RESONANCE PHENOMENA DURING FORCED OSCILLATIONS OF THE INTENSITY OF LASER RADIATION

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Oscillation of the intensity of laser radiation during sinusoidal variation of the resonator Q factor is considered. The dependence of the amplitude of the intensity oscillations on the pump power, frequency, and Q-switching amplitude is derived.

THE problem of forced oscillations of laser intensity arises in connection with the analysis of the behavior of a laser system when its parameters vary with time. Since the laser radiation intensity behaves like an oscillator characterized by a certain frequency and damping coefficient, the values of which are determined by specific parameters of the laser[1,2], temporal variations of these parameters are expected to lead to resonant phenomena.

The process of radiation-intensity oscillation upon modulation of the resonator losses was analyzed by Ratner[3]. Under the conditions of very low modulation amplitude, he reduced the problem to a solution of an ordinary linear differential equation with a driving force, and in the analysis of the behavior of the system at large amplitudes he confined himself to stating several qualitative considerations.

As will be shown below, the process of temporal variation of the intensity of laser radiation when the resonator Q varies in time is described by an oscillatory equation with a specific nonlinearity, which leads to a large number of singularities inherent in non-autonomous systems with asymmetrical linear characteristics[4,5].

Let us consider the case of single-mode generation, assuming the electromagnetic field to be spatially homogeneous. To describe the temporal variation of the number of photons in the resonator, we use the kinetic equations

$$\dot{n}=\frac{n_0-n}{\tau_p}-sDNn, \tag{1a}$$

$$\dot{N}=-\gamma(t)N+DNnL. \tag{1b}$$

Here n is the inverted population per unit length of the active medium, n_0 the inverted population produced in the system by the pump power W in the absence of stimulated emission, D a coefficient proportional to the Einstein coefficient, and L the resonator length. The parameter s assumes different values, depending on whether the generation follows a four-level or a three-level scheme. In the former case s = 1 and in the latter s = 2. The quantity τ_p is determined by the formula

$$\tau_p=\tau/(1+\varkappa W\tau), \tag{2}$$

where κW is the probability of the transition of the active center into the excited state under the influence of the pump, and τ is the time of spontaneous decay. The coefficient $\gamma(t)$ of the loss of photons in the resonator will henceforth be written in the form

$$\gamma(t)=\gamma_0+\Delta\gamma(t), \tag{3}$$

where γ_0 is the time-independent loss.

This system was considered in the self-excitation regime, and therefore Eq. (1b) does not contain a term describing the increment of the number of photons due to the luminescent decay of the upper level.

Introducing in place of N the function

$$N=\frac{Dn_0L-\gamma_0}{s\gamma_0D\tau_p}e^{x-1}=\frac{\alpha-1}{sD\tau_p}e^{x-1}, \tag{4}$$

where $\alpha = Dn_0L/\gamma_0$, and eliminating the variable n from (1), we obtain

$$\ddot{x}+\dot{x}\left(\frac{1}{\tau_p}+\frac{\alpha-1}{\tau_p}e^{x-1}\right)+\frac{\alpha-1}{\tau_p}\gamma_0(e^{x-1}-1)+\frac{\alpha-1}{\tau_p}\Delta\gamma(t)(e^{x-1}-1)=-\Delta\dot{\gamma}(t)-\frac{\alpha}{\tau_p}\Delta\gamma(t). \tag{5}$$

We shall assume that the time variation of the loss is harmonic with an amplitude ξ, i.e.,

$$\Delta\gamma(t)=\xi\sin\omega t. \tag{6}$$

Expanding the function e^{x-1} at a series and confining ourselves to terms of order x^3, which enables us to consider values of x in the range $|x-1|\lesssim 2-3$, we obtain a second-order nonlinear differential equation with an asymmetrical nonlinear characteristic

$$\ddot{x}+\frac{\dot{x}}{3\tau_p}\left[\alpha+2+\frac{\alpha-1}{2}(3x+x^3)\right]+\frac{\omega_0^2}{6}(3x+x^3)+\frac{\xi}{6\gamma_0}\omega_0^2(3x+x^3)\sin\omega t=\frac{2}{3}\omega_0^2-\xi\omega\cos\omega t-\frac{\xi}{3\tau_p}(\alpha+2)\sin\omega t. \tag{7}$$

Here

$$\omega_0^2=(\alpha-1)\gamma_0/\tau_p. \tag{8}$$

We seek the steady-state periodic oscillation regime, which in first approximation can be represented in the form

$$x=c+r\sin(\omega t+\delta)=c+Be^{i\omega t}+B^*e^{-i\omega t}. \tag{9}$$

Strictly speaking, we should seek a solution containing not only harmonics with frequency ω, but also ultraharmonic and subharmonics with frequencies 2ω, 3ω, $\omega/2$, and $\omega/3$, the amplitudes of which may turn out to be appreciable when ω approaches $\omega_0/2$, $\omega_0/3$, $2\omega_0$ or $3\omega_0$[5]. If we confine ourselves henceforth in the analysis of the results to the frequencies $\omega \sim \omega_0$, then the chosen approximation is quite satisfactory.

Recognizing that the parameters of lasers in which the active medium is ruby or neodymium glass lie in the range

$$\gamma_0 \sim 10^7 - 10^9 \text{ sec}^{-1} \quad \tau_p \sim 10^{-3} - 10^{-4} \text{ sec}$$
$$\omega_0 \sim 10^5 - 10^6 \text{ rad/sec} \quad \alpha \sim 1 - 10^2, \tag{10}$$

and in addition $\xi/\gamma_0 \ll 1$ and $(\omega\tau_p)^{-1} \ll 1$, we obtain after discarding the small terms

$$\frac{\xi^2\omega^2}{\omega_0^4} = r^2\left[\left(a - \frac{\omega^2}{\omega_0^2}\right)^2 + \frac{\omega^2}{9\tau_p^2\omega_0^4}b^2\right] \tag{11}$$

$$a = \frac{5c^3+3c+4}{12c}, \quad b = \frac{\alpha-1}{4}(8+3c+c^3)+3;$$
$$r^2 = \frac{2}{3}\,\frac{4-c^3-3c}{c}. \tag{12}$$

Here $r^2 = 4BB^*$. Starting from these equations, let us trace the variation of r as a function of the change of the Q-switching amplitude ξ, the modulation frequency ω, and the pump power level W.

From (11) we get ω/ω_0:

$$\frac{\omega}{\omega_0} = \left(\frac{\xi^2}{4\omega_0^2r^2} - \frac{1}{36\tau_p^2\omega_0^2}b^2 + a\right)^{1/2} \pm \left(\frac{\xi^2}{4\omega_0^2r^2} - \frac{1}{36\tau_p^2\omega_0^2}b^2\right)^{1/2}. \tag{13}$$

Figure 1 shows the resonance plots of r against ω/ω_0 for $\alpha = 11$, $\tau_p = 10^{-3}$ sec, and $\gamma_0 = 10^8$ sec^{-1}. The dashed line shows the curve describing the location of the resonant extrema. The form of this curve is given approximately by the formulas

$$r_{extr} \approx 3\xi\tau_p/b, \tag{14}$$

$$\omega_{extr}/\omega_0 = \sqrt{a}. \tag{15}$$

It is seen from Fig. 1 that when $\xi \lesssim 10^3$ sec^{-1} the curves become analogous to the resonance curves obtained by solving the linear equations. At large values of ξ, regions where certain periodic solutions become unstable appear in the amplitude-frequency curves. The instability limits are determined by the points at which the tangents to the curves are vertical.

On the amplitude-frequency characteristic 3, corresponding to $\xi = 2 \times 10^4$ sec^{-1}, there are observed two instability regions, AB and CD. This means that even in the case of an adiabatic slow growth of ω, the amplitude r increases continuously to the point A ($\omega/\omega_0 = 2.972$), and experiences a jump with further increase ω, to the value r defined by curve BC. This "increase" is far from instantaneous. The steady-state periodic regime with the new value of r will be preceded by a certain transient process, during which the radiation intensity can change in a non-periodic manner. A similar situation occurs also in the region of the point C. Here r changes jumpwise from a value 2.67 to 0.03 (naturally, after the attenuation of the corresponding transient process).

It must be emphasized that even such an insignificant amplitude of the Q modulation as $\xi = 3 \times 10^{-4}\gamma_0$ produces, at the corresponding frequencies, abrupt changes in the value of N. In the case under consideration $r_{extr} = 2.67$, the ratio N_{max}/N_{min} turns out to be ~ 200, the width of the spike ΔT at half the intensity at $r > 1$ is determined by the formula

$$\Delta T = \frac{2}{\omega}\arccos\left(1 - \frac{\ln 2}{r}\right) \tag{16}$$

and reaches here a value 10^{-6} sec, while the phase δ which enters in (10) is $\delta \sim 0$, whereas far from resonance $\delta \sim \pi/2$.

The presence of strong resonant phenomena is typical precisely of solid-state lasers. Here, as a rule, the quantity α/τ_p, which determines the dissipative processes of the system, turns out to be much smaller than ω_0, which determines the region of resonant frequencies. In gas lasers, on the other hand, these quantities, are approximately the same order and the resonant phenomena will not be so clearly pronounced here; in addition, in this case the instability regions should also vanish.

Let us consider now the dependence of r on ξ, shown in Fig. 2. At frequencies $\omega/\omega_0 \leq 0.95$, this dependence turns out to be monotonic, and when $\omega/\omega_0 > 0.95$ a more complicated picture is observed. The amplitude characteristics must contain here the instability region, and for the values of $0.95 < \omega/\omega_0 < 1.00$ there are two such regions. The presented curves demonstrate that there can exist several regimes of system behavior, some of which are stable and some are not.

To study the dependence of r on the pump power at fixed ξ and ω, we find from Eqs. (11) and (12) that

$$\alpha - 1 = \frac{12c\tau_p\omega}{\gamma_0(5c^3+3c+4)}\left\{\omega \pm \left[\frac{\xi^2}{r^2} - \left(\frac{1}{\tau_p} + \frac{\omega^2}{\gamma_0}\,\frac{c^4+3c^2+8c}{5c^3+3c+4}\right)^2\right]^{1/2}\right\}. \tag{17}$$

The obtained equality is best analyzed for the case of rare-earth lasers, since here $\alpha - 1 = W/W_{thr} - 1$, where W_{thr} is the threshold pump power, and τ_p practically coincides with τ.

Figure 3 shows plots of r against W for the values $\gamma_0 = 10^8$ sec^{-1}, $\tau_p = 10^{-3}$ sec, $\omega = 10^5$ rad/sec, and two

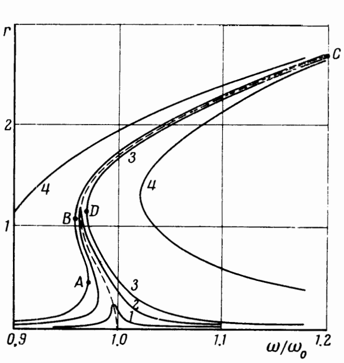

FIG. 1. Dependence of r on the frequency ratio ω/ω_0 for different values of the Q-switching amplitude ξ: 1–$\xi = 2 \times 10^3$ sec^{-1}, 2–$\xi = 10^{-4}$ sec^{-1}, 3–$\xi = 2 \times 10^4$ sec^{-1}, 4–$\xi = 10^5$ sec^{-1}.

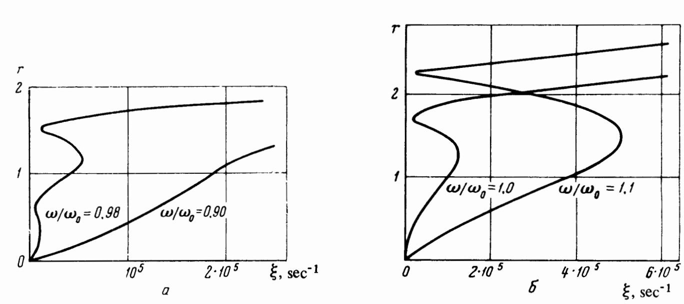

FIG. 2. Dependence of r on the Q-switching amplitude ξ at different frequencies ω/ω_0.

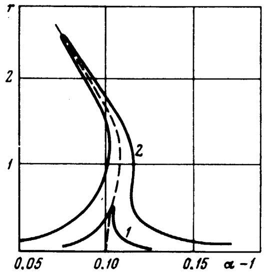

FIG. 3. Plot of r against $\alpha-1$ at $\xi = 5 \times 10^2$ sec^{-1} (curve 1) and $\xi = 2 \times 10^3$ sec^{-1} (curve 2) in the case $\omega = 10^5$ rad/sec.

values of ξ ($\xi = 5 \times 10^2$ sec^{-1} and $\xi = 2 \times 10^3$ sec^{-1}. At $\xi = 2 \times 10^3$ the curve has two regions where the periodic solutions become unstable.

The location of the resonant curve on the frequency scale can be roughly estimated by starting from the formula

$$\alpha - 1 = \omega^2 \tau_p / \gamma_0, \tag{18}$$

which is obtained for the maxima of the curves as $r \to 0$. It follows therefore that at small excesses of the threshold value of the pump power the violation of the stable generation regime is due to the low frequencies $\omega \le \sqrt{\gamma_0/\tau_p}$. In addition, under these conditions the damping time of the transients, if it is assumed that it is of the order of τ_p, is also the largest. The resonant effects at frequencies $\omega \sim 10^6$–10^7 rad/sec for values $\gamma_0 = 10^8$ sec^{-1} and $\tau_p \sim 10^{-3}$–10^{-4} sec become manifest as values $\alpha \sim 10$–10^2, which is rarely attainable in experiment.

Summarizing the foregoing, we note that periodic variation of the resonator Q in time leads to a periodic variation of the intensity of the laser emission. In the region $\omega \sim \omega_0$, even slight Q-switching amplitudes $\sim 10^{-4}\gamma_0$–$10^{-5}\gamma_0$ are capable of causing abrupt changes in intensity. The solution shows that at the same laser-system parameters the oscillations of the intensity may correspond to different periodic regimes, some of which are unstable. The presence of the instability region makes it impossible, in principle, to tune adiabatically the amplitude of the oscillations by slowly varying the parameters of the system.

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