MAGNETIC FLUX QUANTIZATION IN THE NORMAL STATE

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The possibility of appearance of coherent quantum effects in superconductors at temperatures above the critical transition temperature T_c (into the superconducting state) is analyzed. It is shown that in a superconducting ring placed in a constant magnetic field H at a temperature $T > T_c$ a circulating current is induced which varies periodically as a function of the magnetic flux Φ. The amplitude of the current is proportional to the parameter $e^{-L/\xi}$ where L is the length of the perimeter of the ring and $\xi \sim v_0/(T_c(T - T_c))^{1/2}$ is a temperature-dependent coherence radius.

AMONG the essential manifestations of the concept of macroscopic coherence of the London[1] superconducting state are the so-called coherent quantum effects: fluxoid quantization[1,2], oscillation of superconducting currents in time in the presence of an electric field (Josephson effect)[3], quantum interference of superconducting currents (cf. e.g.[4]). The purpose of the present paper is to analyze the possibility of occurrence of such effects under the conditions of "fluctuation pairing"[5], i.e., at temperatures exceeding the critical temperature of the superconductive transition, T_c.[1)] It turns out that coherent quantum effects do indeed occur for $T > T_c$; however, in distinction from the situation below T_c, where the phase coherence propagates over unlimited spatial and temporal intervals, under the conditions of fluctuation pairing, it occurs only over distances of the order of the coherence length $\xi(T) = (D/\Gamma(T))^{1/4}$ and for time intervals of the order of $\Gamma^{-1}(T)$, where $\Gamma(T) = 8\pi^{-1}(T - T_c)$ is the reciprocal of the relaxation time of the Cooper pairs[8] and D is the diffusion coefficient ($D = (1/3)v_0 l$ for an impure alloy and $D \sim v_0^2/T_c$ for $l \gg v_0/T_c$; l and v_0 are respectively the mean free path and the Fermi velocity of the electrons in the normal state). This causes a broadening of the spectrum of Josephson oscillations above T_c[6,7], and for the case of the flux quantization there occurs an exponential decay of the amplitude of the circulating current, under the condition that the length L of the quantization perimeter is considerably larger than the characteristic size $\xi(T)$ (cf. infra).

1. We consider a thin superconducting filament with transverse dimensions d_1 and d_2 small compared to $\xi(T)$, placed in a field with vector potential **A**, produced by a source of magnetic field (a narrow solenoid) of small area (Fig. 1). According to Aharonov and Bohm[9], the action of the vector potential is not eliminated even if the magnetic field vanishes in the region of the metal. In the case illustrated in Fig. 1 the order parameter (the wave function of the pair) ψ depends only on one coordinate x, measured along the length of the filament: $\psi = \psi(x)$. We denote the length of the perimeter of the ring by L ($L = 2\pi R$). Since the function must be single-valued after traversing the closed contour, it admits a representation of the form

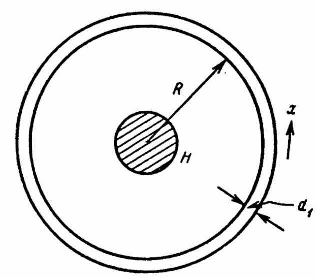

FIG. 1

$$\psi = \sum_{n=-\infty}^{\infty} \psi_n e^{ik_n x}, \tag{1}$$

where $k_n = 2\pi n/L$, $n = 0, \pm 1, \pm 2, \ldots$. In the Ginzburg-Landau the superconducting current is given by the expression ($\hbar = 1$)

$$j = \frac{2e}{m} \operatorname{Im}\left[\psi^*\left(\frac{\partial}{\partial x} - \frac{2ie}{c}A\right)\psi\right], \tag{2}$$

where $A = A_x$ is determined by the total flux Φ of the magnetic field produced by the solenoid:

$$A = \Phi/L. \tag{3}$$

We find the thermodynamic expectation value of j, by averaging with the Gibbs factor $e^{-\beta F}$:

$$\bar{j} = \operatorname{Sp}(je^{-\beta F})/\operatorname{Sp}(e^{-\beta F}). \tag{4}$$

Here β is the reciprocal temperature ($\beta = 1/T$) and F the Ginzburg-Landau free energy[10], which in our case is given by

$$F = \frac{d_1 d_2}{2m}\int_0^L \left\{\left|\left(\frac{\partial}{\partial x} - \frac{2ie}{c}A\right)\psi\right|^2 + \frac{1}{\xi^2}|\psi|^2\right\}dx, \tag{5}$$

where $d_1 d_2$ is the cross sectional area of the filament[2)]. According to the expansion (1), F has the representation in the form of a sum

[1)]The case of temporal coherence of superconducting currents in a Josephson tunnel junction for $T > T_c$ has been considered in a previous paper of the author [6] (cf. also [7]). In the present paper we consider the effect of quantization of the magnetic flux for $T > T_c$.

[2)]We note that the Ginzburg-Landau equation in its usual form is not valid for $T > T_c$. One can only write down a temporal generalization of such an equation, which includes stochastic terms, owing to which ψ is nonzero for temperatures above T_c.

$$F=\sum_{n=-\infty}^{\infty}F_n,\quad F_n=\frac{d_1d_2}{2m}L\left[\left(k_n-\frac{2e}{c}A\right)^2+\frac{1}{\xi^2}\right]|\psi_n|^2,\quad (6)$$

from where it can be seen that the different values of n are independent from one another. The expectation value of the current is defined according to (2) as

$$\bar{j}=\frac{2e}{m}\operatorname{Re}\sum_{n,n'=-\infty}^{\infty}\left(k_n-\frac{2e}{c}A\right)\overline{\psi_{n'}^*\psi_n}\exp\{i(k_n-k_{n'})x\}.\quad (7)$$

The element of phase-space volume in the computation of the trace in Eq. (4) has the quasiclassical product approximation[7]

$$d\Gamma=\prod_n\frac{dN_n\,d\varphi_n}{2\pi},\quad (8)$$

where N_n is the number of particles in the condensate for a given value of n, φ_n is the appropriate phase. Considering $N_n\sim|\psi_n|^2$, we obtain[3)]

$$\overline{\psi_{n'}^*\psi_n}=\frac{2mT}{d_1d_2L}\left[\left(k_n-\frac{2e}{c}A\right)^2+\frac{1}{\xi^2}\right]^{-1}\delta_{nn'}.\quad (9)$$

Substituting (9) into (7) we have for the current

$$\bar{j}=\frac{4eT}{d_1d_2L}\sum_{n=-\infty}\frac{k_n-(2e/c)A}{[k_n-(2e/c)A]^2+\xi^{-2}}.\quad (10)$$

It is obvious that the expression obtained for the expectation value is different from zero. This means that under the action of the static field of the vector potential A a nondecaying current is induced in the ring, even if the temperature T of the ring is above the critical temperature T_c of the superconducting transition. It is clear from (10) that $\bar{j}$ is a periodic function of A, with period $\Delta A=\pi c/eL$, which is equivalent to a change of flux by one quantum $\Phi_0=hc/2e$.

Summing the series (10) by means of the Poisson summation formula (cf. e.g.[12]) we obtain

$$\bar{j}=\frac{4eT}{d_1d_2}\operatorname{Im}\{(e^{L/\xi}e^{2\pi i\Phi/\Phi_0}-1)^{-1}\}=-\frac{4eT}{d_1d_2}\sum_{p=1}^{\infty}e^{-pL/\xi}\sin\left(2\pi p\frac{\Phi}{\Phi_0}\right).\quad (11)$$

It is clear from here that for $L\gg\xi$ the current amplitude $\bar{j}$ is proportional to the exponentially small factor $\exp(-L/\xi(T))$. Therefore the observation of the effect is possible only sufficiently close to T_c, when $\xi(T)$ is large, or in superconducting rings of small diameter ($\sim10^4$ cm); in the latter case the effect will have a significant magnitude even for temperatures which are not very close to T_c (a similar slow decay of the fluctuational component of the current with temperature occurs in the theory of fluctuational conductivity of films, developed by Aslamazov and Larkin[5]).

2. It is not difficult to carry out a similar calculation for a hollow cylinder for which the wall thickness d is small compared to the coherence length $\xi(T)$. In this case ψ depends both on the coordinate x along the perimeter of the cylinder (Fig. 1) and on the coordinate z in the direction of the normal to the plane of the ring, so that in place of (1) one must write the expansion

$$\psi(x,z)=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dq\,\psi_{nq}\,e^{ik_nx}e^{iqz}.\quad (12)$$

The remaining computations are completely analogous to the ones above. We list only the final result. The expectation value of the current is given by the expression:

$$\bar{j}=-\frac{4eT}{\pi d\xi}\sum_{p=1}K_1\left(p\frac{L}{\xi}\right)\sin\left(2\pi p\frac{\Phi}{\Phi_0}\right).\quad (13)$$

Here $K_1(x)$ is a first-order Bessel function of imaginary argument. Since the asymptotic behavior of $K_1(x)$ for $x\to\infty$ has the form $(2/\pi x)^{1/2}e^{-x}$, it is clear that the expression (13) contains the same kind of exponential smallness for large L as the expression (11).

3. Finally, we consider the occurrence of quantum interference effects in a superconducting ring containing a weak coupling: a Josephson tunnel junction T (Fig. 2). For simplicity we consider the case of a filament ($d_1, d_2\ll\xi(T)$), and the critical current of the junction[6,7] is considered small compared to the magnitude of the current in the ring in the absence of the gap (Eq. (11)).

In place of Eq. (2) for the current we get in this case the Josephson expression for j, according to which the magnitude of the current is proportional to the sine of the discontinuity of the phase of ψ at the point of the junction[3,4,6,11]. This can be written in the following form:

$$j=T_{12}\operatorname{Im}[\psi(0)\psi^*(L)],\quad (14)$$

where, according to[6], the magnitude

$$T_{12}=\frac{\pi}{4eRT_c}\frac{8(\pi T_c)^2}{7\zeta(3)N}.\quad (15)$$

(R is the junction resistance in the normal state). Now the function $\psi(x)$ is no longer required to be single-valued after traversing the contour, since at the junction there is a jump of its phase, the magnitude of which is determined from the requirement of minimizing the free energy (for $T<T_c$) and under the conditions of fluctuation pairing, by averaging with respect to the Gibbs factor $e^{-\beta F}$, cf. (14). Since the quantity T_{12} is proportional to a small parameter—the transparency coefficient of the barrier—the function $\psi(x)$ in (14) can be calculated to zeroth order in the transparency, i.e., in the current-free state, in which it satisfies the boundary conditions[10]

$$\left[\left(\frac{\partial}{\partial x}-\frac{2ie}{c}A\right)\psi\right]_{x=0}=\left[\left(\frac{\partial}{\partial x}-\frac{2ie}{c}A\right)\psi\right]_{x=L}=0.\quad (16)$$

Making the substitution

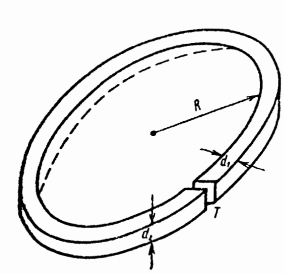

FIG. 2

3)We note that in the book of de Gennes [11] the phase volume is assumed in the calculation of the fluctuations to be proportional to $d\Psi_{n1}$ for which we see no justification. This, however, does not lead to essential differences, since the mean values calculated from (8) and given by de Gennes differ only by a numerical factor of the order of unity.

$$\psi = \exp\left(\frac{2ie}{c}Ax\right)\Psi, \tag{17}$$

we obtain from (14)

$$\bar{j} = T_{12}\,\mathrm{Im}\left[\exp\left\{-2\pi i\frac{\Phi}{\Phi_0}\right\}\overline{\Psi(0)\Psi^*(L)}\right]. \tag{18}$$

The function $\Psi(x)$ now satisfies the boundary conditions

$$\left(\frac{\partial\Psi}{\partial x}\right)_{x=0} = \left(\frac{\partial\Psi}{\partial x}\right)_{x=L} = 0, \tag{19}$$

from which it follows that it can be represented in the form of a series

$$\Psi(x) = \sum_{n=0} \Psi_n \cos\frac{\pi n x}{L}. \tag{20}$$

Making further use of the expression (5) for the free energy and computing the expectation value $\overline{\Psi(0)\Psi^*(L)}$ we easily obtain

$$\bar{j} = T_{12}\frac{2mT}{d_1 d_2 L}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{(\pi n/L)^2 + \xi^{-2}}\sin\left(2\pi\frac{\Phi}{\Phi_0}\right) = j_s \sin\left(2\pi\frac{\Phi}{\Phi_0}\right). \tag{21}$$

The quantity j_s is the amplitude of the Josephson current after summing the series (21) and is written in the form:

$$j_s = T_{12}\frac{2mT\xi}{d_1 d_2}\frac{1}{\mathrm{sh}(L/\xi)}. \tag{22}$$

Thus, the amplitude of the stationary Josephson current in the ring also decreases exponentially with the length of the perimeter L (cf. Eqs. (11), (13)). As already remarked, this is due to the fact that the phase coherence at temperatures above T_c propagates only to distances of the order $\xi(T)$. We note that in the nonstationary Josephson effect above T_c[6,7] the amplitude of the oscillating component of the current does not exhibit an exponentially small factor. However, the temporal behavior of the current in the latter case is described by an exponential $e^{-\Gamma t}$, which leads to a widening of the radiation spectrum, i.e., to the appearance of a lorentzian form for the Fourier component of the current correlation function $\overline{j(t)j(t')}$ with width $\Delta\omega = 2\Gamma$.

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263