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MOTION OF HELIUM IONS NEAR A VAPOR-LIQUID SURFACE

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A description is given of some effects associated with the proximity of helium ions to the liquid–vapor interface.

HELIUM ions that are close to the liquid–vapor interface are repelled from it by the image force

$$F = \left(\frac{e}{2x}\right)^2 \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1(\varepsilon_1 + \varepsilon_2)}; \tag{1}$$

x is the distance from the ion to the surface; ϵ_2 and ϵ_1 are the dielectric constants of the liquid and vapor, $\epsilon_2 - \epsilon_1 = 0.06$; $F > 0$ corresponds to repulsion. By compensating the action of the force (1) by an external field E_0 directed normal to the liquid–vapor surface and forcing the ion toward the surface, we can fix the position of the ion at any given depth x_0 from the surface:

$$x_0 = \frac{1}{2}\left[\frac{e}{E_0}\frac{\varepsilon - 1}{\varepsilon(\varepsilon + 1)}\right]^{1/2}, \quad \varepsilon_1 \approx 1, \quad \varepsilon_2 \equiv \varepsilon. \tag{2}$$

This possibility is convenient for the observation of a number of effects. It should be noted that a similar situation has already been studied experimentally[1] with the aim of making clear the character of the motion of positive and negative (±) ions through the liquid–vapor interface. It was shown in that case that: a) the current of negative ions through the surface has an activation character with activation energy $\sim 25^\circ$K and practically disappears at a temperature $T \sim 1^\circ$K and below, b) it is generally not possible to pull the positive ions from the liquid helium by an electric field E_0 up to $E_0 \sim 400$ V/cm. Thus, in the low temperature region, $T < 1^\circ$K, both negative and positive ions can be strongly forced to the free surface without risking their removal into the gaseous phase.

Let us now estimate the degree of thermal smearing out of the level x_0. If $E_0 \gtrsim 300$ V/cm and $T < 0.5^\circ$K, the spreading Δx_0 is of the order $\Delta x_0 \sim 10^{-7}$ cm. This means that for characteristic distances $x_0 \sim 10^{-6}$ cm, corresponding to fields $E_0 \sim 10^2$–10^3 V/cm and being most typical for the problems considered below, we can neglect this smearing.

SOME RESONANCE EFFECTS

1. The x_0 coordinate (2) determines the position of the bottom of the potential well formed by the potentials of the image force (1) and the electric field E_0. Consequently, small oscillations of ions near x_0 are possible, with a characteristic frequency ω_0 equal to

$$\omega_0{}^2 = \frac{1}{m_\pm}\frac{e^2}{x_0{}^3}\frac{\varepsilon - 1}{\varepsilon(\varepsilon + 1)}, \tag{3}$$

where $m_\pm$ is the effective mass of the ± ions, $m_+ \sim 75 \mathrm{He}^4$, $m_- \sim 245 \mathrm{He}^4$. Setting $x_0 \sim 10^{-5}$–10^{-6} cm, which, for $\epsilon - 1 = 0.06$, corresponds to fields $E_0 \sim 10$–10^3 V/cm, we find for ω_0 the estimate $\omega_0 \sim 10^6$–10^7 sec^{-1}. The experimental determination of the frequency ω_0 gives direct evidence on the effective mass of the ± ions and therefore is of considerable interest.

Observation of the well-delineated resonance at the frequency ω_0 is possible only upon satisfaction of the inequality $\omega_0\tau \gg 1$ (τ is the characteristic time of relaxation). As estimates show, at a depth $x_0 \sim 10^{-5}$–10^{-6} cm, the time τ is of the order of the volume. In pure helium, the quantity τ_{vol} increases monotonically with decrease in the temperature. Therefore, one can always point out a region of temperatures where the inequality $\omega_0\tau > 1$ should begin to be satisfied. In particular, for negative ions, according to the experimental data,[2] the inequality $\omega_0\tau > 1$ for $\omega_0 \sim 10^7$ is satisfied at $T < 0.2^\circ$K.

It should be noted that the expression (3) for ω_0 is obtained under the assumption of the absence of deformations of the free surface. This assumption is valid as long as $(a/x_0)^2 \ll 1$ (a is the ionic radius).

2. The interaction of the ions with the free surface shows the effect on the spectrum of surface waves in liquid helium. To explain the character of this effect, we limit ourselves to consideration of the simplest limiting case of long surface waves with low ionic concentrations: $\lambda \gg l \gg x_0$. λ is the wavelength, l is the mean distance between ions that fill the plane x_0, and x_0 is the distance between the equilibrium free surface and the equilibrium plane filled with ions.

The necessary set of equations for the determination of the spectrum of surface waves in the presence of backing of ions and in the limit $\lambda > l > x_0$ has the form

$$\rho g\zeta + \rho\frac{\partial\varphi}{\partial t} - \alpha\left(\frac{\partial^2\zeta}{\partial y^2} + \frac{\partial^2\zeta}{\partial z^2}\right) - nF(x_0 + \zeta - \zeta_1) = 0, \tag{4}$$

$$m_\pm\frac{d^2\zeta_1}{dt^2} - eE_0 + F(x_0 + \zeta - \zeta_1) = 0, \tag{5}$$

where ρ is the density of liquid helium, g the acceleration due to gravity, φ the hydrodynamic velocity potential, α the coefficient of surface tension, n the surface density of ions, F(x) is from (1), E_0 is the applied external field, ζ the local deviation of the free surface from the equilibrium value $x = 0$ and ζ_1 is the departure of the ions from the equilibrium position x_0.

Equation (4) is the boundary condition for the hydrodynamic problem. This condition differs from the usual ([3], p. 291) by the term nF, which takes into account, in the mean, the effect of the ions on the free surface. The introduction of the averaged pressure nF,

$n \sim l^{-2}$ is possible because of the inequality $\lambda \gg l$, which is assumed to be satisfied.

The oscillations of the free surface in turn lead to the appearance of time-periodic forces acting on the ions in a direction normal to the free surface. Furthermore, there is Coulomb interaction between the ions. Therefore the motion in the general case is described by an infinite set of interlocking equations. However, in view of the inequality $l \gg x_0$, we can neglect the Coulomb interaction, limiting ourselves only to account of the interaction of the ions with the surface. Here the equations of motion of the ions are uncoupled and take the form of the equation of motion for the individual ion (5).

Expanding the force $F(x_0 + \zeta - \zeta_1)$ from (4) and (5) in a series in the small departures $\zeta - \zeta_1$, expressing ζ_1 in terms of ζ by means of (5), substituting this expression for ζ_1 in (4) and then proceeding in the normal fashion, we find the dispersion law for the surface waves:

$$\omega^2 = (g+\gamma)k + (\alpha/\rho)k^3, \quad \gamma = nF(x_0)\frac{2}{\rho x_0}\left(1+\frac{\omega_0{}^2}{\omega_0{}^2-\omega^2}\right); \tag{6}$$

here ω_0 is from (3), and k is the wave vector. The right side of this dispersion law differs from the usual law in the term γk. This term becomes large, in resonance fashion, in the vicinity of $\omega \sim \omega_0$.

The discussions above are valid only in the case of a constant mean concentration of ions along the liquid-vapor surface. Actually, such a situation is unrealistic in view of the fact that the ions are of like charge and can be maintained at a finite distance from one another only with the help of outside external forces (the walls of the vessel and so on), which usually destroys the uniformity of distribution of the charges. This assertion, however, does not apply to stationary current states, in the case of which the constancy of the carrier density along the direction of the lines of the current is one of the necessary conditions for stationarity of the current. Therefore, by introducing additional electrodes in the helium bath, which force the surface ions to move along the surface with some constant velocity, much less than the velocity of the surface waves, and by exciting the ions in the direction of the current state thus created, we obtain the possibility of studying the surface waves of the type described under conditions that are close to those used in the calculation.

3. We shall now describe another scheme which makes it possible to observe resonance phenomena at the frequency ω_0. With this purpose, we consider an oscillatory LC circuit, the principal capacitor C_1 of which consists of two parallel plates placed in the helium tank so that the liquid-vapor interface lies between them. In addition to the principal capacitance C_1, the circuit also contains an auxiliary capacitor C_2, connected in series with C_1 so that one can apply a constant voltage to C_1 without short-circuiting it by the inductance L. Because of the presence of helium temperatures, all the elements of the circuit can be made superconducting in order to give the highest Q to the circuit in the absence of helium ions in the liquid.

Now let ions be introduced into the liquid helium. By means of the constant field intensity E_0 applied at the plates of C_1, these ions are forced to the surface of the liquid, remaining at depth x_0 from (2). The plane x_0 should be about midway between the plates of the capacitor C_1 and should be sufficiently far from them that ions carrying out small oscillations near x_0 do not touch the plates of C_1. Under similar conditions in the excitation of the LC circuit, the ions in the gap of the capacitor C_1 will undergo forced oscillations, absorbing the energy of the oscillations and decreasing the Q of the circuit.

If we assume that in the presence of the ions the Q of the circuit nevertheless remains high, then the ionic current between the plates of the capacitor C_1 can be written in the form[4]

$$j = \frac{Ne^2}{m_\pm}\frac{\omega\exp[i(\pi/2-\phi)]}{[(\omega_0{}^2-\omega^2)^2+(\omega/\tau)^2]^{1/2}}\mathscr{E}_0 e^{i\omega t}, \quad \operatorname{tg}\phi = \frac{\omega/\tau}{\omega_0{}^2-\omega^2}; \tag{7}$$

N is the number of ions reaching a unit surface of the capacitor, ω_0 is from (3), ω is the eigenfrequency of the LC circuit, $\mathscr{E}_0 \exp(i\omega t)$ is the alternating part of the electric field at the plates of C_1.

We now define the Q of the circuit as the ratio $Q = W_C/(2\pi q/\omega)$. Here $W_C = \mathscr{E}_0^2 d/8\pi$ is the initial energy of the circuit, carried to a unit surface of the plate capacitor, d is the distance between the plates of the capacitor, q the power loss associated with the ionic current, computed per unit surface area, $q = \operatorname{Re}(j\mathscr{E})$, where j is taken from (7) and $\mathscr{E} = \mathscr{E}_0 \times \exp(i\omega t)$. As a result, we obtain the following expression for Q:

$$Q = \frac{d}{16\pi^2}\frac{m_\pm}{Ne^2}\frac{[(\omega_0{}^2-\omega^2)^2+\omega^2/\tau^2]^{1/2}}{\cos(\pi/2-\phi)}. \tag{8}$$

It is seen from (8) that the Q of the circuit has a sharp minimum for $\omega \sim \omega_0$.

MOBILITY NEAR THE LIQUID-VAPOR INTERFACE

The nearness of the free surface influences the mobility of the ions along the surface. This effect is especially marked in the low temperature region, where the mobility of the ions begins to experience the free interface at distances much greater than the dimensions of the ions themselves.

1. Volume mobility at low temperatures. For calculation of the surface mobility of ions and a comparison of it with the corresponding volume mobility, we need some information on the mechanism of retardation of the ions in the bulk of the liquid at low temperatures. All the necessary considerations on the mechanism of mobility at low temperatures is contained in [2, 5-8].

In the low temperature region T < 0.5 °K, the rotons in the liquid helium are practically frozen out and the wave length of the characteristic phonons begin to exceed the dimensions of the ± ions. Under these conditions, the value of the drift velocity $U_D^\pm$ is determined from the relation[2]

$$eE = \frac{\hbar}{6\pi^2}U_D^\pm\int_0^\infty\frac{\sigma^\pm(k)\,k^3dk}{\exp(c\hbar k/k_BT)-1}, \tag{9}$$

where E is the applied field moving the ion along the liquid-vapor surface, c the sound velocity, $\sigma^\pm(k)$ the total transport scattering cross section of long-wave phonons by a single ± ion. (In the long-wave limit, the scattering is spherically symmetric, and therefore the difference between the simple cross section and the transport cross section tends to zero.) Thus, the calculation of $U_D^{\pm}$ reduces to the determination of $\sigma^{\pm}$.

The value of $\sigma^{(-)}$ in the temperature range $T < 0.4\,^\circ K$ that is of interest to us can be written down in the form[5, 9]

$$\sigma^{(-)} = \sigma_s^{(-)} + \sigma_a^{(-)},$$
$$\sigma_s^{(-)} = \frac{4\pi a^2}{(\omega_s^2/\omega^2 - 1)^2 + \delta^2}, \quad \sigma_a^{(-)} = \sigma_s^{(-)}\frac{\delta}{ka}, \tag{10}$$

where

$$\omega_s = 8\alpha/\rho a^3, \quad \delta = ka.$$

ω_s is the characteristic frequency of a bubble, α the coefficient of surface tension, ρ the density of liquid helium, δ the damping constant. According to (10), the characteristic frequency ω_s for $\alpha = 0.36$, $a = 2 \times 10^{-7}$ cm and $\rho = 0.142$ g/cm^3 is of the order $\omega_s \approx 5 \times 10^{10}$ sec^{-1}. On the other hand, the characteristic thermal frequency ω_T for $T \sim 0.4\,^\circ K$ is also of the order $\omega_T \sim 5 \times 10^{10}$ sec^{-1}. This means that an important part of the thermal phonons for $T \sim 0.4\,^\circ K$ will be scattered by the bubble in resonant fashion. In addition to the s resonance, the bubble also possesses higher resonances, the closest frequency of which ω_d, according to [7, 8], is equal to $\omega_d \approx \pi\omega_s/\sqrt{2}$. However, according to estimate, for $T < 0.4\,^\circ K$, the principal contribution to the mobility is given by the s-resonance.

We proceed to the case of positive ions. Here we have the assertion[6] that the scattering cross section of long-wave phonons by a positive ion has the form $\sigma^{(+)} \backsim \omega^4$, i.e., the positive ion is equivalent to a solid sphere. Actually, there is a condensation of the liquid helium around the positive charge. This falls off with distance by a power law. Systematic account of this fact shows that the problem of the scattering of long-wave phonons by a positive ion does not reduce to the problem of scattering by a solid sphere. Therefore, the question of the phonon scattering cross section by a positive ion still remains in the discussion stage.

2. Mobility of negative ions close to the free surface. Using information on the volume mobility of the ions, we can obtain definite results on the mobility of the ions close to the interface. It is natural that specific results can be obtained only for negative ions at the present time.

Let us bring a negative ion to a finite distance x_0 from the free surface of liquid helium. The expected change in the mobility here has the following origin. The total pressure at the free surface is equal to zero. This shunting action of the surface should also appear in the interior of the liquid at distances of the order of a wavelength of the characteristic phonons. Thus, if we bring the ion to a layer of depth of the order of the characteristic length of the phonons, and recall that the retardation of the ion takes place because of the resonance scattering of the phonons by the characteristic oscillations of the bubble,[1] we can then expect that the shunting action of the surface, which decreases the pressure amplitude in the phonon waves, leads to a change in the value of the mobility of the negative ions in comparison with the volume. And, since the characteristic lengths of the phonons at low temperatures greatly exceeds the dimensions of the ions, the effect of the surface on the mobility of the ions begins to be seen at distances much greater than the dimensions of the ions themselves.

Let the phonon incident on the free surface have the potential

$$\varphi_0 = \frac{P_0}{i\omega\rho}e^{i(\omega t - \mathbf{kr})}.$$

Then the pressure at a given point of the liquid at depth x_0 from the free surface has the form

$$P_0(t) = P_0 2i \sin(kx_0\cos\gamma)\, e^{i\omega t}, \tag{11}$$

where γ is the angle between the normal to the surface and the direction of the wave vector of the phonon $\mathbf{k}$. For the calculation of the scattering cross section of the phonons by a negative ion close to the free surface, $\sigma(k, x_0, \gamma)$, we also need the potential of the spherical wave induced by the oscillations of the sphere located at a distance x_0 from the free surface. Taking into account the boundary condition at the liquid–vapor interface $P|_{x=0} = 0$, we write down this potential in the form

$$\varphi_1 = Be^{i\omega t}\left[\frac{e^{-ikr}}{r} - \frac{e^{-ikr_1}}{r_1}\right] \tag{12}$$
$$r_1^2 = r^2 + (2x_0)^2 - 4rx_0\cos(\widehat{\mathbf{rx}_0}).$$

On the boundary of the bubble, the pressure and the radial velocity ought to be continuous:

$$P_0(t) + P_1(t)\,|_a = P_{\text{el}}(t), \quad U_r^{\text{I}}|_a = U_r^{\text{II}}|_a; \tag{13}$$

Here $P_0(t)$ is from (11), $P_1(t) = i\omega\rho\varphi_1|_{r=a}$, $U_r^1|_a = \partial\varphi_1/\partial r|_a$, φ_1 is from (12), P_{el} is the electron pressure inside the bubble. It is necessary to add the coupling between P_{el} and the radial velocity of the bubble U_r^{II} to these equations. This coupling, as in the volume case, has the form

$$i\omega P_{\text{el}} = (8\alpha/a^2)\,U_r^{\text{II}} \tag{14}$$

and is obtained under the assumption of the instantaneous adjustment of the electron pressure to the given deformation of the bubble.

By solving the set of Eqs. (13) and (14) relative to the unknown quantities B, P_{el}, and U_r^{II}, we find them all as functions of $P_0(t)$. Using the definition of the total scattering cross section and the solution of the set (13)–(14), we find the sought-for expression $\sigma(k, x_0, \gamma)$:

$$\sigma(k, x_0, \gamma) = \sigma_s + \sigma_a,$$
$$\sigma_s = \frac{4\pi a^2\cdot 4\sin^2(kx_0\cos\gamma)}{(\omega_s^2/\omega^2 - 1)^2 + \delta_1^2}, \quad \sigma_a = \frac{\delta_1}{ka}\sigma_s, \tag{15}$$

where

$$\delta_1(k) = ka - \frac{a}{2x_0}\sin 2kx_0 + \frac{\omega_s^2}{\omega^2}\left[\frac{a^2}{4x_0^2}\sin 2kx_0 - \frac{a^2k}{2x_0}\cos 2kx_0\right],$$

ω_s is from (10), $\omega = kc$.

The cross section (15) contains the expected factor $\sin^2(kx_0\cos\gamma)$, which effectively decreases the scattering cross section for $kx_0\cos\gamma < 1$. Moreover, for $kx_0 < 1$, the damping constant δ_1 changes form, in comparison with the volume case.

The resultant cross section $\sigma(k, x_0, \gamma)$ must be substituted in the general expression for the balance of forces acting on the ion in the external field **E**, applied

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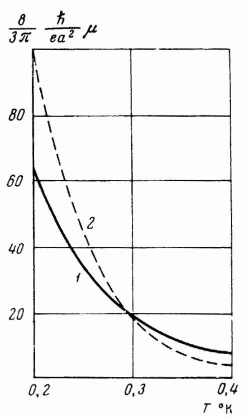
[1] It should be noted that although the resonance scattering cross section $\sigma^{(-)}$ decreases near the surface, it still remains large. Therefore, the possible contributions to the total scattering cross section, connected with the presence of surface phonons on the free surface of liquid helium are not important. These contributions cannot change the resonant parts of the scattering, since the surface oscillations take place under the condtion div **V** = 0, i.e., **P** = const.



along the liquid-vapor interface. We have

$$eE=\frac{U_D\hbar}{4\pi^3}\int_0^\infty dk\int_0^{\pi/2} d\theta\int_0^\pi d\varphi\, k^3\sin\theta\cos^2\theta\, n\,(kT)\,\sigma(k,x_0,\gamma).$$
$$\cos\gamma=-\sin\theta\sin\varphi.$$

The polar axis of the spherical system of coordinates is directed along **E**, n(kT) is the Bose distribution function, $\sigma(k, x_0, \gamma)$ is from (15), x_0 is from (2). After integration over the angles, we obtain

$$eE=\frac{U_D\hbar}{\pi^2}\int_0^\infty dk\, k^3 n(kT)\frac{4\pi a^2(1+\delta_1/ka)}{(\omega_s^2/\omega^2-1)^2+\delta_1^2}\left[\frac{1}{6}+\frac{\Gamma(^3/_2)}{4}\frac{J_{^3/_2}(2kx_0)}{(kx_0)^{^3/_2}}\right], \tag{16}$$

where $\Gamma(^3/_2)$iis the gamma function, $J_{3/2}$ is the Bessel function of order $^3/_2$.

In the limit when the characteristic wave numbers $\bar{k}$ satisfy the inequality $\bar{k}x_0 \gg 1$, the relation (16) transforms into (9). In the opposite limiting case $\bar{k}x_0 \gg 1$, which begins to be satisfied for $x_0 \approx 10^{-6}$ cm at temperatures $T \lesssim 0.5$°K, the equation of the balance of forces becomes qualitatively different from (9). For this region of temperatures, and for $x \approx 10^{-6}$ cm, the mobility of the negative ions $\mu = U_D/E$, following from (16) is written on drawing 1 (curve 2). For comparison, the graph for the volume mobility (curve 1) is plotted on this same drawing, constructed with the help of Eqs. (9) and (10). As is seen from the drawing, curves 1 and 2 differ significantly from one another.

POSSIBILITIES FOR THE EXISTENCE OF SURFACE ELECTRONS

In the previous sections, we have discussed the various properties of the helium ions near the liquid–vapor interface. Here, the helium ions are moved to the interface from the liquid side. No less interesting is the variant in which the charged particles, more precisely, electrons, move to the liquid–vapor interface from the side of the gaseous phase. Several properties of the electrons under similar conditions are discussed in the present section.

1. We again return to the expression for the force F from (1). If the difference $\epsilon - 1$ in this formula is understood in the sense $\epsilon_1 - \epsilon$, where ϵ_1 is the dielectric constant of the medium in which the charge is located, then it is evident that for $\epsilon_1 > \epsilon_2$ the charge is repelled from the interface, and for $\epsilon_1 < \epsilon_2$ it is attracted to it. Thus, if the charge is located above the surface of liquid helium (the situation $\epsilon_1 < \epsilon_2$), then it will be attracted to the interface. This attraction, after the transition through the interface, changes to repulsion from the interface and therefore, in the field of force (1) the charged particles, wherever they would be located at the initial instant (above the boundary or below it), should pass into the depth of the liquid helium. However, this assertion is valid in full measure only for positive charges. In the case of electrons, the situation is somewhat different. The difference is that for free electrons the boundary of the liquid helium is a potential barrier of exchange origin of height 1.4 eV.[10] Therefore, the electrons which have energies < 1.4 eV are also attracted to the free surface of the liquid helium by the forces of (1); they cannot pass through the boundary into the depth of the liquid helium and should be localized on its surface. Such electrons, brought up to the liquid helium from the side of the gaseous phase and localized on the liquid–vapor boundary, we shall call surface electrons. This special term enables us to avoid the confusion between surface electrons and surface ions (positive and negative) brought to the liquid–vapor interface from the side of the liquid phase. It is evident that the properties of the surface electrons will be intermediate between the properties of the negative ions in purely liquid or purely gaseous media.

2. For clarification of the degree of localization of the surface electrons, we consider the solution of the Schrödinger equation for an electron in the corresponding potential well. The potential well consists of the potential force (1) and the potential barrier, located at x = 0. The gaseous phase corresponds to x > 0. We have

$$\psi(x,y,z)=f(x)\exp[i(k_y y+k_z z)]$$
$$\frac{d^2f}{dx^2}+\frac{2m}{\hbar^2}\left(E-\frac{\hbar^2k^2}{2m}+\frac{\beta}{x}\right)f=0,$$
$$\beta=\frac{e^2}{4}\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1(\varepsilon_1+\varepsilon_2)},\quad k^2=k_y^2+k_z^2,$$
$$f(x)\,|_{x\to+\infty}\to 0,\quad f(x)\,|_{x=0}=0. \tag{17}$$

In writing down (17), it is assumed that the surface electron weakly deformed the free surface of the liquid helium and therefore, these deformations can be neglected in the zeroth approximation.

In Coulomb units (m = mass, $\hbar^2/m\beta$ = length, $m\beta^2/\hbar^2$ = energy), Eq. (17) is rewritten thus:

$$\frac{d^2f}{dx^2}+2\left(E-k^2+\frac{1}{x}\right)f=0.$$

After the single substitution $n=[-2(E-k^2)]^{-1/2}$ and $X = 2x/n$, we obtain

$$f''+[-{}^1/_4+n/X]f=0\quad f''\equiv d^2f/dX^2. \tag{17a}$$

The solution of (17a) which satisfies the boundary conditions (17) is sought in the form

$$f(X)=X^l e^{-X/2}w(X). \tag{18}$$

As a result, we get the following equation for w(X):

$$Xw''+(2l-X)w'+w[n-l+l(l-1)/X]=0.$$

This equation reduces to the hypergeometric one if we set l = 1. The solution of the latter equation with l = 1, which is finite for X = 0, and which increases no more rapidly than a polynomial of finite degree as X = 0, has the form ([11], p. 158)

$$w(X) = \Phi(-n+1, 2, X), \quad n = 1, 2, \ldots, \tag{19}$$

Φ is the confluent hypergeometric function. The relations (18) and (19) solve the given problem.

The energy levels of the surface electron in dimensional units are determined by the relation

$$E - \frac{\hbar^2 k^2}{2m} = -\frac{m\beta^2}{2\hbar^2 n^2}, \quad n = 1, 2, 3, \ldots. \tag{20}$$

The energy of the ground state with account of $|\epsilon_1 - \epsilon_2| = 0.06$ and $m \sim 10^{-27}$ g, is equal to $E_1 = -0.003$ eV.

The wave function of the n-th state is determined by Eqs. (18), (19). In the ground state, n = 1, this function has the form

$$f(x) = \text{const} \cdot x e^{-\gamma x}, \quad \gamma = m\beta / \hbar^2.$$

Its maximum is located at the distance $x_0 = \gamma^{-1} \approx 10^{-6}$ cm from the free surface. This distance is much greater than atomic distances and therefore use of the macroscopic expression for the force (1) in the construction of the potential energy of the surface electron can be regarded as sufficiently correct.

Let us estimate the value of the depth of the free surface under the electron. For this purpose, we shall assume that the total force acting from the side of the surface electron on the free surface is equal to F from (1), where x as a coordinate means the average distance of the electron from the surface, i.e., $x_0 \approx 10^{-6}$ cm. This force should be compensated by the forces of surface tension, which arise in the deformation of the surface. It is easy to show that the area of the depressed surface S is approximately equal to $S \approx 4\pi x_0 \xi$, where x_0 is the radius of the depression, ξ the maximum deviation of the free surface from its equilibrium position in the absence of the electron. Setting F = PS, where $P = 2\alpha/x_0$, and α is the coefficient of surface tension, we get the equation for the determination of ξ. As a result,

$$\xi = \frac{1}{8\pi\alpha}\left(\frac{e}{2x_0}\right)^2 \frac{\varepsilon - 1}{\varepsilon(\varepsilon + 1)} \lesssim 10^{-8}\,\text{cm}.$$

Thus, $\xi \ll x_0$ and consequently the assumption used above as to the smallness of the deformation of the free surface is actually valid in practice. Although the deformation of the free surface is seen to be small, which fact is connected essentially with the distribution of the electron pressure on the surface over a large area, it should be noted that even such bends are sufficient to cause the effective mass of the surface electron to be of the order of the mass not of the free electron, but of the helium atom.

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