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MODE EXCITATION, SPATIAL COHERENCE, AND FREE OSCILLATION KINETICS IN A RUBY LASER

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Spatial coherence and free oscillation kinetics are investigated by high-speed-camera scanning using the Young and Linnik techniques. When irregular spikes are being generated, the radiation in each of them is spatially coherent and a single transverse mode is excited. Incoherence of the radiation is due to the simultaneous excitation of a large number of transverse modes and regular pulses of radiation that decay to a stationary level. The conditions for spatially coherent or incoherent generation are derived on the basis of some assumptions concerning the effect of dielectric-constant inhomogeneities and its changes on "hole burning" and losses in the modes, and agreement of these with experiment is obtained. From these conditions it follows that the generation characteristics considered depend on the transverse distribution of the population inversion in the active rod, on the type of cavity used, and on the pump power.

As is known, the radiation dynamics of free oscillation in solid (and, in particular, ruby) lasers depends on the type of cavity used and the pumping conditions: in spherical (and some other) cavities the dynamics is regular, obeying the simplest balance equations,[1] but in resonators with plane mirrors generation is unsteady and has the character of irregular pulsations. It was shown in[2] that nonstationary radiation can appear during generation of a single mode because of a kind of parametric action on the active medium arising as a consequence of a burning out of the population inversion by stimulated emission of a single mode. But from the calculation carried out in[3] it follows that the observed irregularities cannot be completely explained solely by longitudinal inversion inhomogeneities, and in[4] it was suggested and in[5] shown by a calculation of specific models, that transverse inhomogeneities in the population inversion have a marked effect on the generation dynamics and excitation of transverse modes. The irregularity of the spikes is due to the excitation of modes with a single transverse index (or, as they say, of a "single transverse mode") in each spike, which means spatial coherence of the radiation. Experimentally this was shown for a laser with plane mirrors in[6]. Regularity of the pulses is associated with simultaneous generation of many transverse modes.[4]

Thus, inhomogeneities in the population inversion will have a significant effect on the excited transverse modes; based on this idea, we shall in this paper derive the conditions for the generation of a single mode or of several modes simultaneously (irregular and regular regimes, respectively) and compare them with experiment.

SPATIAL COHERENCE AND CHARACTER OF THE GENERATION DYNAMICS

We shall derive the conditions for excitation of only one transverse mode (more exactly, of modes with one transverse index) in free generation. If only one transverse mode is excited, the radiation is spatially coherent. Otherwise, if several modes are excited simultaneously, the radiation will be spatially incoherent, whereas, on the other hand, this should lead to regular and damped radiation kinetics. The conclusion pertains, strictly speaking, to the first spike, and for subsequent spikes the condition may be at most only necessary, but not sufficient.

We use the following assumptions in the derivation: a) the active substance is a three-level system and has a homogeneously broadened luminescence line; b) in the region of transition frequencies there are no other transitions, other than the laser, and there is no nonlinearity of the absorption coefficient (or gain) of the saturable filter type; c) the number of possible modes of generation is much greater than one.

Under these conditions, neglecting effects associated with mode interaction, the generation will be described by the simplest balance equations (of the Statz–de Mars type)[7]

$$\partial(|E|^2 / dt = -\omega(\varepsilon_1'' + \varepsilon_L'')|E|^2, \quad (1a)$$

$$\tfrac{1}{2}\partial\varepsilon_1'' / \partial t = -w - b|E|^2\varepsilon_1'', \quad (1b)$$

where E is the amplitude of the radiation field, ω is the generated frequency; $\epsilon_1'' = -\epsilon_0''n/n_0 = -\lambda K_0 n/2\pi n_0$ is the imaginary part of the dielectric constant due to inversion of the population n, averaged over the resonator; the term ϵ_L'' describes the resonator losses; w is the pump power, expressed in units of change of dielectric constant per second; $b = K_0\lambda/2\pi n_0 h$; n_0 is the concentration of active atoms; $K_0 = (2\pi/\lambda)\epsilon_0''$ is the absorption coefficient for the unexcited substance; $|E|^2 = |E_1|^2 + |E_2|^2 + \ldots + E_1E_2^* + E_1^*E_2 + \ldots$, where $E_1, E_2, \ldots$ are the fields of the various modes. Neglect of mode interaction is possible, for example, when $|E_1| \gg |E_2|, \ldots$, i.e., when only one mode is excited. Equation (1) is applicable also if we are interested only in quantities averaged over times of the order of the length of a spike, in the case when the period of the beats between nearest modes is much smaller than the spike length (then terms of the type $\overline{E_1E_2^*}$ give zero when averaged and the mode losses are close).

Different transverse modes will be discriminated by their losses and will have different thresholds. When generation begins, the first mode to reach threshold and come up to an intensity sufficient for significant "burning" of the population inversion (according to Eq. (1b)) is the mode with the highest Q. If the second mode is unable to attain any significant intensity by the instant when this mode exhausts the population inversion to the value at threshold, then the generation will consist of only one mode. Thus, for generation in only one mode, i.e., for spatial coherence of the laser radiation, it is necessary that the inversion be burned up in a time τ that is less than the difference in the times for attainment of the maximum inversion by the modes Δt. Starting from Eq. (1), it is easy to show that this difference is approximately equal to the difference of the threshold times and depends on the difference of the losses between nearest modes $\Delta\psi''$ (which we shall call the "discrimination losses"):

$$\Delta t = \frac{c}{Lw\omega}\Delta\psi'' \tag{2}$$

(L is the resonator length).

Similarly, from (1), for the time τ, as well as for the maximum change in dielectric constant $|\Delta\epsilon''_{max}|$ in this time, we obtain

$$\tau \approx \frac{1}{\sqrt{w\omega}}\frac{1}{2}\left(\ln\frac{\Phi_1}{\Phi_0}\right)^{-1/2}\ln\left(4\ln\frac{\Phi_1}{\Phi_0}\right) \approx \frac{0{,}4}{\sqrt{w\omega}} = \frac{c}{Lw\omega}\psi_0, \tag{3}$$

$$|\Delta\varepsilon''_{max}| \approx 2\left(\frac{w}{\omega}\ln\frac{\Phi_1}{\Phi_0}\right)^{1/2} \approx 12\sqrt{\frac{w}{\omega}}, \tag{4}$$

where $\psi_0 = 0.4\,Lc^{-1}(w\omega)^{1/2}$; $\Phi_1 = wn_0/\epsilon_0''\epsilon_L''\omega$ is the number of photons in the mode per cm^3 at the moment of inversion maximum; $\Phi_0 = \epsilon_0''/s_0L\epsilon_L''$ is the number of photons of spontaneous emission in one mode of the resonator per cm^3; s_0 is the size of the transverse section of the mode. In calculating the numerical coefficients in (3) and (4) we took the typical value $w = 5 \times 10^{-4}\ s^{-1}$ for the pump power in a ruby laser, which gives $\ln(\Phi_1/\Phi_0) = 35$. For other values of w the numerical coefficients are almost unchanged because of their very slow logarithmic dependence on the parameters. Upon introduction of the "spatial coherency parameter"

$$\zeta = \frac{\Delta t}{\tau} = \frac{\Delta\psi''}{\psi_0}$$

the condition for spatial coherence is expressed as

$$\zeta > 1. \tag{5}$$

And if $\zeta < 1$ (when the losses of the various modes is sufficiently close), then the excitation of several modes simultaneously is possible, i.e., spatial incoherence of the radiation is possible. As follows from the solutions of Eq. (1), there then exists a regime of regular pulsations of the radiation that decay to a stationary level, upon fulfillment of the condition $1/\Delta\omega' \ll \tau$, i.e.,

$$\Delta\omega' \gg 2.5\sqrt{w\omega}, \tag{6}$$

where $\Delta\omega'$ is the difference in the frequencies of the nearest transverse modes (the frequency of the mode beats). This condition is usually fulfilled in a laser (we shall have more to say about this later).

In calculating the losses of the modes, it is necessary to keep in mind that usually, and particularly in the first spikes, the size of the excited modes is less than the sizes of the diaphragms (frequently natural—the edges of the ruby or of the mirrors, etc.) inside the resonator.[8] This means that diffraction at the diaphragm edges does not play a role in discriminating among the modes. The basic factor influencing the losses of the transverse modes will be the nonuniformity of the transverse distribution of population inversion. When the active element takes up a small part of the resonator length, it can be represented as an amplifying sheet perpendicular to the resonator axis with a nonuniform distribution of gain g in the plane of the sheet. As shown in[9] the calculation of the modes of a resonator of this type can be made analytically, if the distribution of gain g follows a Guassian law ("Gaussian diaphragm")

$$g = g_0 \exp(-x^2/b^2). \tag{7}$$

This case corresponds to a quadratic distribution of the population inversion, or the absorption coefficient of the medium K, in terms of the transverse coordinate x, is:

$$K = K_1 + \chi x^2; \tag{8}$$

here $\chi = 1/b^2 l$, where l is the length of the active rod. In[10] it was shown experimentally that under usual pumping conditions just this type of transverse distribution of the inversion is created in lasers, particularly upon excitation of lower order modes, which would usually be the case in the first spikes.

The Gaussian diaphragm acts on the light beam as a lens with an imaginary focal distance. The calculation in this case reduces to a calculation of the frequencies of the modes of a resonator with a lens of imaginary focus $f = ikb^2$; the imaginary part of the frequency will represent the loss. From[9,11] is easily found the following expression for the frequencies of the resonator modes ω with a lens and two identical spherical mirrors having radius of curvature R:

$$\omega = \frac{c}{L}\left\{q\pi + \left(m+\frac{1}{2}\right)\arccos\frac{\delta-1}{\delta+1}\left(\left[1-\frac{(\delta-\gamma)(1-\gamma)}{2(\delta-1)}\varphi\right]\right.\right.$$
$$\left.\left.\times\left[1-\frac{(\delta+\gamma)(1+\gamma)}{2(\delta-1)}\varphi\right]\right)^{1/2}\right\}, \tag{9}$$

where the parameter $\delta = 2R/L - 1$ is a characteristic of the resonator[1)]; the parameter $\gamma = 2L_1/L - 1$ is a characteristic of the position of the lens or Gaussian diaphragm in the resonator; L_1 is the separation of the lens from the mirror:

$$\varphi = L/f = -iL/kb^2 = -iL\chi l/k;$$

the integer q is the axial index of the mode, and m is the transverse index (for simplicity we do not write the second transverse index). From this we can obtain approximate expressions for the discrimination loss $\Delta\psi''$:

$$\Delta\psi'' =$$

[1)]This parameter δ is connected with the parameter g usually employed in the theory of spherical resonators [11] by the relation $\delta = (1 + g)/(1 - g)$.

$$
= \begin{cases}
2\sqrt{|\delta|}, & -\gamma^2 < \delta < -\gamma^2 \frac{|\varphi|}{4} \\
\sqrt{2}\gamma\left(1 + \frac{\delta}{2\gamma^2} - \frac{2\delta}{\gamma^2 |\varphi|}\right)\sqrt{\frac{|\varphi|}{4}}, & -\gamma^2 \frac{|\varphi|}{4} < \delta < \gamma^2 \frac{|\varphi|}{4} \\
\left(\sqrt{\delta} + \frac{\gamma^2}{\sqrt{\delta}}\right)\frac{|\varphi|}{4}, & \gamma^2 \frac{|\varphi|}{4} < \delta < 1 - \frac{|\varphi|}{2} \\
\frac{1}{2}\left\{1 + \delta + \gamma^2\left(3 - \delta - 4\left[1 + 4\left|\frac{\delta - 1}{\varphi}\right|^2\right]^{-1}\right)\right\}\frac{|\varphi|}{4}, & 1 - \frac{|\varphi|}{2} < \delta < 1 + \frac{|\varphi|}{2} \\
\left(\sqrt{\delta} + \frac{\gamma^2}{\sqrt{\delta}}\right)\frac{|\varphi|}{\sqrt{4}}, & 1 + \frac{|\varphi|}{2} < \delta < \frac{4}{|\varphi|} \\
\sqrt{2}\left(1 - \frac{1 - \gamma^2}{2\delta} - \frac{2}{\delta |\varphi|}\right)\sqrt{\frac{|\varphi|}{4}}, & \frac{4}{|\varphi|} < \delta
\end{cases} \quad (10)
$$

The dependence of $\Delta\psi''$ on the resonator parameter $\delta' = \delta/(1 + \delta)$ is shown in Fig. 1 for the value $|\varphi| = 8 \times 10^{-3}$, usually realized in our experiments (see below). It follows from these graphs that the spatial coherency parameter ξ depends strongly on the quantities δ and γ. In addition, ξ depends on φ, as well as on w.

The derived condition (5) is actually for the first spike. In subsequent peaks the inversion distribution changes on account of burning. Since the configuration and losses of the modes depends on the inversion distribution, both the modes themselves and their losses will change, leading to irregularity of the pulsations. Thus the condition (5) is also the condition under which a regime of irregular spikes is possible. Burning up of the inversion, on the average, equalizes the inversion distribution and hence in the generation process the parameter ξ will probably diminish (if the pump w does not diminish). The greater the magnitude of ξ in the first peak, the more probable that condition (5) will be maintained in subsequent peaks, and the radiation in them will be spatially coherent. In the opposite case, one may suppose that if the pump w is not decreased, this condition will be maintained and the kinetics will remain regular.

When several modes are excited (not only with different transverse, but with different axial indices) it can be assumed that all modes will appear whose threshold times do not exceed the threshold time of the first mode by the quantity τ. From this we can estimate the width of the spectrum of the generating modes Δ_g in the first spike, if we assume that the gain coefficients of the modes depend on frequency in accordance with the Lorentzian shape of the luminescence line,[12] and their losses are close[2)]:

$$
\Delta_g = \Delta_l/4\left(1 + \frac{\varepsilon_L''}{6}\sqrt{\frac{\omega}{w}}\right)^{1/2}, \quad (11)
$$

where Δ_l is the width of the luminescence line.

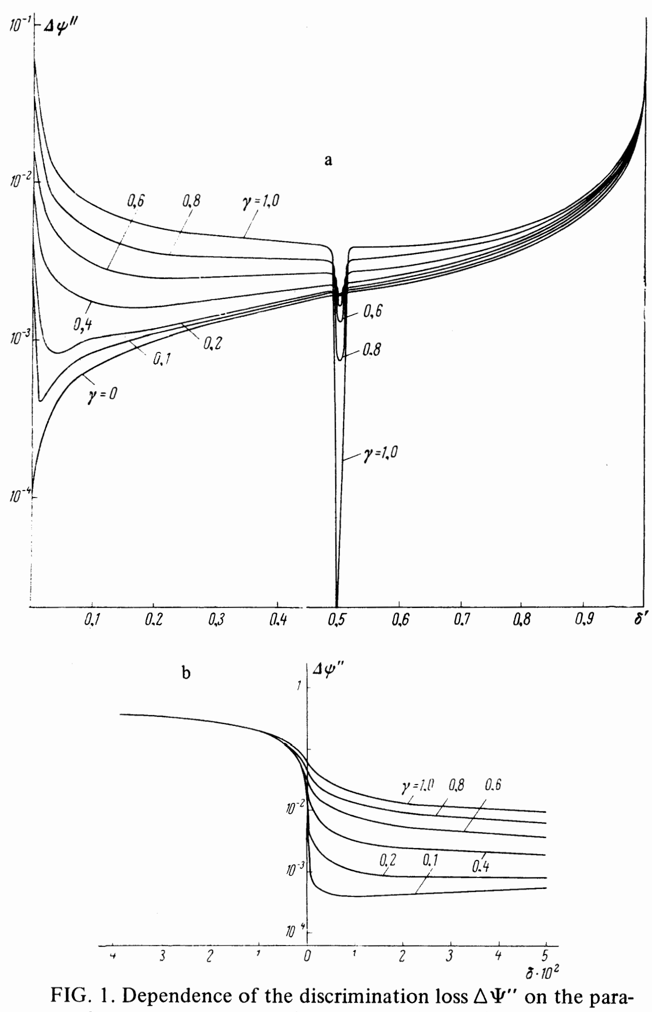

FIG. 1. Dependence of the discrimination loss $\Delta\Psi''$ on the parameter of a spherical resonator δ: a—for $0 < \delta < \infty$ ($\delta' = \delta(1 + \delta)$); b—for δ close to zero.

EXPERIMENT

These ideas give only a qualitative indication of the spatial coherence or incoherence of the radiation and do not permit a quantitative calculation of the degree of coherence. Hence in the experiment we studied the spatial coherence only qualitatively, with a simultaneous investigation of the generation dynamics. These results were compared with the magnitude of the parameter ξ calculated from the quantities δ, γ according to (10), as well as from w and $|\varphi|$, which were measured experimentally.

The spatial coherence was studied using the Young scheme—a recording was made of the pattern of bands obtained by Fraunhofer diffraction by a diaphragm of two apertures of diameter 0.3 mm and spacing 1.25 mm, placed immediately in front of the laser mirror. The pattern was scanned by a slit in an SFR fast photoregistration apparatus, which gave information on the generation dynamics, as well as on the mode beats. A fast scan permitted registration of the simultaneous excitation of modes with frequency differences up to 70 MHz. For greater frequency differences the beats were not resolved, and the interference bands were smeared out (spatial incoherence). Sharp horizontal bands indicated spatial coherence, i.e., excitation of one transverse mode.

In some cases, we scanned the spectrum obtained with a Fabry-Perot interferometer or the pattern in the far zone simultaneously and on the same plate with the Young pattern.

Besides the Young scheme, for the study of the shape of the wave front we used a Linnik interferometer,[14] both with slit scanning in the SFR and frame scanning ("time magnifier"). In this arrangement, the wave under study, passing through a partially transparent aluminum layer, interfered with a spherical "standard"

[2)]This formula agrees with the estimates obtained in [13].

wave created as a result of diffraction by a small aperture of diameter 0.3 mm in this layer. The aluminum layer was deposited on a glass plate which was placed immediately in front of the laser mirror. The interference pattern taken at a distance of 150 cm from the plate was a band of equal height of the studied wave front with respect to the spherical standard wave. The arrangement as a whole was checked with a gas laser, with which very sharp patterns were obtained.

The laser was a ruby rod 72 mm long and 6 mm in diameter, with an atomic concentration of chromium of 0.03% and spherical mirrors with R = 50 cm and reflection coefficient 98%; sometimes plane mirrors were used. The distance L between the mirrors was from 43 to 106 cm. Pumping was accomplished by two IFP-2000 linear flashtubes in a "tight" illuminator.

The population distribution was determined by photometery of the distribution of luminescence light over the end of the ruby rod, without scanning.3) This does not give the instantaneous distribution of inversion, but one which is averaged over a single pump pulse; however, as shown in[10], these are approximately the same. Examples of the distributions obtained are presented in Fig. 2. The magnitudes of χ obtained for the maximum of the distribution of absorption coefficient (see (8)) were usually from 0.5 to 1.5 cm^{-3}. This corresponded to $|f| = (0.7-1.8) \times 10^4$ cm and $|\varphi| = (3-17) \times 10^{-3}$.

The pump power was measured in two ways. In the first, the pump power was found from oscillograms of the light output of the pump lamp according to the formula

$$w = \frac{\lambda K_0}{4\pi} y_{\text{thr}} \Big/ \int_0^{t_{\text{thr}}} y\,dt, \qquad (12)$$

where y(t) is the oscillogram of the lamp output, y_{thr} its ordinate at the moment of initiation of generation. This formula was derived under the assumption that almost all the pump energy from the moment the lamp lights to the threshold of generation goes into the transfer of approximately half of all the chromium ions into the upper level.

In the second way, the frequency of generated pulses N was measured; this, as follows from Eq. (1), depends on the pump: $N = (2\omega w)^{1/2}/2\pi$. (This method was used only when the pulsations were sufficiently regular.)

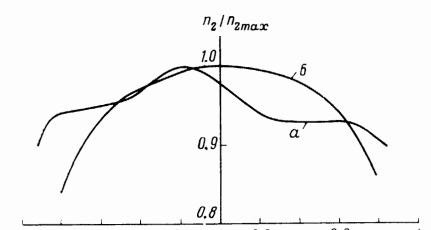

FIG. 2. Relative distribution of the population over a cross section of a ruby: a—along a line passing through the maximum in population parallel to the optic axis c and a plane passing through the pump lamp; b—along a line passing through the population maximum and perpendicular to the c axis; n_2—population of the upper chromium levels, r—radius of the ruby rod.

3)We thank A. M. Mozharovskiĭ for making these measurements.

The experimental results on the pump power w as a function of the voltage U to which the condensers were charged (1200 μF) are presented in Fig. 3. It is seen that the first method gives values that are too high, which is explained by neglect of the losses from the upper levels due to luminescence. The consistency of the values obtained by the second method indicates, as was already shown, for example, in[15], that the generation kinetics in the case of regular regimes is rather well described by the balance equations (1).

RESULTS

Slit-scan Young patterns were taken at various pumping rates w, with configurations of the resonator corresponding to parameter values: $\delta = -0.09$; 0.031; 0.22; 0.27; 0.30; 1.13; 1.51; ∞, for $\gamma = 0$, and $\delta = 0.031$ for $\gamma = 0.020$; 0.103; 0.207; 0.290.

Examples for different values of ζ, δ, and γ are given in Fig. 4. The patterns obtained bear a completely different character for different ζ, and this depends precisely on the magnitude of ζ—for the same values of δ and γ and different values of ζ the patterns bear a different character. For $\zeta \ll 1$ (Fig. 4a) (usually in resonators close to concentric, the least $\zeta = 0.17$) the radiation is always incoherent (the interference fringes are smeared out), the mode beats are not seen (since in these resonators the closest modes are much more than 70 MHz apart), and the pulsations are regular and decay to a steady level. Scanning of the Fabry-Perot interferogram (e.g., Fig. 4a) showed that the width of the generation spectrum in the first spike was 0.3—0.5 cm^{-1} for pumping of $w = (2-8) \times 10^{-4}$ s^{-1}. Assuming the loss in one double passage of the resonator to be about 10%, we obtain from (11) a value of $\Delta_g = 1.1-1.5$ cm^{-1} for the width of the spectrum. This difference between the measured and calculated widths should not be considered significant in view of the crudity of the assumptions in deriving (11) and the inaccuracy of measuring the spectral width.

When ζ is close to unity, the scan shows coherent portions, mode beats are sometimes seen, and the character of the pulsations begins to change toward incoherence (Fig. 4d). In Fig. 4b it is seen how several

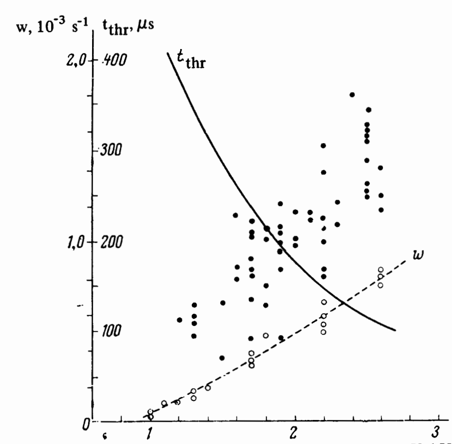

FIG. 3. Graphs of pump power w and time for initiation of generation t_{thr} as functions of the initial voltage on the condensers in the pump power supply U; ●—pump power from the oscillographic method, ○—from the spike frequency.

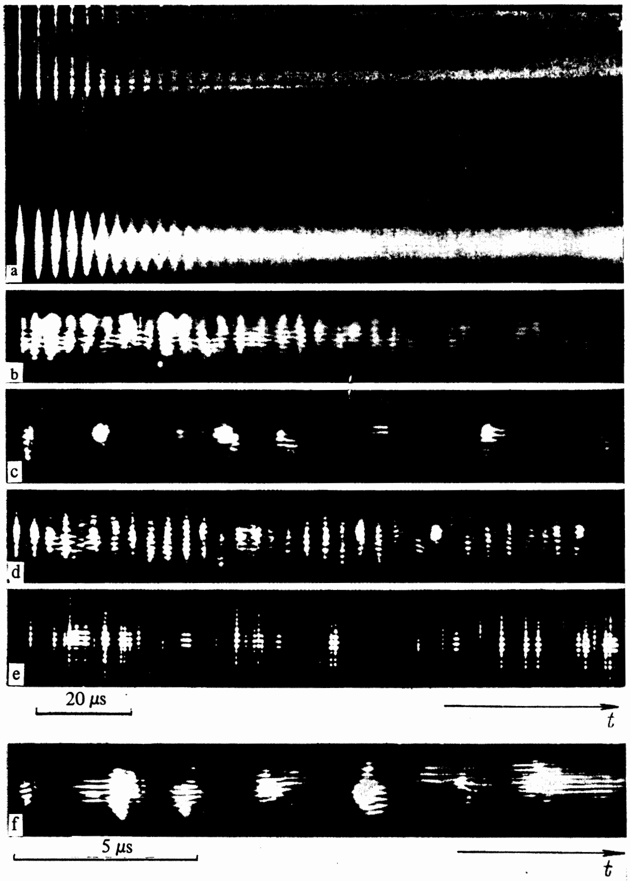

FIG. 4. Slit-can by SFR of the Young pattern during generation in different resonators and at different values of the spatial coherence parameter: a–ζ = 0.19, δ = 0.031, γ = 0.21; b–ζ = 0.81, δ = 0.031, γ = 0.21; c–ζ = 0.95, δ = 0.031, γ = 0; d–ζ = 1.37, δ = 1.13; γ = 0; e–ζ = 40, $\delta = \infty$, γ = 0; f–ζ = 0.88, δ = 1.51, γ = 0.

modes enter successively into the first spikes. With approach to a plane resonator, when $\zeta \gg 1$ (to $\zeta = 40$), the fringes are very sharp—the radiation is always spatially coherent. Jumping of modes in frequency from spike to spike increases and reaches 0.5 cm^{-1}, which is explained not only by a change in the transverse indices, but also in the longitudinal.[3,16] The pulsations in this case have a disordered character.

Some results of the investigation of the wave front of the radiation by the Linnik method with frame scanning are presented in Fig. 5. Sharp patterns, as is to be expected, are obtained only with ζ close to and greater than unity (Fig. 5b, c). The wave front is, as a rule, spherical, with some irregularities. Sometimes, usually not in the first spikes, one obtains patterns corresponding to interference of several spherical waves from different centers (Fig. 5c). From these patterns it is possible to find the distance between the centers of the spherical waves, which permits computation of the mode order m, using the approximate formula from the theory of the spherical resonator:

$$m \approx \pi d^2 / 2\lambda L\sqrt{\delta}. \qquad (13)$$

In our experiments we obtained d = 2–3 mm and m = 15–25, which agrees with the data obtained in[4].

DISCUSSION

In treating the mode interaction we considered only transverse modes, and irregularities in generation were attributed only to jumping of these modes. At the same time it is known that in actuality[16] there also appear irregularities due to jumping of axial mode indices. At first sight it seems that these jumps ought to appear also in a spherical resonator. The good regularity of the pulsations and the tendency to steady state is evidence that these jumps do not appear. This is probably explained by the following circumstances. As can be derived, for example, from Eq. (9), in spherical resonators with $\delta < 1$ (in "beyond confocal" resonators) the nearest transverse modes in frequency and losses do not belong to the same axial index, but to neighboring ones. In such modes the number of nodes and antinodes in the field in the longitudinal direction differs. When they oscillate simultaneously the inversion is used up uniformly, and the conditions for axial jumping do not arise.

The experimental results of our work show that the spatial coherence, as well as the kinetics, of the radiation depends strongly on the transverse distribution of the population inversion, i.e., on the distribution of the imaginary part of the dielectric constant. Thus, in the formation and excitation of modes in a laser it is not only the distribution of the real part ϵ' inside the resonator, i.e., the refractive index, as is usually assumed,[17,18] that is important, but also the imaginary part ϵ'', i.e., the population inversion. The effect of the population distribution on laser modes was considered in[19,20], but sometimes this effect is underestimated.[19]

In this connection, let us discuss the effect of inhomogeneities of both kinds on the formation of modes and their losses in the example of a plane-parallel resonator completely filled with an active medium. We assume that the mode is formed just on account of inhomogeneities, and the effect of the edges of mirrors and diaphragms in the resonator is neglected. Let the inhomogeneity in the distribution of ϵ have the form of a hump in the transverse section, of height $\Delta\epsilon$ and width a. The question whether this inhomogeneity will

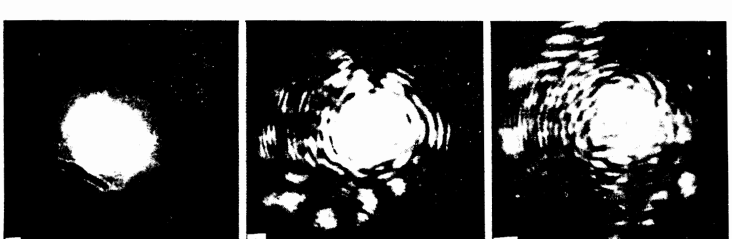

FIG. 5. Frame-scan by SFR of Linnik interferograms: a–ζ = 0.31; b–ζ = 0.81; c–ζ = 0.81.

sustain at least one mode is analogous to the question of maintaining an electromagnetic field by means of the total internal reflection of a dielectric layer of width a, the dielectric constant of which is greater by $\Delta\epsilon$ than that of the surrounding medium. It is necessary to fulfill the condition (see[21])

$$a\mathrm{Re}k_x > \pi/2, \tag{14}$$

where k_x is the transverse component of the wave vector.[4)]

From the conditions for total reflection we have

$$k_x = \omega c^{-1}\sqrt{\Delta\varepsilon}, \tag{15}$$

whence we obtain the condition for retention of at least one mode in the transverse dimension a:

$$a\omega c^{-1}\mathrm{Re}(+\sqrt{\Delta\varepsilon}) > \pi/2. \tag{16}$$

These modes, as is easily seen in the example of the dielectric layer,[21] will have losses less than the losses of modes in the surrounding space, only if the imaginary part $\Delta\epsilon'' < 0$, i.e., the population inversion has a maximum. It is only this case that is of practical interest, since in generation only the modes of highest Q are excited. The losses of these modes, as can be seen from the formulas given in[21] for the dielectric layer, will depend on the size of $\Delta\epsilon''$, i.e., on the height of the hump in the population inversion. The mode configuration, their near and far zones (and angular divergence) is determined mainly by the distribution of ϵ', i.e., the refractive index, since the quantity Re $k_x = \omega c^{-1}\mathrm{Re}\,(\Delta\epsilon)^{1/2}$, on which the mode configuration depends, is more sensitive to the real part $\Delta\epsilon'$ than to the imaginary part $\Delta\epsilon''$ (if only $\Delta\epsilon' < 0$):

$$\mathrm{Re}\sqrt{\Delta\varepsilon} = [{}^1\!/_2(\Delta\varepsilon' + \sqrt{\Delta\varepsilon'^2 + \Delta\varepsilon''^2})]^{1/2}. \tag{17}$$

Only in unstable resonators ($\Delta\epsilon' < 0$) will the mode configuration be sensitive also to the distribution of the population inversion.

In deriving the coherence condition (5) we assumed that the modes themselves would remain unaltered in the generation process. In actuality exhaustion of the population inversion can change the mode configuration, and coherence may not be maintained. It is easy to obtain the condition for no change in the mode for a plane-parallel resonator with a medium, when $|\Delta\epsilon'| < |\Delta\epsilon''|$, and the mode is determined by the distribution of the population inversion. It is obvious that the mode will change significantly when the change $\Delta\epsilon$ when the inversion is used up satisfies a condition like (16). The mode is unchanged in the opposite case, from which, for a quadratic medium of the type (8), we can derive the condition

$$\Delta\omega' > 2.4\sqrt{w\omega}.$$

In the derivation we used Eq. (4) and set, according to the theory of modes of a plane-parallel resonator with an imaginary lens (9),

$$a^2 = \frac{1}{k}\sqrt{2L|f|} = \sqrt{\frac{2}{k\chi}}, \quad \Delta\omega' = c\sqrt{\frac{\chi}{2k}}.$$

4)For a hump in ϵ, of arbitrary shape the mode is a superposition of waves with different k_x, and relations (14)–(16) are approximately true for moderate values of $|k_x|$.

The condition obtained coincides with (6). When (6) is not fulfilled, the balance equations (1), as already said, are inapplicable, and for calculation of the generation dynamics it is necessary to revert to more exact equations of the type employed in[5]. It is easy to show that the condition (6) will be fulfilled when

$$\chi > 11.5\ w\omega^2/c^3. \tag{18}$$

It follows from this that in a ruby laser with the usual pump ($w \approx 10^{-3}\ s^{-1}$), χ should be greater than $4 \times 10^{-3}\ cm^{-3}$ (size of the first mode a less than 0.3 cm). In our work we obtained $\chi \approx 1\ cm^{-3}$, and for $1/L = 0.1$, $\chi \approx 0.1\ cm^{-3}$ averaged over the resonator, i.e., condition (6) is fulfilled. In spherical resonators the inversion distribution scarcely affects the mode configuration, and the difference in frequency between the modes $\Delta\omega'$ is usually much greater than in a plane-parallel resonator, so that the modes are not altered during generation and condition (6) is always fulfilled.

The authors thank M. D. Galanin for a discussion of the work and M. N. Popova for assistance and discussions.

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