SOME FEATURES OF KINETIC PHENOMENA IN METALS UNDER MAGNETIC BREAKDOWN CONDITIONS

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The influence of magnetic breakdown on the electron kinetics of metals is considered for the case when the spread of the energy levels is much smaller than the characteristic distance between them. A general method is proposed for calculating arbitrary kinetic coefficients; the method yields closed analytic expressions for the coefficients. It is shown that in the case under consideration multiple coherent electron scattering by magnetic breakdown regions in momentum space is appreciable not only for the oscillating part of the kinetic coefficients (with respect to magnetic field) but also for the smooth part. General properties of resonance and galvanomagnetic phenomena are investigated, and giant magnetoresistance oscillations are predicted for some types of Fermi surfaces. It is shown that the "magnetic breakdown" energy spectrum, which in a certain sense is "accidental," is unstable with respect to weak external fields that induce stochastic motion of electrons, even when the fields possess a regular structure (e.g., the field of an ultrasound wave). "Giant" nonlinear interaction effects between an ultrasound wave and direct electric current and other nonlinear effects are predicted.

1. INTRODUCTION. DYNAMIC COHERENT EFFECTS DUE TO MAGNETIC BREAKDOWN

THE magnetic breakdown phenomenon[1,2], namely interband tunnel transitions of conduction electrons in a magnetic field **H**, changes significantly many macroscopic properties of metals at intensities $H \sim 10^4-10^6$ Oe. In this paper we shall investigate the influence of magnetic breakdown on kinetic phenomena in metals. A direct solution of the problem and an investigation of different macroscopic effects will be carried out in Secs. 2–4. In this section we formulate those features of electrodynamics which are due to magnetic breakdown and are of importance in what follows.

Under breakdown conditions, the quasiclassical parameter $\kappa = e\hbar H/cb_0^2$ is always small (b_0 is the characteristic dimension of the unit cell of the reciprocal lattice, e is the electron charge, and c is the velocity of light). This means that interband transitions are significant only in small regions of p-space, where the distance between the classical trajectories $\epsilon_{1,2}(p) = E$, $p_z = \text{const}$ (E—electron energy, p_z—projection of its momentum on the direction $\mathbf{H} = \{0, 0, H\}$, $\epsilon_{1,2}(p)$—dispersion laws of the energy bands 1 and 2 taking part in the breakdown) become equal to $\lesssim \sqrt{\kappa}\, b_0$. (Certain typical configurations of the approaching trajectories of different bands are shown in Figs. 1–3, the regions of interband transitions are schematically noted on the figures by dashed lines, while the arrows denote the directions of motion of the electron, and the indices ± are explained below.) The well known[3] quasiclassical description of electron dynamics holds everywhere outside the breakdown regions.

The one-electron Hamiltonian $\hat{\mathscr{H}}$ with allowance for the interband transitions can be written in the form

$$\hat{\mathscr{H}} = \varepsilon_r(\hat{p})\,\delta_{rr'} + \sum_\alpha \hat{\mathscr{H}}_\alpha, \quad \hat{p} = \hat{P} - (e/c)\,\hat{A}. \qquad (1)$$

Here $\epsilon_r(\hat{p})$ is the Hamiltonian in the absence of breakdown[3]; **p** and **P** are the kinematic and generalized momenta; **A** is the vector potential, which we choose in the form $\mathbf{A} = \{-Hy, 0, 0\}$; r is the number of the band; the index α numbers the breakdown regions, and the summation in (1) is over all possible α; $\hat{\mathscr{H}}_\alpha$ is a local but not small perturbation, which acts in the α-th breakdown region and causes a unique quantum scattering of the electron, accompanied by a change of the number of the band[1]). This scattering process is de-

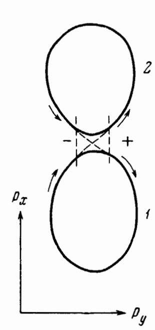

FIG. 1

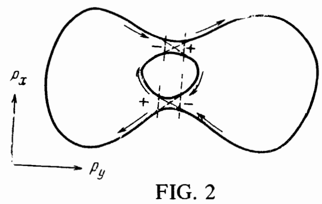

FIG. 2

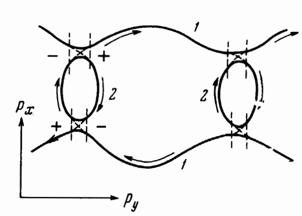

FIG. 3

1)The form of the operator $\hat{\mathscr{H}}_\alpha$ was found in the author's paper [4] for arbitrary values of **H** and for an arbitrary dispersion law.

scribed by a unitary matrix $\hat{\tau}_\alpha$[4], which in the stationary case connects the quasiclassical functions $\Psi^{(-)}_{r,\alpha}$ and $\Psi^{(+)}_{r,\alpha}$, which represent the eigenvector of the Hamiltonian $\mathscr{H}$ in the "quasiclassical" p-space regions adjacent to different sides to the "α-th" breakdown section (see Figs. 1—3). The rigorous definition of $\hat{\tau}_\alpha$ and $\Psi^{(\pm)}_{r,\alpha}$ is[4]:

$$\Psi^{(\pm)}_{r,\alpha} = c^{(\pm)}_{r,\alpha} \exp\left\{\frac{i}{\sigma}(P_{x0}p_y) - \int\limits_{p_\alpha}^{p_y} p^{(\pm)}_{x,r}(p_y')\,dp_y'\right\} \times \left(\frac{c}{eH}\right)^{1/2} \left|\frac{\partial \varepsilon_r}{\partial p_x}\right|^{-1/2} \delta_{P_x P_{x0}} \delta_{p_z p_{z0}}, \quad (2)$$

$$\varepsilon_r(p^{(\pm)}_{x,r}(p_y), p_y, p_{z0}) = E, \quad \sigma = e\hbar H/c;$$

$$c^{(+)}_{r,\alpha} = \sum_{r'=1}^{2} \tau^{(\alpha)}_{rr'} c^{(-)}_{r',\alpha}. \quad (2a)$$

The indices − and + pertain here and throughout to the sections of the classical trajectories which enter the breakdown region and leave it, respectively; p_α is the y-component of the center of the "magnetic breakdown" region; P_{x0} and p_{z0} are the values of the conserved quantities P_x and p_z; the constant coefficients $c^{(-)}_{r,\alpha}$ play the role of the amplitudes of the incident wave, and $c^{(+)}_{r,\alpha}$ are the scattering amplitudes. According t to[4], the matrices $\hat{\tau}^{(\alpha)}$ are given by

$$\hat{\tau}^{(\alpha)} = \begin{pmatrix} \tau_\alpha e^{i\theta_\alpha} & -\rho_\alpha \\ \rho_\alpha & \tau_\alpha e^{-i\theta_\alpha} \end{pmatrix}, \quad \tau_\alpha^2 + \rho_\alpha^2 = 1,$$

$$\theta_\alpha = \frac{\pi}{4} + \frac{\gamma_\alpha}{2} - \frac{\gamma_\alpha}{2}\ln\frac{\gamma_\alpha}{2} + \arg\Gamma\left(\frac{i\gamma_\alpha}{2}\right),$$

$$\rho_\alpha = \exp\left(-\frac{\pi\gamma_\alpha}{2}\right), \quad \gamma_\alpha = \frac{H(E, p_{z0})}{H}. \quad (3)$$

Here the parameter H_α[4], which is proportional to the square of the interband energy gap in the region α, is a smooth function of E with a characteristic variation interval $\sim\epsilon_0$, which is the width of the energy band. The quantity $\rho^2_\alpha = W_\alpha$, which determines the probability of the transition with change of the number of the band, is called the probability of the magnetic breakdown.

The wave eigenfunction Ψ of the Hamiltonian (1), if we neglect the dimensions of the breakdown region, can be represented in the form of the following sum:

$$\Psi = \sum_{r,\alpha} c^{(+)}_{r,\alpha} \exp\{iP_{x0}p_y/\sigma\}\psi_{r,\alpha}(p_y)\delta_{P_x P_{x0}}\delta_{p_z p_{z0}}. \quad (4)$$

The summation in (4) extends over r = 1, 2 and over all α; $c^{(+)}_{r,\alpha}$ are defined by formulas (2); $\psi_{r,\alpha}$ is the quasiclassical stationary wave function, which is constructed in terms of the classical motion over the section of the trajectory $\epsilon_r(p) = E$, $p_z = p_{z0}$, which emerges from the region of the magnetic breakdown with index α and enters into the region $\beta = \beta(r, \alpha)$ uniquely defined by specifying r and α. (See Figs. 1—3; in particular, as seen from Fig. 1, the case $\beta = \alpha$ is possible.) The function $\psi_{r,\alpha}(p_y)$ differs from zero only on the segment which is the projection of the section (r, α) on the p_y axis. The form of $\psi_{r,\alpha}$ in the vicinity of the α-th region is determined by formula (2), the behavior of $\psi_{r,\alpha}$ in other sections of p-space is determined by the requirement of continuity and by the known conditions for the "joining together" near the classical turning points. Thus, the wave function Ψ is determined uniquely by specifying the scattering amplitudes $c^{(+)}_{r,\alpha}$.

The connection between $c^{(+)}_{r,\alpha}$ can be easily obtained by combining Eqs. (2a) with the phase relations

$$\frac{c^{(-)}_{r,\beta}}{c^{(+)}_{r,\alpha}} = \exp\left\{i\left(\frac{S_{r,\alpha}}{\sigma} + \frac{m_{r,\alpha}\pi}{2}\right)\right\}, \quad S_{r,\alpha} = -\int\limits_{p_\alpha}^{p_\beta} p_{x,r}\,dp_y', \quad \beta = \beta(\alpha, r), \quad (5)$$

that follow from the general properties of the quasiclassical function $\psi_{r,\alpha}$ ($m_{r,\alpha}$ is the number of turning points in the section (r, α), $S_{r,\alpha}(E, p_{z0})$ is the integral over the contour of this section). Elimination of $c^{(-)}_{r,\alpha}$ yields for $c^{(+)}_{r,\alpha}$ a system of linear homogeneous equations in the form

$$c^{(+)}_{r,\alpha} = \sum_{r',\alpha'} U^{r,\alpha}_{r',\alpha'}(E, p_z)\,c^{(+)}_{r',\alpha'}, \quad U^{r,\alpha}_{r',\alpha'} = V^{r,\alpha}_{r',\alpha'} \exp\left\{i\left(\frac{S_{r',\alpha'}}{\sigma} + \frac{m_{r',\alpha'}\pi}{2}\right)\right\}. \quad (6)$$

The matrices $U^{r,\alpha}_{r',\alpha}$ and $V^{r,\alpha}_{r',\alpha}$ are unitary. In each row (r, α) of the matrix V there are only two nonzero elements, which coincide with the elements of the r-th row of the matrix $\tau^{(\alpha)}$. As is clear from Figs. 1—3, the indices of the columns containing these elements coincide with the indices of the sections that enter in the breakdown region α. The system of equations (6) actually describes multiple coherent scattering of an electron by all regions of the magnetic breakdown. This phenomenon is the physical gist of the problem considered here and determines many of the singularities of the energy spectrum, which play an important role in the kinetic phenomena.

We consider first closed configurations (Figs. 1, 2) with a finite number of breakdown regions. In this case the energy levels at fixed p_z form a discrete set of values of $E_n(p_z)$, determined from the condition for the solvability of the final system (6):

$$\mathrm{Det}\,\|\delta^{r,\alpha}_{r',\alpha'} - U^{r,\alpha}_{r',\alpha'}(E, p_z)\| = 0. \quad (7)$$

The determinant (7), which is a linear combination of different products of rapidly oscillating quasiperiodic functions $\exp\{iS_{r,\alpha}(E, p_z)/\sigma\}$ with periods $\Delta E_{r,\alpha} = \hbar/T_{r,\alpha} \ll \epsilon_0$; $T_{r,\alpha}$ is the total time of classical motion over the section (r, α). In the general case, since $\Delta E_{r,\alpha}$ are not commensurate, the periods of the different oscillating terms in (7) are also not commensurate, and the zeroes $E_n(p_z)$ of the determinant (7) are arranged quite irregularly, in fact randomly[2)]. The frequency of the transition $\Omega_l = (E_{n+l} - E_n)/\hbar$ is then of the order of Ω_0, the characteristic Larmor frequency ($l \sim 1$). Similarly, owing to the fact that $\partial S_{r,\alpha}/\partial p_z$ are not commensurate, the quantity Ω_l is an aperiodic function of p_z with a small characteristic variation interval $\sim\kappa b_0$.

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2)The irregularity of the spectrum takes place when $W(\gamma)$ are not too close to zero or to unity, i.e., under conditions when the coherent scattering, "destroys" the motion on any fixed trajectory by "entangling" (in accordance with (6)) all the sections (r, α). In the limiting cases $W \to 0$ and $W \to 1$, the system of equations (6) breaks up into independent subsystems corresponding to motion over definite trajectories: when $W \to 0$, these are the trajectories of the individual bands, when $W \to 1$, "cooperating" orbits appear, consisting of sections from different bands. In both limiting cases, the indicated subsystems correspond to independent sets of equidistant levels.

In the case of open one-dimensional periodic configurations (Fig. 3), α is a double index in the form (l, α), where the arbitrary integer l is the number of the unit cell of the reciprocal lattice, and the index a numbers the regions of breakdown inside the l-th unit cell. When account is taken of the equivalence of all the cells, the matrix $\hat{U}$ can be written in terms of a and l in the form

$$\hat{U} = U^{r,a}_{r',a'}(l-l'),$$
$$U^{r,a}_{r',a'}(l) = V^{r,a}_{r',a'}(l)\exp\left\{i\left(\frac{S_{r',a'}}{\sigma} + \frac{m_{r',a'}\pi}{2}\right)\right\}. \quad (8)$$

Here $\hat{U}(l) \neq 0$ only for $|l| \leq 1$; the matrix V is made up of the matrix elements τ_a in accordance with the rules indicated above. The solutions of the system (6) with matrix (8) take the form of plane waves:

$$c_{r,a}(l) = \tilde{c}_{r,a}(k)e^{ikl}, \quad k = P_{x0}b_y/\sigma, \quad \tilde{c}_{r,a}(k) = \tilde{c}_{r,a}(k+2\pi); \quad (9)$$
$$\tilde{c}_{r,a}(k) = \sum_{r',a'} \tilde{U}^{r,a}_{r',a'}(k)\tilde{c}_{r',a}(k), \quad \tilde{U}^{r,a}_{r',a'} = \tilde{V}^{r,a}_{r',a'}(k)\exp\{iS_{r',a}\sigma^{-1}\},$$
$$\tilde{V}(k) = \sum_{l=-1}^{1} \hat{V}(l)e^{ikl}. \quad (10)$$

Here b_y is the period of the reciprocal lattice along the openness direction; the connection between k and P_{x0} follows from the requirement that $\Psi(p_y) = \Psi(p_y + b_y)$ be periodic; the constant phase factors $\exp(im_{r,\alpha}\pi/2)$ are included in $\hat{V}$. The index + over $c_{r,\alpha}$ will henceforth be omitted.

The energy spectrum corresponding to the coherent states (9) and (10) depends on P_x (the index 0 will henceforth be omitted) and constitutes a discrete set of "magnetic" bands $E_n(P_x, p_z) = E_n(P_x + \sigma b_y^{-1}, p_z)$, in which the characteristic width and the characteristic interband gap are $\sim\hbar\Omega_0$. In analogy with the case of closed configuration, the dependence of E on n and p_z is random. In quantum states with given n, p_z, and P_x, the average velocity along the x axis differs from zero and equals $\partial E_n/\partial P_x$; in the general case $\partial E_n/\partial P_x$ is of the order of v_0 - the characteristic velocity of the electron in the absence of a magnetic field.

2. FORMULATION OF PROBLEM

The singularities of the "magnetic breakdown" energy spectrum noted in Sec. 1 becomes strongly manifest in kinetic effects under the condition that the characteristic lifetime of the coherent states (4) is much larger than Ω_0^{-1}. This inequality, which makes it possible to neglect the "broadening" of the energy levels and to represent the equilibrium one-electron density matrix $\hat{\rho}_0$ in the form

$$\hat{\rho}_0 = f_0(\hat{\mathcal{H}}), \quad f_0(E) \equiv [1 + \exp\{(E-\zeta)/kT\}]^{-1} \quad (11)$$

(T—temperature, ζ—Fermi energy) will henceforth be assumed satisfied.

The specific properties of the kinetic phenomena under conditions of magnetic breakdown are determined by the coherent effects considered above, and do not depend on the details of the relaxation mechanism. Therefore, to calculate the kinetic coefficients it is possible to introduce a certain single relaxation time t_0, which coincides in order of magnitude with the lifetime of the coherent states (4). In this approximation, an arbitrary kinetic coefficient χ_{AB} is determined (with allowance for (11)) by the following formula:

$$\chi_{AB} = -\,\mathrm{Sp}\int_0^\infty e^{-i\omega t - \nu_0 t}\hat{B}\hat{\dot{A}}(t)\frac{d\hat{f}_0}{d\mathcal{H}}\,dt$$
$$= -\sum_{\xi,\xi'} \frac{\langle\xi|\hat{B}|\xi'\rangle\langle\xi'|\hat{\dot{A}}|\xi\rangle}{i\omega + i(E_\xi - E_{\xi'})\hbar^{-1} + \nu_0}\frac{df_0}{dE_\xi}; \quad (12)$$
$$\nu_0 = t_0^{-1}, \quad \hat{A}(t) = \exp\left(\frac{i\mathcal{H}t}{\hbar}\right)\hat{A}\exp\left(-\frac{i\mathcal{H}t}{\hbar}\right), \quad \hat{\dot{A}} = \frac{1}{i\hbar}[\hat{A}, \mathcal{H}].$$

Here $\hat{B}$ is the operator of a physical quantity that deviates from its own equilibrium value under the influence of a perturbation proportional to $\hat{A}e^{i\omega t}$ (ω—frequency of external field); ξ denotes the complete set of quantum numbers of the stationary state $|\xi\rangle$ defined by formula (4). In writing down (12) it was assumed that $kT \gg \hbar\Omega_0$. This limitation, not being essential in what follows, simplifies the calculations. For simplicity we also assume that the classical analogs of the operators $\hat{A}$ and $\hat{B}$ depend only on the kinematic momentum $\hat{p}$.

In the ξ-representation, the matrix elements of an arbitrary physical quantity $\varphi_r(p)$ can be written with quasiclassical degree of accuracy as follows:

$$\langle\xi|\hat{\varphi}|\xi'\rangle = \delta_{p_z p_z'}\delta_{P_x P_x'}\sum_{r,i} c_{r,i}(\xi)c^*_{r,i}(\xi')\cdot$$
$$\times \int_0^{T_{r,i}} \varphi_r(p(t_{r,i}))\exp\left\{-\frac{i}{\hbar}(E_\xi - E_{\xi'})t_{r,i}\right\}dt_{r,i}; \quad (13)$$
$$\sum_{r,i}|c_{r,i}|^2 T_{r,i} = 1. \quad (14)$$

Here the index i is α or a, depending on the type of configuration (closed or open), $\xi = \{n, p_z, P_x\}$; $p(t_{r,i})$ determines the classical motion of the particle in p-space along the section (r, i), with energy E_ξ, and the origin for the time $t_{r,i}$ is chosen such that $t_{r,i} = 0$ corresponds to the "departure" of the particle from the region i; formula (14) follows from the normalization of $|\xi\rangle$ to unity. We note that the dependence of $c_{r,i}$ and E_ξ on P_x exists only in the case of open periodic configurations.

In view of the irregularity of the spectrum, noted in Sec. 1, the matrix elements $\langle\xi|\hat{\varphi}|\xi'\rangle$ which enter in formula (12) turn out to be rather complicated functions of n, n′, p_z, and P_x. Therefore an attempt to sum the series (12), using the explicit dependence of $\langle\xi|\hat{\varphi}|\xi'\rangle$ on the indicated quantum numbers, encounters considerable difficulties. In Sec. 3 we shall formulate a general approach that makes it possible to circumvent these difficulties and to obtain compact closed expressions for χ_{AB}, which take into account the influence of the coherent processes both on the oscillatory effects of the Shubnikov-de Haas type, and on the smooth part of the kinetic coefficients. An analysis of the obtained expressions will be presented later for the case of resonant and galvanomagnetic phenomena.

The essentially non-equidistant distribution of the terms $E_n(p_z)$ and $E_n(p_z, P_x)$ and the instability of the spectrum with respect to small ($\sim\kappa b_0$) changes of p_z and P_x cause the coherent macroscopic effects to turn out to be exceedingly sensitive to weak external fields, causing a change of E, p_z, and P_x. An analysis of the situation that arises in this case is given in Sec. 4. In particular, we investigate there the unique stochastic effects that arise if the perturbing field is sufficiently inhomogeneous. It must be emphasized that the stochastization of the electron dynamics, which will be considered later, is due to the random nature of the "magnetic breakdown" spectrum and can appear in weak fields with a simple regular structure (e.g., in the deformation field of a periodic utrasonic wave).

3. CALCULATION OF KINETIC COEFFICIENTS

A. Let us consider first an arbitrary closed configuration. We introduce for convenience a single index $\mu \equiv (r, \alpha)$, which assumes 2N values, where N is the number of breakdown regions in the given configuration. The main contribution to χ_{AB} (formula (12)) is made by the values $|E_n - \zeta| \sim kT \ll \epsilon_0$ (n is the number of the level). For these energies, accurate to quantities $\sim(kT/\epsilon_0)^2 \kappa^{-1} \ll 1$, we have

$$\exp\left\{\frac{iS_\mu(E, p_z)}{\sigma}\right\} \approx \exp\left\{\frac{iS_\mu(\zeta, p_z)}{\sigma}\right\} + \exp\{i\omega_n T_\mu(\zeta, p_z)\}, \quad E_n = \zeta + \hbar\omega_n; \tag{15}$$

here S_μ is determined by formula (5), and T_μ is the total time of motion over the section μ. With the aid of (15), the system (6) is written as follows:

$$c_\mu(n) = \sum_{\mu'=1}^{2N} U_{\mu\mu'}(\zeta)\exp\{i\omega_n T_{\mu'}\} c_{\mu'}(n), \tag{16}$$

$$\hat{U}(\zeta) = V_{\mu\mu'}(\zeta, p_z)\exp\left\{\frac{iS_{\mu'}(\zeta, p_z)}{\sigma}\right\}.$$

The factors $\exp(im_{r,\alpha}\pi/2)$ are included in $\hat{V}$.

We now express χ_{AB} in terms of the coefficients c_μ. Substituting the expression (13) for the matrix elements A and B in (12) and writing df_0/dE_n in the form

$$\frac{df_0}{dE_n} = \int_0^\infty dE \frac{df_0}{dE}\int_{-\infty}^\infty \exp\left\{\frac{-it}{\hbar}(E - \hbar\omega_n - \zeta)\right\} dt,$$

we obtain after regrouping of the terms

$$\chi_{AB} = -\frac{eH/c}{(2\pi\hbar)^3}\sum_{\mu,\mu'=1}^{2N}\int dp_z \int_0^\infty dE \frac{df_0}{dE}\int_{-\infty}^\infty dt' \exp\left\{-\frac{it'(E-\zeta)}{\hbar}\right\}$$
$$\times\int_0^\infty dt\, e^{-i\tilde{\nu}t}\int_0^{T_\mu} dt_\mu B(t_\mu, \zeta)\int_0^{T_{\mu'}} dt'_{\mu'} A(t'_{\mu'}, \zeta) G_{\mu\mu'}(t + t_\mu - t'_{\mu'}) G^*_{\mu\mu'}$$
$$\times(t + t_\mu - t'_{\mu'} + t'),$$
$$\tilde{\nu} = \omega - i\nu_0; \tag{17}$$

$$G_{\mu\mu'}(t) = \sum_{n=-\infty} c_\mu^*(n; \zeta, p_z) c_{\mu'}(n; \zeta, p_z) e^{-i\omega_n t}. \tag{17a}$$

The kinetic coefficient χ_{AB} pertains here to a unit volume. The definitions of $B(t_\mu)$ and $A(t_\mu)$ are clear from (13). As shown by (17) and (17a), the remaining problem consists of finding the universal matrix $G_{\mu\mu'}(t)$. Using (16), we can easily verify that $G_{\mu\mu'}(t)$ satisfies the finite-difference equation

$$G_{\mu\mu'}(t) = \sum_{\mu''=1}^{2N} U_{\mu'\mu''}(\zeta, p_z) G_{\mu\mu''}(t - T_{\mu''}). \tag{18}$$

It will be shown in the Appendix that $G_{\mu\mu'}(t)$ is that particular solution of Eq. (18), which satisfies the following initial conditions:

$$G_{\mu\mu'}(t) = \delta_{\mu\mu'}\delta(t), \quad -T_{\mu'} < t \leqslant 0. \tag{19}$$

Noting that the ratios $T_\mu(\zeta, p_z)/T_{\mu'}(\zeta, p_z)$ $(\mu \neq \mu')$ are irrational for all values of p_z (with the exception of a set of zero measure), and taking (19) into account, we can represent the matrix $G_{\mu\mu'}(t)$ for $t > -T_{\mu'}$ in the form

$$G_{\mu\mu'}(t) = \sum_{l_1 \ldots l_{2N} \geqslant 0}^{\infty} A_{\mu\mu'}(\mathbf{l})\delta(t - \mathbf{l}\mathbf{T}), \quad \mathbf{T} = \{T_1 \ldots T_{2N}\},$$
$$\mathbf{l}\mathbf{T} = \sum_{\mu=1}^{2N} l_\mu T_\mu, \quad \mathbf{l} = \{l_1 \ldots l_{2N}\}, \quad l_\mu = 0, 1, \ldots, \infty. \tag{20}$$

The coefficients $A_{\mu\mu'}(\mathbf{l})$ are connected with one another by recurrence relations, which are obtained directly from (18):

$$A_{\mu\mu'}(\mathbf{l}) = \sum_{\mu''=1}^{2N} U_{\mu'\mu''}(\zeta) A_{\mu\mu''}(\mathbf{l} - \mathbf{e}^{(\mu'')}), \quad e^{(\mu)}_{\mu'} = \delta_{\mu\mu'}. \tag{21}$$

Equation (21) is valid for all $l_1, \ldots, l_{2N} \geq 0$ with the exception of $\mathbf{l} = 0$, and then $A_{\mu\mu'}$ satisfy the boundary conditions

$$A_{\mu\mu'}(0) = \delta_{\mu\mu'}, \quad A_{\mu\mu'}(\mathbf{l} - (l_{\mu'} + 1)\mathbf{e}^{(\mu')}) = 0 \quad (\mu, \mu' = 1 \ldots 2N), \tag{21a}$$

which follow from the initial conditions (19) and from the fact that all the T_μ are not commensurate.

The system of recurrence relations (21) together with the boundary conditions (21a) can be represented in compact form, using the generating functions

$$F_{\mu\mu'}(\boldsymbol{\varphi}) = \sum_{l_1 \ldots l_{2N} \geqslant 0}^{\infty\;\prime} A_{\mu\mu'}(\mathbf{l})\exp\left\{i\mathbf{l}\left(\boldsymbol{\varphi} - \frac{\mathbf{S}}{\sigma}\right)\right\},$$
$$\boldsymbol{\varphi} = \{\varphi_1 \ldots \varphi_{2N}\}, \quad \mathbf{S}(\zeta, p_z) = \{S_1 \ldots S_{2N}\} \tag{22}$$

(the prime at the summation sign denotes that the term with $\mathbf{l} = 0$ has been omitted; φ_μ are arbitrary complex numbers). Taking into account (21) and (21a), and also the connection between $\hat{U}$ and $\hat{V}$ (see (6)), we find for $F_{\mu\mu'}$ the following system of linear equations:

$$F_{\mu\mu'}(\boldsymbol{\varphi}) = \sum_{\mu''=1}^{2N} V_{\mu'\mu''}(\zeta, p_z)\exp\{i\varphi_{\mu''}\} F_{\mu\mu''}(\boldsymbol{\varphi}) + V_{\mu'\mu}(\zeta, p_z)\exp\{i\varphi_\mu\}. \tag{23}$$

Expression (17) for the coefficient χ_{AB} can be written directly in terms of $F_{\mu\mu'}$. Omitting straightforward but rather cumbersome transformations, which use formulas (20) and (22) and the relation $G^*_{\mu\mu'}(t) = G_{\mu'\mu}(-t)$, we present the final result

$$\chi_{AB} = -\frac{eH/c}{(2\pi\hbar)^3}\int dp_z \int dE \frac{df}{dE}\left\{\sum_{\mu=1}^{2N}\int_0^{T_\mu} A(t_\mu)\, dt_\mu\right.$$
$$\times\int_0^{t_\mu} B(t_\mu')\exp\{i\tilde{\nu}(t_\mu' - t_\mu)\}\, dt_\mu' L_\mu(E, p_z)$$
$$+\sum_{\mu,\mu'=1}^{2N}\int_0^{T_\mu} B(t_\mu)\exp\{i\tilde{\nu}t_\mu\}\, dt_\mu \int_0^{T_{\mu'}} A(t_{\mu'})\exp\{-i\tilde{\nu}t_{\mu'}\}\, dt_{\mu'}\left[M_0^{(\mu,\mu')}(\zeta, p_z)\right.$$
$$\left.\left.+\sum_{\mathbf{m}\neq 0} M_{\mathbf{m}}^{(\mu,\mu')}(\zeta, p_z)\exp\left\{\frac{i\mathbf{m}\mathbf{S}(E, p_z)}{\sigma}\right\}\right]\right\}; \tag{24}$$

$$L_\mu = 1 + 2\,\mathrm{Re}\, F_{\mu\mu}\left(\frac{\mathbf{S}(E, p_z)}{\sigma}\right),$$

$$M_0^{(\mu,\mu')} = \frac{1}{(2\pi)^{2N}}\int_{\Delta_{2N}} F_{\mu\mu'}(\boldsymbol{\varphi} - i\tilde{\nu}\mathbf{T}) F^*_{\mu\mu'}(\boldsymbol{\varphi} + i0)\, d\boldsymbol{\varphi},$$

$$M_{\mathbf{m}\neq 0}^{(\mu,\mu')} = \frac{1}{(2\pi)^{2N}}\int_{\Delta_{2N}} [F_{\mu\mu'}(\boldsymbol{\varphi} - i\tilde{\nu}\mathbf{T}) F^*_{\mu\mu'}(\boldsymbol{\varphi} + i0)$$
$$+ F_{\mu\mu'}(\boldsymbol{\varphi} - i\nu\mathbf{T}) F_{\mu'\mu}(\boldsymbol{\varphi} + i0)]\, e^{-i\mathbf{m}\boldsymbol{\varphi}}\, d\boldsymbol{\varphi}, \tag{24a}$$

$$d\boldsymbol{\varphi} = d\varphi_1 d\varphi_2 \ldots d\varphi_{2N}, \quad \mathbf{m} = \{m_1 \ldots m_{2N}\}, \quad m_\mu = 0, \pm 1, \ldots, \pm\infty.$$

Here the region of integration Δ_{2N} is a 2N-dimensional cube with side 2π, the symbol i0 indicates the rule for going around the poles of $F_{\mu\mu'}(\varphi)$, which coincide with the zeroes of $\mathrm{Det}\,\|\,\delta_{\mu\mu'} - V_{\mu\mu'}\exp(i\varphi_{\mu'})\,\|$ (see (23)).

In the case of open one-dimensional periodic configurations, it is necessary to take into account the dependence of $c_{r,a}$ and E_ξ on the parameter k (see (9) and (10)). In all other respects the calculations are analogous to the preceding ones and lead to an expression for χ_{AB} coinciding in form with (24). Now, however, there appears in L_μ and $M_{\mathbf{m}}^{(\mu,\mu')}$ an additional integration with respect to k:

$$L_\mu = \frac{1}{2\pi}\int_0^{2\pi} L_\mu(k)\,dk, \qquad M_{\mathbf{m}}^{(\mu,\mu')} = \frac{1}{2\pi}\int_0^{2\pi} M_{\mathbf{m}}^{(\mu,\mu')}(k)\,dk; \tag{24b}$$

$$L_\mu(k) = 1 + \frac{1}{2}\operatorname{Re} F_{\mu\mu}^{(k)}\left(\frac{\mathbf{S}}{\sigma}\right),$$

$$M_0^{(\mu,\mu')}(k) = \frac{1}{(2\pi)^{2N}}\int_{\Delta_{2N}} F_{\mu\mu'}^{(k)}(\varphi - i\tilde{\nu}\mathbf{T})F_{\mu\mu'}^{(k)*}(\varphi + i0)\,d\varphi,$$

$$M_{\mathbf{m}\neq 0}^{(\mu,\mu')}(k) = \frac{1}{(2\pi)^{2N}}\int_{\Delta_{2N}} [F_{\mu\mu'}^{(k)}(\varphi - i\tilde{\nu}\mathbf{T})F_{\mu\mu'}^{(k)*}(\varphi + i0)$$

$$+ F_{\mu\mu'}^{\prime(k)}(\varphi - i\nu\mathbf{T})\,F_{\mu'\mu}^{(k)}(\varphi + i0)]\,e^{-i\mathbf{m}\varphi}\,d\varphi, \tag{25}$$

$$F_{\mu\mu'}^{(k)}(\varphi) = \sum_{\mu''=1}^{2N}\tilde{V}_{\mu'\mu''}(k;\zeta,p_z)\exp(i\varphi_{\mu''})F_{\mu\mu''}^{(k)}(\varphi) + \tilde{V}_{\mu'\mu}(k;\zeta,p_z)\exp(i\varphi_\mu).$$

Here $\mu \equiv (r, a)$, N is the number of breakdown regions in the unit cell, the matrix $\hat{V}(k)$ is defined by formula (10). Formulas (24)—(24b) together with (23) and (25) make it possible to find the arbitrary kinetic coefficient under conditions of magnetic breakdown. From the definition of $\hat{V}$ and $\hat{V}(k)$ and from the equalities (23) and (25) it follows that the generating functions $F_{\mu\mu'}$ and $F_{\mu\mu'}^{(k)}$ depend only on the scattering matrices $\hat{\tau}^{(i)}(\zeta, p_z)$ ($i = \alpha$, a), i.e., they are smooth functions of H. Thus, the rapid oscillations in H are determined only by exponentials of the form $\exp\{i\mathbf{m}\cdot\mathbf{S}(E, p_z)/\sigma\}$, where m is an arbitrary whole-number vector that differs from zero. The influence of coherent scattering on the smooth part of χ_{AB} is expressed by the matrix $M_0^{(\mu,\mu')}$.

B. In order to present a physical interpretation of the results, let us consider (with the configuration of Fig. 1 as an example) the time development of a quasiclassical packet $|\,0\,\rangle = |\mathbf{R}_0, \mathbf{P}_0, r_0\rangle$ at $t = 0$, having a definite band number r_0 and localized in r- and p-spaces near $\mathbf{R}_0$ and $\mathbf{P}_0$ ($\langle 0|0\rangle = 1$). So long as the interband transitions are small, the quasiclassical approximation is valid, and the vector of state $|\,t\,\rangle$, which results from $|\,0\,\rangle$, is determined, accurate to details of no consequence, by the following expression[5]:

$$|\,t\,\rangle = e^{i\mathscr{S}(t)/\hbar}|\,\mathbf{R}_{r_0}(t),\ \mathbf{P}_{r_0}(t),\ r_0\,\rangle,\ \mathscr{S}(t) = \int_{\mathbf{R}_0}^{\mathbf{R}(t)}\mathbf{P}\,d\mathbf{r} - E_0 t, \tag{26}$$

$$E_0 = \varepsilon_{r_0}(\mathbf{p}_0).$$

The center of the packet $\mathbf{R}_{r_0}(t)$, $\mathbf{P}_{r_0}(t)$ shifts in accordance with the classical equations of motion of the band r_0; $\mathscr{S}(t)$ is the classical action

After passing through the magnetic-breakdown region, the packet splits under the influence of the perturbation $\hat{\mathscr{H}}_\alpha$ (see (1)), going over into a superposition of states with different band numbers:

$$|t\rangle = \sum_{r=1}^{2}\tau_{rr_0}\exp\left\{\frac{i\mathscr{S}(\bar{t})}{\hbar}\right\}|t; r\rangle,\ \langle t; r|t; r\rangle = 1. \tag{27}$$

Here $\bar{t}$ is the instant of "scattering," determined from the coincidence of $\mathbf{p}_{r_0}(\bar{t})$ with the center of the "magnetic breakdown" region, $|\,t;\bar{r}\,\rangle$ is a localized state, constituting the packet $|\,\mathbf{R}_{r_0}(\bar{t}),\ \mathbf{P}_{r_0}(\bar{t}),\ r\,\rangle$ at $t = \bar{t}$. The packets $|\,t;\,r\,\rangle$ again experience scattering at the instants of time $\bar{t} + T_r$ (T_r is the Larmor period of the r-th band), generating two new localized states each, etc. As the result of such a multiple scattering, occurring at the instants of time $t_1 = \bar{t} + l_1T_1 + l_2T_2$ (the integers $l_{1,2} \geq 0$), the number of packets increases exponentially. After the instant of scattering t_1, there are produced in the band r simultaneously $C_{l_1+l_2}^{l_1}$ packets that interfere with each other:

$$\begin{gathered}|\,t;\nu,r,\mathbf{l}\rangle = a_{\nu,r}(\mathbf{l})\,|\,t;r,\mathbf{l}\rangle, \qquad |\,t_1;r,\mathbf{l}\rangle = |\,\mathbf{R}_1,\mathbf{P}_1,r\rangle,\\ \mathbf{P}_1 = \mathbf{P}_{r_0}(\bar{t}), \qquad \mathbf{R}_1 = \{X_{r_0}(\bar{t}), Y_{r_0}(\bar{t}), Z_{r_0}(\bar{t}) + \mathbf{l}\Delta\mathbf{Z}\},\\ \Delta Z_r = (\partial\varepsilon_r/\partial p_z)_S\,T_r, \qquad a_{\nu,r} = b_{\nu,r}(\mathbf{l})\exp(i\mathscr{S}_1\hbar^{-1}).\end{gathered} \tag{28}$$

The index $\nu = 1, 2, \ldots C_{l_1+l_2}^{l_1}$ numbers the states (28). To each index there is set in correspondence a certain path described by the classical particle emerging from the point $\mathbf{R}_0$ and $\mathbf{P}_2$ and making, in a definite sequence, l_1 turns in the first band and l_2 in the second; $\mathscr{S}_1$ is the summary increment of the classical action during the entire time of motion along the path ν ($\mathscr{S}_1$ is the same for all paths). As seen from (27), the quantities $b_{\nu,r}(\mathbf{l})$ constitute different products of $l_1 + l_2 + 1$ factors, namely the elements of the matrix $\hat{\tau}$. These factors appear whenever the path ν crosses the region of magnetic breakdown in p-space. With (28) taken into account, the state $|\,t\,\rangle$ for an arbitrary t is written as follows:

$$\begin{gathered}|t\rangle = \sum_r\sum_{l_1,l_2\geqslant 0}^{\infty} B_{rr_0}(\mathbf{l})\,|t;r,l\rangle\ \vartheta_r(t - t_l),\\ B_{rr_0}(\mathbf{l}) = \sum_\nu a_{r,\nu}(\mathbf{l}), \qquad \vartheta_r(t) = \begin{cases}1, 0 < t < T_r,\\ 0,\ t < 0,\ t > T_r.\end{cases}\end{gathered} \tag{29}$$

The quantity $B_{rr_0}(\mathbf{l})$ is determined by the sum over all ν. From the definition and from Fig. 1 it follows directly that the summary amplitude B_{rr_0} satisfies the same recurrence relations (21) and boundary conditions (21a) as the quantities $A_{\mu\mu'}(\mu \equiv r)$[3)]. Thus, the coefficients of the generating functions acquire the physical meaning of amplitudes describing the multiple scattering of a quasiclassical packet.

Formula (29) can be used to calculate the kinetic coefficients, by changing over to the packet representation in (12). This gives rise to mean values of the form $\langle\,t\,|\hat{\varphi}|\,t\,\rangle$, where $\hat{\varphi} = \varphi_r(\hat{\mathbf{r}}, \hat{\mathbf{P}})$ is an arbitrary physical operator. Taking into account the quasiclassical relation $\hat{\varphi}|\,\mathbf{R},\ \mathbf{P},\ r\,\rangle = \varphi_r(\mathbf{R}, \mathbf{P})|\,\mathbf{R},\ \mathbf{P},\ r\,\rangle$ neglecting in the general case the crossing terms with $\mathbf{l} \neq \mathbf{l}'$, which are not commensurate with T_1 and T_2, we find $(t > \bar{t})$[4)]

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3)The author is indebted to V. L. Ginzburg for this remark.

4)The rapidly oscillating phase factors $\exp\{i\mathscr{S}_l - \mathscr{S}_{l'})/\hbar\}$ and further integration with respect to p_z and E make it possible to neglect those crossing terms for which accidentally $t_1 \approx t_1'$.

$$\langle t\,|\hat{\varphi}|\,t\rangle = \sum_{r;l_1,l\geq 0} |B_{rr_0}(\mathbf{l})|^2 \varphi_r(\mathbf{R}_r(t), \mathbf{P}_r(t))\,\vartheta_r(t-t_1);\quad \mathbf{R}_r(t_1)=\mathbf{R}_1,\ |\mathbf{P}_r(t_1)=\mathbf{P}_1. \tag{30}$$

After integrating with respect to t and averaging over $\mathbf{R}_0$ and $\mathbf{P}_0$, we can again obtain the smooth part of the expression (24).

C. The use of formulas (24)–(24b) for a detailed analysis of the different kinetic effects is beyond the scope of one article and will be published in subsequent papers. We confine ourselves here to an examination of the general properties of the resonant and galvanomagnetic phenomena.

In order for some resonance effects to occur in an alternating external field, it is necessary that the configurations of the electron orbits be closed. A discrete set of randomly disposed levels $E_n(p_z)$ corresponds to the quasicontinuous spectrum of the possible resonant frequencies $\Omega = [E_n(p_z) - E_{n'}(p_z)]/\hbar$. Mathematically, the quasicontinuity follows from the fact that the density of the distribution of the particles Ω on the Fermi surface, which is proportional to the "correlation" function

$$A(\Omega;\zeta,p_z) \equiv g(\zeta,p_z)\,g(\zeta-\hbar\Omega,p_z),\quad g(\zeta,p_z)\equiv\sum_{n=-\infty}^{\infty}\delta(\zeta-E_n(p_z)),$$

contains in addition to the rapidly oscillating terms also a part $\bar{\Lambda}(\Omega)$ which is regular in Ω and in p_z.[5)] We shall prove this statement with the aid of the equation

$$g(\zeta,p_z)=\frac{1}{2\pi\hbar}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i\omega_n t}\,dt=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}dt\sum_{\mu}T_\mu G_{\mu\mu}(t) \tag{31}$$

($\hbar\omega_n = E_n - \zeta$), which follows from the definitions of g and $G_{\mu\mu}$ and from the normalization condition (14).

Using expressions (21), (22), and (31) we obtain after transformations similar to those that led to formula (24)

$$\bar{\Lambda}(\Omega;\zeta,p_z)=\frac{2}{(2\pi)^{2N+2}\hbar^2}\sum_{\mu,\mu'}T_\mu T_{\mu'}\,\mathrm{Re}\int_{\Delta_{2N}}F_{\mu\mu}(\varphi+i0)F^{*}_{\mu'\mu'}(\varphi+\Omega\mathrm{T})\,d\varphi. \tag{32}$$

The integrals with respect to φ are analytic functions of Ω, thus proving the statement made above. In the general case $\Lambda(\Omega)$ differs from zero on a certain sequence of intervals with a characteristic width $\sim\Omega_0$. Thus, under magnetic-breakdown conditions, even when $\Omega_0 t_0 \gg 1$ (but $\Omega_0 t_0 \ll \kappa^{-1}$), to each value of p_z there correspond broad ($\sim\Omega_0$) absorption bands, thus qualitatively changing the entire picture of the resonant effects. Formulas (24) and (24a) take this circumstance into account automatically.

In the investigation of the general properties of galvanomagnetic phenomena, it is possible to start directly from formula (12), where $\hat{B} = e\hat{v}_i$, $\hat{A} = e\hat{v}_k$ ($\hat{v}_i$ is one of the components of the velocity operator), $\omega = 0$, and $\chi_{AB} = \sigma_{ik}$ is the electric-conductivity coefficient. Since the magnetic-breakdown regions made a small contribution to the matrix elements, the operator $\hat{v}$ is connected with $\hat{p}$ by the same relations as in the absence of breakdown:

$$\hat{\mathbf{p}} = (e/c)\,[\hat{\mathbf{v}}\mathbf{H}],\quad \hat{p}_{x,y} = (i\hbar)^{-1}[\hat{p_{x,y}},\hat{\mathscr{H}}]. \tag{33}$$

If the configuration of the electron orbits is closed, then the motion in p-space is finite, and according to (33) $\langle\xi|\hat{v}_x|\xi\rangle = \langle\xi|\hat{v}_y|\xi\rangle = 0$. In this case the transverse electric conductivity is $\sigma_{xx}\sim\sigma_{yy}\sim\sigma_0/(\Omega_0 t_0)^2$, where σ_0 is the characteristic value of the electric conductivity at H = 0. To calculate σ_{xy} we use the relation

$$-i\hbar\sum_{\xi'}\frac{\langle\xi|\hat{v}_y|\xi'\rangle\langle\xi'|\hat{v}_x|\xi\rangle}{E_\xi - E_{\xi'}} = -\frac{c}{eH}\langle\xi|\hat{v}_y\hat{p}_y|\xi\rangle, \tag{34}$$

which follows from (33). Taking into account (34), (13), and formula (A.5) of the Appendix, we obtain after several simple transformations, carried out in the zeroth approximation in $(\Omega_0 t_0)^{-1}$, that $\sigma_{xy} = -\sigma_{yx} = (c/eH)(n_+ - n_-)$ (n_- and n_+ is the number of electrons and holes), i.e., the magnetic breakdown leaves σ_{xy} and σ_{yx} unchanged.

In the general case $W \sim 1 - W$, open one-dimensional configurations correspond to current states $|\xi\rangle$, in which $\langle\xi|\hat{v}_x|\xi\rangle\sim v_0$ (see Sec. 1). According to (12), the contribution of these states to σ_{xx} is of the order of $\sigma_0(\delta p/b_0)$, where δp is the width of the interval of values of p_z at which $\partial E_n/\partial P_x \neq 0$. At the same time $\langle\xi|v_y|\xi\rangle = 0$, so that $\sigma_{yy}\sim\sigma_0(\Omega_0 t_0)^{-2}$; the components σ_{xy} and σ_{yx}, unlike the case of closed configurations, depend significantly on the breakdown parameters. Concrete calculations carried out on the basis of (24) and (25), confirm the drawn conclusions. Formulas (24) and (25) show also that under breakdown conditions the oscillation effects of the Shubnikov-de Haas type inevitably arise also for open configurations. In this case the oscillating part of the electric conductivity is $\tilde{\sigma}_{xx}\sim\sqrt{\kappa}\sigma_0$. (It is assumed that inside the layer δp there is located at least one of the extremal areas of the type $\mathbf{m}\cdot\mathbf{S}$.) When $\delta p\sim\sqrt{\kappa}b_0$ we have $\tilde{\sigma}_{xx}\sim(\delta p/b_0)\sigma_0$; if at the same time $(\Omega_0 t_0)^{-2}\ll\delta p/b_0$, then the contribution of the closed configurations to σ_{xx} can be neglected, and consequently $\sigma_{xx}\sim\tilde{\sigma}_{xx}$. The latter denotes that the electric conductivity experiences "giant" oscillations in the magnetic field. Such an effect was recently observed in the investigation of magnetic breakdown in beryllium[6], the Fermi surface of which has a geometry that ensures the required smallness of δp.

4. INFLUENCE OF WEAK PERTURBATIONS ON COHERENT EFFECTS

The analysis that will be carried out in this section is valid for arbitrary small perturbations $\hat{\mathscr{H}}'$. However for concreteness we shall assume that $\hat{\mathscr{H}}' = \delta\epsilon\sin(\mathbf{k}\cdot\mathbf{r})$ ($\delta\epsilon/\epsilon_0\ll 1$). The spatial inhomogeneity $\hat{\mathscr{H}}'$ is governed by the parameter kd_0 (d_0 is the characteristic Larmor radius). If $kd_0 \gtrsim 1$, then it is simplest to carry out the investigation in terms of the packets $|\mathbf{R}_0, \mathbf{P}_0, \mathbf{r}_0\rangle$ introduced in Sec. 3. In view of the smallness of $\hat{\mathscr{H}}'$, the vector of state $|t\rangle$, which represents the time development of $|\mathbf{R}_0, \mathbf{P}_0, \mathbf{r}_0\rangle$, can be constructed by directly generalizing formula (29) and neglecting, where possible, the quantities $\sim\delta\epsilon/\epsilon_0$. For the configuration of Fig. 1 we obtain

5)The integral $\int_{\Omega}^{\Omega+\Delta\Omega} d\Omega'\int_{p_z}^{p_z+\Delta p_z}\Lambda\,dp_z'\ (\Delta p_z\gg\kappa b_0,\ \Delta\Omega\gg\kappa\Omega_0)$ is proportional, accurate to corrections $\sim\kappa$, to the number of states with $E = \zeta$ contained in the layer Δp_z and characterized by transition frequencies Ω belonging to $[\Omega, \Omega+\Delta\Omega]$.

$$|t\rangle=\sum_{r=1}^{2}\sum_{l_1,l_2\geqslant 0}^{\infty}\sum_{\nu}a_{r,\nu}(\mathbf{l})\,|t;\nu,r.\mathbf{l}\rangle\,\vartheta_r(t-t_1),$$
$$a_{r,\nu}(\mathbf{l})=b_{r,\nu}(\mathbf{l})\exp(i\Phi_{\nu,\mathbf{l}}), \tag{35}$$
$$|t_1;\nu,r,\mathbf{l}\rangle=|\mathbf{R}_{\nu,\mathbf{l}},\mathbf{P}_{\nu,\mathbf{l}},r\rangle,\ \ \Phi_{\nu,\mathbf{l}}=\mathcal{S}_{\nu,\mathbf{l}}/\hbar.$$

Here $|t;\ \nu,\ r,\ \mathbf{l}\rangle$ is the analog of $|t;\ r,\ \mathbf{l}$ of formulas (28) and (29); the index ν and the quantities $b_{\nu,r}(\mathbf{l})$ are defined in Sec. 3. The dependences of $\mathbf{R}_{\nu,l}$, $\mathbf{P}_{\nu,l}$, and of the total increment of the action on the form of the path ν are due to the strong spatial inhomogeneity ($kd_0 > 1$) of the perturbation along the z axis. In this case $\mathbf{R}_{\nu,\mathbf{l}}$ and $\mathbf{P}_{\nu,\mathbf{l}}$ have a scatter relative to $\mathbf{R}_1$ and $\mathbf{P}_1$ of formula (28), by amounts equal respectively $\gtrsim(\delta\epsilon/\epsilon_0)R_0$ and $(\delta\epsilon/\epsilon_0)b_0$; for the phase difference we have

$$|\Phi_{\nu,\mathbf{l}}-\Phi_{\nu',\mathbf{l}}|\gtrsim\lambda=\delta\varepsilon/\hbar\Omega_0,\quad \nu\neq\nu'. \tag{36}$$

The mean value of the operator $\varphi_r(\hat{\mathbf{r}},\ \hat{\mathbf{P}})$ in the state (35) is determined by an expression of the type (30), where now

$$|B_{rr_0}(\mathbf{l})|^2=\sum_{\nu,\nu'}b_{r,\nu}b^*_{r,\nu'}\exp[i(\Phi_{\nu,\mathbf{l}}-\Phi_{\nu',\mathbf{l}})]\langle t;r,\nu,\mathbf{l}\,|\,t;r,\nu',\mathbf{l}\rangle \tag{37}$$

and $\mathbf{R}(t)$, $\mathbf{P}(t)$ correspond, neglecting the terms $\sim\delta\epsilon/\epsilon_0$, to the unperturbed classical motion. In the case $\lambda \gg 1$, the quantities $\exp[i(\Phi_\nu - \Phi_{\nu'})]$ $(\nu \neq \nu')$ are, in accordance with (36), rapidly oscillating functions of $\mathbf{R}_0$ and $\mathbf{P}_0$, so that all the terms in (37) with $\nu \neq \nu'$ vanish after the averaging over $\mathbf{R}_0$ and $\mathbf{P}_0$ needed for the calculation of the kinetic coefficients. Taking this circumstance into account, we can write

$$|B_{rr_0}(\mathbf{l})|^2=\sum_{\nu}|b_{r,\nu}(\mathbf{l})|^2. \tag{38}$$

The vanishing of the interference "crossing" terms in (38) makes it possible to regard $\langle t\,|\,\hat{\varphi}\,|\,t\rangle$ as the mathematical expectation of the random function $\varphi_r(\mathbf{R}_r(t),\ \mathbf{P}_r(t))$, which is specified by the classical motion of the electron over the random paths ν; the quantity $|b_{r,\nu}(\mathbf{l})|^2$, which is equal to the product of different powers of W and $1 - W$, is the probability of realizing a given type of path. Thus, in the case when $\lambda \gg 1$, the breakdown probability acquires a stochastic meaning, and the calculation of the kinetic coefficients that do not depend on λ is carried out in terms of discrete random wanderings of the particle[6]. As a result it turns out that the dissipative part of most kinetic coefficients remains finite, even if $t_0 \to \infty$. This significantly distinguishes the case $\lambda \gg 1$ from the coherent situation of Sec. 3. For example, in the stochastic case open configurations (Fig. 3) correspond to the electric conductivity $\sigma_{xx} \sim (\delta p/b_0)\sigma_0(\Omega_0 t_0)^{-1}$ [7], whereas the "coherent" value is $\sigma_{xx} \sim (\delta p/b_0)\sigma_0$ (see Sec. 3).

When $\lambda \sim 1$ it is necessary to retain in (37) the "crossing" terms for which $|\Phi_\nu - \Phi_{\nu'}| \sim 1$. Under these conditions the kinetic coefficients begin to depend on the parameter λ. In the case $\lambda \ll 1$, simple estimates show that the kinetic coefficients are determined, accurate to quantities $\sim\lambda$, by formulas (24)–(24b).

[6]Such calculations were actually performed in [7], where the influence of the coherent processes on the smooth part of the kinetic coefficients was not considered at all, and the limits of the stochastic interpretation of W were not stipulated. The analysis presented here can serve as the basis of the stochastic approach [7]. We note that $\lambda \lesssim 1$ the results of [7] are no longer correct.

If $kd_0 \ll 1$, then the randomization of the dynamics of the electron arises only after very long times $\sim\Omega_0^{-1}\exp(1/kd_0) \gg t_0$. However, in the case of weak spatial inhomogeneity, the perturbation $\hat{\mathscr{H}}'$ has a strong influence on the coherent effects. By way of an example let us consider open one-dimensional periodic configurations (Fig. 3), putting $\mathbf{k} = \{k,\ 0,\ 0\}$ (x—direction of the openness in coordinate space). In the zeroth approximation in $(\delta\epsilon/\hbar\Omega_0)kd_0 \ll 1$ we can neglect transition between the "magnetic" bands $E_n(p_z,\ P_x)$ and to describe the dynamics of the electron by the classical Hamiltonian $\mathscr{H}_n = E_n(p_z,\ P_x) + \delta\epsilon\sin kx$. From the conservation law $\mathscr{H}_n(x,\ P_x) = E$ and from the periodicity of $E_n(p_z,\ P_x)$ (see Sec. 1) it follows that when $\lambda \gtrsim 1$ the motion of the electron in the direction of the x axis becomes finite. The characteristic dimension of the resulting quantum "traps" $\Delta l \sim (\hbar\Omega_0/\delta\epsilon)k^{-1} \gg d_0$. In the case $\Delta l \ll l_0$ (the characteristic mean free path), the electric conductivity is $\sigma_{xx} \sim (\Delta l/l_0)^2\sigma_0 \ll \sigma_0$.

If the perturbation $\hat{\mathscr{H}}'$ is produced by ultrasound, then $\Delta l \sim \kappa(I/s^3\rho_0)^{-1/2}(s/\omega)$ (I—density of the energy flux of the ultrasonic wave, s and ω are the velocity and frequency of the sound, and ρ_0 is the density of the metal), and consequently, at sufficiently large I the electric conductivity (and other kinetic coefficients) begin to depend strongly on the intensity of the ultrasound. In order for these "giant" nonlinear effects to arise, it is necessary to have $I > 1$ W/cm^2 sec and $\omega \gtrsim 10^5$ sec^{-1} (at $l_0 \sim 10^{-10}$ cm). The randomization effects become manifest when ω becomes $\gtrsim 10^8$ sec^{-1}. A detailed analysis of the nonlinear effects (including in particular, the case $\lambda \lesssim 1$, $kd_0 \gtrsim 1$) will be published in a separate article.

The source of perturbation $\hat{\mathscr{H}}'$ can be also a weak inhomogeneity of the strong magnetic field $\mathbf{H}(\mathbf{r})$, which acquires, owing to the de Haas-van Alfen effect inside the metal, an addition of the form

$$\Delta\mathbf{H}=\mathbf{H}_1\sin\mathbf{kr}\ \ (H_1\sim\exp(-2\pi^2kT/\hbar\Omega_0)H\sqrt{\varkappa},\ |\mathbf{k}|\sim|\nabla H|/H\varkappa).$$

In this case $\delta\epsilon \sim (H_1/H)\epsilon_0$ and the randomization becomes significant when $kT \lesssim \hbar\Omega_0$ and $H/|\nabla H| < 10$ cm. If $d_0|\nabla H|/H \ll 1$, formation of quantum magnetic "traps," considered by the author and Kadigrobov in[8], also becomes possible.

In conclusion we note that for experimental observation of the phenomena connected with multiple scattering it is necessary to have a small concentration of the dislocations whose deformation potential, being long-range, can cause randomization of the electron dynamics, in analogy with ultrasound[7)]. Estimates in which the foregoing results are used show that the role of the dislocations becomes significant if the characteristic distance between them is much smaller than or are of the order of d_0.

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[7)]The influence of dislocations on coherent effects under breakdown conditions was observed experimentally by Stark in an investigation of "giant" oscillations of the magnetoresistance of Mg [2].

my indebtedness to L. M. Falicov, who made many valuable remarks.

APPENDIX

1. The function $G_{\mu\mu'}(t)$, defined by formulas (18)–(20), can always be written in the form of a series

$$G_{\mu\mu'}(t)=\sum_{n'=-\infty}^{\infty}\beta_\mu(n')c_{\mu'}(n')\exp(-i\omega_{n'}t). \quad (A.1)$$

We shall prove that $\beta_\mu(n') = c_\mu^*(n')$. Multiplying (A.1) by $T_{\mu'}c_{\mu'}^*(n)\exp(i\omega_n t)$ ($\omega_n = \omega_n + i\gamma$, $\gamma = +0$) and integrating with respect to t from zero to infinity, we obtain with allowance for (20)–(22) and (14)

$$\sum_{\mu'}T_{\mu'}c_{\mu'}^*(n)F_{\mu\mu'}\left(\tilde{\omega}_n\mathrm{T}+\frac{\mathrm{S}}{\sigma}\right)=\frac{\beta_\mu(n)}{\gamma}+O(1). \quad (A.2)$$

We perform the operation of complex conjugation on (16) and multiply these equations line by line by the equations of the system (23), in which we put $\varphi = \tilde{\omega}_n\mathrm{T} + \mathrm{S}/\sigma$. Using the unitarity condition

$$\sum_{\mu}U_{\mu\mu'}U^*_{\mu\mu''}=\delta_{\mu'\mu''}$$

and taking into account the connection between $\hat{V}$ and $\hat{U}$, we obtain after introducing a pole singularity with respect to ν the sought result

$$\sum_{\mu'}T_{\mu'}c_{\mu'}^*(n)F_{\mu\mu'}\left(\tilde{\omega}_n\mathrm{T}+\frac{\mathrm{S}}{\sigma}\right)=\frac{c_\mu^*(n)}{\gamma}+O(1). \quad (A.3)$$

2. Let us consider the sum

$$\sum_{n=-\infty}^{\infty}\varphi(E_n(p_z))\,|c_{n,\mu}|^2$$

$$=\frac{1}{2\pi\hbar}\int dE\,\varphi(E)\sum_{n=-\infty}^{\infty}\int dt\exp\left\{\frac{-it(E-E_n)}{\hbar}\right\}|c_\mu(n)|^2, \quad (A.4)$$

where $\varphi(E)$ is an arbitrary sufficiently smooth function. Taking into account the definition (17a) and formulas (20)–(22), we get

$$\sum_{n=-\infty}^{\infty}=\frac{1}{2\pi\hbar}\int\varphi(E)dE\ (1+\text{(plus rapidly oscillating terms)}). \quad (A.5)$$

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