even at the highest ion velocities we have investigated, and only in the cases with a small number of electrons making part in the collision, the probability of collectivizing them becomes so small that the incident  $Li^{3+}$  ion can be considered as a structureless particle with charge  $z_{eff} = 3q$ .

As has already been noted, the quantity  $\gamma_1^2$  at energies higher than 1 MeV is approximately constant in all the gases investigated and is equal to 1.7 on average. In accordance with Bethe's formula, this quantity can be regarded as the ratio of the square of the effective charge of the Li<sup>+</sup> ion to the square of the proton charge. Starting from this, the effective charge of the Li<sup>+</sup> ion is given numerically by the quantity  $\gamma_1 = \sqrt{1.7}$ . Thus, the effective charge in the case of singly charged lithium ions is equal on average to  $z_{eff} = 1.3q$ .

A fact worthy of attention is that for Li<sup>+</sup> ions, as in the cases of ionization by He<sup>+</sup> and N<sup>+</sup> ions investigated earlier,  $z_{eff}$  is approximately equal to  $Z^{1/4}q$ , where Z is the atomic number of the fast particle. These results, along with those obtained earlier<sup>[18]</sup>, enable us to obtain semi-empirical relations for the cross sections for pure ionization of gaseous atoms and molecules by fast singly charged light ions; however, this problem is the subject of a separate investigation to be made later.

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