EFFECT OF A QUANTIZING MAGNETIC FIELD ON ELECTRON-ELECTRON COLLISIONS IN SEMICONDUCTORS AND NONLINEAR GALVANOMAGNETIC PHENOMENA

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The density matrix method, with electron-electron collisions taken into account, is used to obtain an expression for the current in semiconductors that are located in crossed strong electric and quantizing magnetic fields. It is shown that the effectiveness of electron-electron collisions is greatly reduced by the quantization of cyclotron orbits. A criterion is derived for the applicability of the effective electron temperature concept. It is noted that the Hall field (if present in the problem) must be taken into account in the energy balance.

1. Electron-electron collisions play an important role in the theory of "hot" electrons. Heating of electrons in semiconductors can occur in relatively low electric fields **E** and leads to nonlinear dependence of the current on **E**.[1] In describing nonlinear galvanomagnetic phenomena subject to these conditions the concept of an effective electron temperature, dependent on the electric field, is often used. This approximation is usually based on the hypothesis that the frequency of electron-electron collisions exceeds the frequency of electron-phonon collisions. In the case of a sufficiently low applied magnetic field **H**, permitting us to neglect the quantization of electron orbital motion and the influence of the field on the collision process, the appropriate criterion is well known.[2,3]

For the case of a quantizing magnetic field **H** the concept of an effective electron temperature $T_e(E)$ has been used in [4,5], although a quantitative test of the applicability of this concept when the orbital motion of the carriers is quantized has nowhere been discussed; therefore the limits of validity of the results are not clear. In [4,5] it is assumed that the electrical conductivity can be calculated from Titeica's equation[6] with the electron temperature replaced by an effective temperature $T_e(E)$ that differs from the lattice temperature T. The justification for this procedure is far from being obvious.

The present work has been performed to establish quantitative tests, in the presence of a mutually orthogonal high electric field and quantizing magnetic field, for the validity of the effective electron temperature concept and of Titeica's electrical conductivity formula with $T_e \neq T$ in nondegenerate semiconductors.

It will be shown here that for the case of Maxwellian statistics the frequency ν_{ee} of electron-electron collisions decreases exponentially as the parameter $\hbar\Omega/T_e$ increases (Ω is the cyclotron frequency of the carriers). This result can be understood from the following considerations. When $\hbar\Omega/T_e \gg 1$ in the nondegenerate electron case the very great majority of the electrons occupy the lowest Landau level N = 0, and the number of electrons on level N = 1 is smaller by the factor $\exp(\hbar\Omega/T_e)$. Electron-electron collisions on the N = 0 level without change of the quantum number N do not affect the distribution, and result in vanishing of the collision integral when the distribution function includes any dependence on the momentum projection in the magnetic field direction. This means that in such cases the scattering becomes elastic and one-dimensional. It therefore becomes necessary to consider collisions of electrons on different Landau levels (such as N = 0 and N = 1). We find immediately that the frequency of electron-electron collisions becomes proportional to $\exp(-\hbar\Omega/T_e)$, which is small in the quantum limit $\hbar\Omega/T_e \gg 1$.

The frequency ν_{ep} of electron-phonon collisions is known to be proportional to the square of the magnetic field. This is easily understood from the fact that, in order of magnitude, $\nu_{ep} \sim \Gamma/\hbar$ (where Γ is the electron-phonon collision width of the Landau level). The width Γ depends on H through the volume of phase space occupied by the momenta of phonons interacting with electrons, and through the density of electron states, which is obviously proportional to H if we neglect oscillations. The phase-space volume of phonons interacting with electrons is also proportional to H. In the quantum limit the de Broglie wavelength of an electron moving along H or in a plane perpendicular to H is of the order $\hbar/\sqrt{mT_e}$ or l, respectively (where $l^2 = c\hbar/eH$ is the magnetic length). For electron-phonon scattering momentum conservation accepts out of the entire phase space of phonon momenta a cylinder with base area $\sim(\hbar/l)^2$ and height $\sqrt{mT_e}$. We obtain, finally, $\nu_{ep} \sim H^2$.

In a quantizing magnetic field the ratio ν_{ee}/ν_{ep} is smaller than the classical ratio by the factor $(\hbar\Omega/T_e)^2 \times \exp(\hbar\Omega/T_e)$. Therefore the region in which the effective temperature concept can be applied is shifted strongly toward high concentrations of current carriers.

It will also be shown that for $T_e \neq T$ the electrical conductivity differs from the calculation based on Titeica's equation with substitution of the effective electron temperature, by terms that are proportional to the parameter

$$\xi = \left(\frac{ms^2}{T}\right)^{1/2}\left(\frac{\hbar\Omega}{T}\right)^{1/2}\frac{T_e - T}{T_e}$$

where s is the velocity of sound and m is the effective carrier mass. It follows herefrom that Titeica's equation can be used to calculate electrical conductivity for

$T_e \neq T$ only when ξ is small. In obtaining these results we analyzed the solutions of the kinetic equation for a one-particle density matrix in the Landau representation. It was here assumed that the work, of the order eEl, done by the electric field **E**, orthogonal to **H**, on an electron during the time between two successive collisions with a scatterer is considerably smaller than the characteristic electron energy $\bar{\epsilon}$; a quantizing magnetic field was also assumed, i.e., $\Omega\tau \gg 1$ (where τ is the electron relaxation time) and $\hbar\Omega \gg T_e$. Under these conditions a diagonal element of the electron matrix density in the Landau representation greatly exceeds the off-diagonal elements; this permitted an integration by iterations of the kinetic equation for a one-particle density matrix.

2. We now consider a semiconductor placed in crossed electric (**E**) and quantizing magnetic (**H**) fields. The Hamiltonian of the system is

$$\hat{H} = \hat{H}_e + \hat{H}_{ee} + \hat{H}_p + \hat{H}_{ep} + \hat{H}_{ei} + \hat{H}_E, \tag{1}$$

where

$$\hat{H}_e = \sum_{\nu} \varepsilon_\nu a_\nu^+ a_\nu, \quad \hat{H}_{ee} = \sum_{\mu\mu'\nu\nu'} B_{\nu'\nu}^{\mu'\mu} a_\nu^+ a_\mu^+ a_{\nu'} a_{\mu'},$$

$$\hat{H}_p = \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} b_{\mathbf{q}}^+ b_{\mathbf{q}}, \quad \hat{H}_{ei} = \sum_{\mathbf{q}\nu\nu'} G_{\mathbf{q}} I_{\nu'\nu}^{\mathbf{q}} a_{\nu'}^+ a_\nu,$$

$$\hat{H}_{ep} = \sum_{\nu\nu'\mathbf{q}} (C_{\mathbf{q}} I_{\nu'\nu}^{\mathbf{q}} a_{\nu'}^+ a_\nu b_{\mathbf{q}} + C_{\mathbf{q}}^* I_{\nu'\nu}^{-\mathbf{q}} a_{\nu'}^+ a_\nu b_{\mathbf{q}}^+),$$

$$\hat{H}_E = \sum_{\nu'} eE_i r_{\nu'\nu}^{(i)} a_{\nu'}^+ a_\nu. \tag{2}$$

Here $\hat{H}_e$ and $\hat{H}_p$ are the Hamiltonians of free electron and phonon systems, respectively; $\hat{H}_{ee}$, $\hat{H}_{ep}$, $\hat{H}_{ei}$, and $\hat{H}_E$ are the energy operators of interactions between electrons and either other electrons, phonons, impurities, or the electric field; the subscript ν designates states in the Landau representation $\{n, p_z, X\}$; C_q and G_q are the Fourier transforms of the interaction energy for an electron interacting with phonons and impurities; $B_{\nu\nu'}^{\mu'\mu}$ is the matrix element of an electron-electron interaction; $I_{\nu'\nu}^{-\mathbf{q}} = \langle \nu' | e^{i\mathbf{q}\mathbf{r}} | \nu \rangle$; a_ν^+, a_ν and $b_{\mathbf{q}}^+$, $b_{\mathbf{q}}$ are the creation and annihilation operators of electrons and phonons, respectively.

The electric field **E** acting on carriers in the Hamiltonian (1) is the effective field including a Hall term, and depends on the boundary conditions. Since the applied electric field is orthogonal to the magnetic field, which is along the z axis, the assumed isotropy of the semiconductor makes the resultant field **E** lie in a plane that is perpendicular to **H**.

The equation of motion of the one-electron density matrix $f_{\nu\nu'} = \langle a_{\nu'}^+ a_\nu \rangle$ (where averaging has been performed over the full density matrix of the system) is

$$\left(i\hbar \frac{\partial}{\partial t} + \varepsilon_{\nu'} - \varepsilon_\nu \right) f_{\nu\nu'} = \sum_{\mu\mu'\rho\rho'} B_{\rho'\rho}^{\mu'\mu} (g_{\nu'\rho'\rho\mu}\delta_{\nu\mu'} - g_{\mu'\nu'\rho\mu}\delta_{\rho'\nu}$$
$$+ g_{\mu'\rho'\nu\mu}\delta_{\nu'\rho} - g_{\mu'\rho'\nu\rho}\delta_{\mu\nu'}) + \sum_{\mathbf{q}\mu} \{C_{\mathbf{q}} (I_{\nu\mu}{}^{\mathbf{q}} h_{\mu\nu'\mathbf{q}} - I_{\mu\nu'}{}^{\mathbf{q}} h_{\nu\mu\mathbf{q}})$$
$$+ C_{\mathbf{q}}^* (I_{\nu\mu}{}^{-\mathbf{q}} \overset{*}{h}_{\mu\nu'\mathbf{q}} - I_{\mu\nu'}^{-\mathbf{q}} \overset{*}{h}_{\nu\mu\mathbf{q}}) + G_{\mathbf{q}} (I_{\nu\mu}{}^{\mathbf{q}} f_{\mu\nu'} - I_{\mu\nu'}^{\mathbf{q}} f_{\nu\mu})\}$$
$$\sum_{\mu} \quad {}^{(i)} \qquad {}^{(i)} \tag{3}$$

where

$$g_{\nu\nu'\mu\mu'} = \langle a_\mu^+ a_{\mu'} a_\nu^+ a_{\nu'} \rangle, \quad h_{\nu\mu\mathbf{q}} = \langle a_\mu^+ a_\nu b_{\mathbf{q}} \rangle, \quad \overset{*}{h}_{\nu\mu\mathbf{q}} = \langle a_\mu^+ a_\nu b_{\mathbf{q}}^+ \rangle.$$

From the Hamiltonian (1) with approximations of the higher correlation functions by products of one-particle functions:

$$\langle a_{\nu'}^+ a_\nu a_{\mu'}^+ a_\mu \rangle \approx f_{\nu\nu'} f_{\mu\mu'} + f_{\mu\nu'}(\delta_{\nu\mu'} - f_{\nu\mu'}),$$
$$\langle a_{\nu'}^+ a_\nu b_{\mathbf{q}'}^+ b_{\mathbf{q}} \rangle \approx f_{\nu\nu'} N_{\mathbf{q}\mathbf{q}'},$$
$$\langle a_\nu^+ a_\mu^+ a_{\nu'} a_\gamma^+ a_{\mu'} a_{\gamma'} \rangle = f_{\gamma'\nu} f_{\nu'\mu} f_{\mu'\gamma} + f_{\nu'\nu} f_{\mu'\mu} f_{\gamma'\gamma}$$
$$+ f_{\mu'\nu} f_{\gamma'\mu} (\delta_{\nu'\gamma} - f_{\nu'\gamma})$$

we easily derive equations of motion for the g and h correlation functions. We present here only the equation for an h in the case of Maxwellian statistics for the electrons:

$$\left(i\hbar \frac{\partial}{\partial t} + \varepsilon_{\nu'} - \varepsilon_\nu - \hbar\omega_q \right) h_{\nu\nu'\mathbf{q}} = eE_i \sum_{\mu} (r_{\nu\mu}^{(i)} h_{\mu\nu'\mathbf{q}} - r_{\mu\nu'}^{(i)} h_{\nu\mu\mathbf{q}})$$
$$+ \sum_{\mu\mu'} C_{\mathbf{q}} I_{\mu\mu'}^{-\mathbf{q}} [(1 + N_{\mathbf{q}}) f_{\mu'\nu'} \delta_{\mu\nu} - N_{\mathbf{q}} f_{\nu\mu} \delta_{\mu'\nu'}].$$

In the stationary case we replace $\hbar\partial/\partial t$ with the adiabatic parameter γ; then in the linear approximation with respect to **E** we obtain

$$h_{\nu\nu'\mathbf{q}} = h_{\nu\nu'q}^{(0)}(f, N_{\mathbf{q}}) + eE_i h_{\nu\nu'\mathbf{q}}^{(i)}(f, N_{\mathbf{q}}),$$

where

$$h_{\nu\nu'\mathbf{q}}^{(0)} = \frac{1}{\varepsilon_{\nu'} - \varepsilon_\nu - \hbar\omega_{\mathbf{q}} + i\gamma} \sum_{\mu} C_{\mathbf{q}} \{ I_{\nu\mu}{}^{-\mathbf{q}} f_{\mu\nu'} (1 + N_{\mathbf{q}}) - f_{\nu\mu} I_{\mu\nu'}^{-\mathbf{q}} N_{\mathbf{q}} \},$$

$$h_{\nu\nu'\mathbf{q}}^{(i)} = \frac{1}{\varepsilon_{\nu'} - \varepsilon_\nu - \hbar\omega_{\mathbf{q}} + i\gamma} \sum_{\mu\mu'} C_{\mathbf{q}} \Bigg\{ \frac{r_{\nu\mu}^{(i)}}{\varepsilon_{\nu'} - \varepsilon_\mu - \hbar\omega_{\mathbf{q}} + i\gamma}$$
$$\times [I_{\mu\mu'}^{-\mathbf{q}} f_{\mu'\nu'} (1 + N_{\mathbf{q}}) - f_{\mu\mu'} I_{\mu'\nu'}^{-\mathbf{q}} N_{\mathbf{q}}]$$
$$- \frac{r_{\mu\nu'}^{(i)}}{\varepsilon_\mu - \varepsilon_\nu - h\omega_{\mathbf{q}} + i\gamma} [I_{\nu\mu'}^{-\mathbf{q}} f_{\mu'\mu} (1 + N_{\mathbf{q}}) - f_{\nu\mu'} I_{\mu'\mu}^{-\mathbf{q}} N_{\mathbf{q}}] \Bigg\}$$

The equation for a g is solved imilarly. Substituting for h and g in (3), we obtain a closed equation for the one-electron density matrix in the lowest approximation for the interactions:[1)]

$$\left(i\hbar \frac{\partial}{\partial t} + \varepsilon_{\nu'} - \varepsilon_\nu \right) f_{\nu\nu'} = I_{\nu\nu'}^{ee}(f) + I_{\nu\nu'}^{ep}(f) + I_{\nu\nu'}^{ei}(f) + I_{\nu\nu'}^{\mathrm{E}}(f). \tag{4}$$

The term $I_{\nu\nu'}^{\mathrm{E}}(f)$, which is linear with respect to the electric field **E**, contains zeroth and second order terms in the scattering potentials.

We seek a solution without diffusion currents, assuming that the electron distribution is spatially uniform. This means that elements of the f matrix which are diagonal with respect to the quantum number N are also diagonal with respect to p_z and are independent of the quantum number X, the x coordinate of the center of a Landau oscillator. From the properties of the $I_{\nu\nu'}^{\mathbf{q}}$ functions and the coordinate matrix elements it follows that the terms $f_{NN'}$ ($N \neq N'$) determined by (4) are independent of X and diagonal with respect to p_z.

The mean electric current is given by

$$j_i = \mathrm{Sp}(\hat{f}\hat{j}_i) = \sum_{\nu\nu'} f_{\nu\nu'} (\hat{j}_i)_{\nu'\nu};$$

[1)]For simplicity we assume that the phonon system is in equilibrium. In the absence of this approximation we would have to write an equation for the phonon distribution function.

$$\hat{j}_i = -\frac{e}{2}(\hat{v}_i\hat{N} + \hat{N}\hat{v}_i), \quad \hat{v}_i = \left(-i\hbar\frac{\partial}{\partial r_i} + \frac{e}{c}A_i\right)\frac{1}{m}, \qquad (5)$$

where $\hat{v}_i$ is the velocity operator and $\hat{N}$ is the density operator of the number of particles. Using the aforementioned properties of the electron density matrix, we obtain

$$j_x = \frac{e\hbar}{ml}\sum_{N,\,p_z,\,X}\sqrt{2(N+1)}\,\mathrm{Im}\,f_{N,\,N+1}(p_z),$$
$$j_y = -\frac{e\hbar}{ml}\sum_{N,\,p_z,X}\sqrt{2(N+1)}\,\mathrm{Re}\,f_{N,\,N+1}(p_z). \qquad (6)$$

The equation for the diagonal elements of the density matrix in the stationary case is

$$I_{\nu\nu}^{ee}(f) + I_{\nu\nu}^{ep}(f) = 0. \qquad (7)$$

In the right-hand side of (4) the $\nu = \nu'$ terms arising out of impurity scattering will vanish (the diagonal elements $f_{\nu\nu}$ are assumed to depend on the energy ϵ_ν); this means that elastic scattering on impurities does not affect the relaxation of electron energy. The zeroth-order scattering term in $I^{E}_{\nu\nu}(f)$ vanishes identically. The second-order phonon scattering term in $I^{E}_{\nu\nu}(f)$ can be neglected, since it contains an additional small factor of the order eEl/ϵ as compared with $I^{ep}_{\nu\nu}(f)$.

For the electron-electron collision integral in the quantizing magnetic field we use an expression, derived in [7], that takes into account the polarization of the medium and also quantum effects (it is not required that the de Broglie wavelength be small as compared with the characteristic lengths in the problem). In a spatially uniform medium, for which[7,8]

$$\varepsilon_{ij}(\omega, \mathbf{k}, \mathbf{k}') = \varepsilon_{ij}(\omega, \mathbf{k})\delta(\mathbf{k} - \mathbf{k}')$$

where $\epsilon_{ij}(\omega, k)$ is the complex dielectric constant tensor, the electron-electron collision integral is

$$I_{\nu\nu}^{ee}(f) = \frac{4\pi}{\hbar}\sum_{\nu'\mu\mu'}\left|4\pi e^2\int d\mathbf{k}\, I^{\mathbf{k}}_{\nu'\nu}I^{-\mathbf{k}}_{\mu'\mu}\left[k_ik_j\varepsilon_{ij}\left(\frac{\varepsilon_{\nu'}-\varepsilon_\nu}{\hbar}, \mathbf{k}\right)\right]^{-1}\right|^2$$
$$\times(f_{\nu'}f_{\mu'} - f_\nu f_\mu)\delta(\varepsilon_{\nu'} + \varepsilon_{\mu'} - \varepsilon_\nu - \varepsilon_\mu). \qquad (8)$$

Summing over momenta with the aid of δ functions, we obtain

$$I_{\nu\nu}^{ee}(f) = \frac{4\pi}{\hbar}\sum_{N_{\nu'}N_\mu N_{\mu'}k_zp_z^\mu}\frac{2m}{(2\pi l)^2}[f_{N_{\nu'}}(p_z^\nu + \hbar k_z)f_{N_{\mu'}}(p_z^\mu - \hbar k_z)$$
$$- f_{N_\nu}(p_z^\nu)f_{N_\mu}(p_z^\mu)]\,\delta(2m\hbar\Omega(N_{\nu'} + N_{\mu'} - N_\nu - N_\mu) + 2\hbar^2k_z^2$$
$$+ 2\hbar k_z(p_z^\nu - p_z^\mu))\int_0^\infty k_\perp dk_\perp\left|4\pi e^2[k^2\varepsilon\left(\frac{\varepsilon_{\nu'}-\varepsilon_\nu}{\hbar}, k_\perp, k_z\right)]^{-1}\right|^2\exp\{-l^2k_\perp^2\}$$
$$\times\frac{(\overline{N}_\nu!)^2(\overline{N}_\mu!)^2}{N_\nu!\,N_{\nu'}!\,N_{\mu'}!\,N_\mu!}\left(\frac{l^2k_\perp^2}{2}\right)^{|N_{\nu'}-N_\nu|+|N_{\mu'}-N_\mu|}$$
$$\times\left[L^{|N_\nu-N_{\nu'}|}_{\overline{N}_\nu}\left(\frac{l^2k_\perp^2}{2}\right)L^{|N_\mu-N_{\mu'}|}_{\overline{N}_\mu}\left(\frac{l^2k_\perp^2}{2}\right)\right]^2. \qquad (9)$$

Here $\overline{N}_\nu = \min(N_\nu, N_{\nu'})$, $L^m_n(t)$ is a generalized Laguerre polynomial, $k_\perp^2 = k_x^2 + k_y^2$, and $\epsilon(\omega, k_\perp, k_z)$ is the longitudinal dielectric constant defined by[8]

$$\varepsilon(\omega, k_\perp, k_z) = k^{-2}k_ik_j\varepsilon_{ij}(\omega, k).$$

In the case $\hbar\Omega \gg T$ only the lowest Landau levels are important. Let us consider electron-electron scattering of the principal carrier group in the $N_\nu = 0$ state. It is easily proved that the main contribution to the collision integral comes from terms with $N_\nu = N_{\nu'} = 0$ and $N_\mu = N_{\mu'} = 1$ in the summation. We note, secondly, that if all the Landau quantum numbers N_ρ are zero, scattering is ineffective (the collision integral vanishes identically). This results from the fact that the scattering is essentially one-dimensional, and that a scattering event does not change the state of a system consisting of two colliding particles. The situation is similar for $N_\nu = N_{\mu'} = 0$, $N_{\nu'} = N_\mu = 1$. In this case we again have $p_z^\mu = p_z^\nu + \hbar k_z$, and the equation $f_1(p_z^\nu + \hbar k_z) \times f_0(p_z^\mu - \hbar k_z) = f_0(p_z^\nu)f_1(p_z^\mu)$ also causes vanishing of the collision integral without dependence on the form of the electron distribution function. Secondly, all values of the quantum numbers besides $N_\nu = N_{\nu'} = 0$, $N_\mu = N_{\mu'} = 1$ make an exponentially small contribution either because of large N (when $N_{\nu'} + N_{\mu'} - N_\mu - N_\nu = 0$ and $p_z \sim \sqrt{2mT_e}$) or because of the large value of k_z that is needed to elevate a particle to a higher Landau level (when $N_{\nu'} + N_{\mu'} - N_\nu - N_\mu \neq 0$). Retaining only the main term and summing over p_z^μ with the aid of a δ function, we obtain

$$I_{\nu\nu}^{ee}(f) = \frac{2m}{(2\pi\hbar)^3l^2}\int_{-\infty}^{+\infty}\frac{dk_z}{|k_z|}[f_0(p_z + \hbar k_z)f_1(p_z) - f_0(p_z)f_1(p_z + \hbar k_z)]$$
$$\times\int_0^\infty k_\perp dk_\perp\left|4\pi e^2\left(k^2\varepsilon\left(\frac{\varepsilon_{\nu'}-\varepsilon_\nu}{\hbar}, k_z, k_\perp\right)\right)^{-1}\right|^2$$
$$\times\exp\{-(lk_\perp)^2/2\}\left[L_1\left(\frac{l^2k_\perp^2}{2}\right)\right]^2. \qquad (10)$$

Since the transverse wave vectors $k_\perp > l^{-1}$ do not contribute to (10), calculations can be performed using (A.4) of the Appendix [neglecting the dependence of the dielectric constant on $(\epsilon_{\nu'} - \epsilon_\nu)/\hbar$]. Because of the long-range scattering potential, electron collisions can occur involving large impact parameters, and it can be assumed that the longitudinal component of the momentum will undergo only a small change as a result of scattering. Expanding (10) to the second order in $\hbar k_z$ (inclusively) and integrating, we obtain

$$I_{\nu\nu}^{ee}(f) = \frac{(-1)me^4}{4\pi\hbar l^2}\{(2+\varkappa)^2e^\varkappa\,\mathrm{Ei}(-\varkappa) + \varkappa + 3\}\frac{d}{dp_z}\left(f_1\frac{df_0}{dp_z} - f_0\frac{df_1}{dp_z}\right),$$
$$\varkappa = (lk_D)^2. \qquad (11)$$

If the Debye radius is large enough to give $\kappa \ll 1$, we have

$$I_{\nu\nu}^{ee}(f) \approx \frac{me^4}{\pi\hbar l^2}\ln\left(\frac{1}{\gamma_0\varkappa}\right)\frac{d}{dp_z}\left(f_1\frac{df_0}{dp_z} - f_0\frac{df_1}{dp_z}\right) \qquad (12)$$

($\gamma_0 = 1.781$ is Euler's constant).

We now proceed to calculate the integral of electron collisions with acoustic phonons. In the lowest approximation with respect to $(\Omega\tau)^{-1}$ this integral is given by

$$I_{\nu\nu}^{ep}(f) = \frac{2\pi}{\hbar}\sum_{\mathbf{q},\,\nu'}C_q^2\{|I^{-\mathbf{q}}_{\nu'\nu}|^2[N_qf_{\nu'} - (1+N_q)f_\nu]\delta(\varepsilon_{\nu'} - \varepsilon_\nu + \hbar\omega_q)$$
$$+ |I^{\mathbf{q}}_{\nu'\nu}|^2[(1+N_q)f_{\nu'} - N_qf_\nu]\delta(\varepsilon_{\nu'} - \varepsilon_\nu - \hbar\omega_q)\}, \qquad (13)$$

where $C_q^2 = C_0^2\hbar q/2\rho s$ (C_0 is the constant of the deformation potential and ρ is the density); an equilibrium distribution of the phonons N_q is assumed.

For not very low lattice temperatures $(ms^2\hbar\Omega)^{1/2} \ll T$, Eq. (13) can be transformed into

$$I_{\nu\nu}^{ep}(f) = \frac{s^2}{l_{ep}^3T}2m^2\hbar\Omega\frac{\varepsilon_{N,\,p_z}}{p_z}\sum_{N'}[p_z^2 - 2m\hbar\Omega(N' - N)]^{-1/2}$$

$$\times \frac{\partial}{\partial p_z}\left[\frac{1}{p_z^2 - 2m\hbar\Omega(N'-N)}\left(1+\frac{mT}{p_z}\frac{\partial}{\partial p_z}\right) f_{N,p_z}\right], \qquad (14)$$

where $l_{ep} = \pi\rho\hbar^4 s^2 (C_0^2 m^2 T)^{-1}$ is the mean free path of electrons scattered on phonons in the absence of a magnetic field.

In the quantum limit, Eq. (7) for the diagonal elements of the density matrix, using (12) and (14) in dimensionless variables, becomes

$$\beta_e \frac{d}{dx}\left(\psi_1(x)\frac{d\psi_0(x)}{dx} - \psi_0(x)\frac{d\psi_1(x)}{dx}\right) + \beta_p \frac{d}{dx}\left(\frac{1}{x^2}\left[1+\frac{T}{2T_e}\frac{d}{x\,dx}\right]\psi_0(x)\right) = 0, \qquad (15)$$

where $x = p_z/\sqrt{2mT_e}$,

$$\beta_e = \frac{2\pi m n_1 e^4}{(2mT_e)^{3/2}}\ln\left(\frac{r_D^2}{\gamma_0 l^2}\right), \quad \beta_p = \frac{s^2}{l_{ep}T}(2mT_e)^{1/2}\left(\frac{\hbar\Omega}{2T_e}\right)^2;$$
$$\psi_N(x) = \frac{2\sqrt{2mT_e}}{(2\pi l)^2\hbar}\frac{1}{n_N} f_N(p_z), \qquad (16)$$

$n_N = n_0 e^{-\hbar\Omega N/T_e}$, n_0 is the electron concentration, and

$$\sum_{N=0}^{\infty}\int_{-\infty}^{+\infty} dx\,\psi_N(x) = 1.$$

The second term of (15) is independent of the electron concentration, and for a sufficiently large concentration $n_0 > n_{cr}$ the electron-electron collisions dominate, determining the diagonal elements of the density matrix. In this case the solution of (15) is

$$\psi_N = C\exp(-x^2 - \hbar\Omega N/T_e), \qquad (17)$$

and the formally introduced parameter T_e has the significance of an effective electron temperature. The lowest concentration n_{cr} for which this solution is valid is determined from the condition

$$n_{cr} = n_{cr}^{cl.}\left(\frac{\hbar\Omega}{T_e}\right)^2 \frac{\exp(\hbar\Omega/T_e)}{\ln(r_D^2/\gamma_0 l^2)}\ln\left(\frac{r_D^2}{b^2}\right), \qquad (18)$$

where $n_{cr}^{cl} = ms^2T_e^2[4\pi e^4 l_{ep} T \ln(r_D/b)]^{-1}$ is the lowest critical electron concentration in the classical case when no quantizing magnetic field is present;[2,3] $b \sim e^2/T$ is the impact parameter at which the kinetic energy of the colliding particles becomes comparable with their mutual interaction energy.

It follows from (18) that the efficiency of electron-electron scattering is greatly reduced in the quantum limit $\hbar\Omega \gg T, T_e$ as compared with the classical case.

3. We shall assume that the electron concentration n exceeds the value n_{cr} represented by (18). Then the diagonal element of the density matrix is given by (17). Electron-electron collisions now drop out of (4) for off-diagonal elements of the density matrix ($\tilde{f}_{\nu\nu'}$) that are much smaller than the diagonal elements represented by (17). The remaining equations for $\tilde{f}_{\nu\nu'}$) can be integrated by iterations with respect to the small parameter $(\Omega\tau)^{-1}$.

Of interest to us are the elements $f_{\nu\nu'}$, linear in the parameter $eEl/\bar{\epsilon}$, to the second order in the constants of interactions with scatterers. The zeroth-order term in $(\Omega\tau)^{-1}$ describes nondissipative currents; however, because of the nonequilibrium diagonal elements of the density matrix, this term contains an implicit dependence (through the electron temperature) on the relaxation frequencies. The first-order terms of $f_{\nu\nu'}$ with respect to $(\Omega\tau)^{-1}$ contain the needed statistical information regarding dissipative currents associated with impurity and phonon scattering ($\sim|G_q|^2$ and $\sim|C_q|^2$, respectively). As the result of our calculations the dissipative component σ_{xx} of the electrical conductivity is represented by

$$\sigma_{xx} = \frac{2\pi}{\hbar}e^2\sum_{q\nu\mu}|I_{\nu\mu}^{q}|^2\left(-\frac{\partial f_\nu(T_e)}{\partial\varepsilon_\nu}\right)(l^2 q_y)^2 \times\left\{|G_q|^2\delta(\varepsilon_\nu-\varepsilon_\mu)\frac{N_d}{2}+|C_q|^2[1+N_q(T)] \times\left[1+\frac{p_z^\mu}{\hbar q_z}\left(1-\exp\left\{\hbar\omega_q\left(\frac{1}{T_e}-\frac{1}{T}\right)\right\}\right)\right]\delta(\varepsilon_\nu-\varepsilon_\mu-\hbar\omega_q)\right\}. \qquad (19)$$

This result agrees with Titeica's equation[6] only if we neglect terms that are proportional to ξ (see Section 1) when this parameter is much smaller than unity.

The quantity σ_{xx} represented by (19) is an implicit function of the electric field **E** because the effective electron temperature is dependent on E. This dependence $T_e(E)$ is usually derived from the energy balance equation by equating the Joule power $W = \mathbf{jE}$ to the energy P that is transferred per unit time by electrons to the lattice as a heat reservoir. The Joule power W depends on the boundary conditions. In the absence of a Hall current we have $W = \sigma_{xx}^{-1}(\sigma_{xx}^2 + \sigma_{xy}^2)E^2$; in the absence of a Hall field, $W = \sigma_{xx}E^2$. We determine P by means of the electron-phonon collision integral

$$P = \frac{2\pi}{\hbar}\sum_{\nu\nu'q}|C_q|^2|I_{\nu'\nu}^{q}|^2\hbar\omega_q[(1+N_q)f_{\nu'} - N_q f_\nu]\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q). \qquad (20)$$

We must emphasize that the dependence of the effective electron temperature T_e on external fields in the presence of a Hall field (but in the absence of a Hall current) is represented by an expression differing from that given in [9]. For weak heating (the electron momentum relaxation is associated with impurities, and the energy relaxation is associated with acoustic phonons), it is shown in [5] that

$$T_e \approx T\left[1+2\left(\frac{cE}{sH}\right)^2\frac{\Omega^2}{\nu_i(T,H)\nu_{ac}(T,H)}\right], \qquad (21)$$

where ν_i and ν_{ac} are the frequencies of momentum relaxation associated with impurities and acoustic phonons, respectively. When the electric field reaches the value

$$E_{cr} = \frac{sm}{e}[{}^2/_{27}\nu_i(T,H)\nu_{ac}(T,H)]^{1/2}, \qquad (22)$$

the system becomes thermally unstable. The excitation of phonons can suppress this instability[5] (in either crossed or parallel fields).

We note in conclusion that the concept of an effective electronic temperature has a wider range of applicability in a quantizing magnetic field than in the classical (nonquantum) case. In [9] the basis of the effective temperature concept did not include electron-electron collisions. It has been shown in the present work that the concept can also be introduced at high electron concentrations when electron-electron collisions are dominant over electron-phonon collisions. It is easily shown that the equation

$$I_{\nu\nu}^{ei}(f) + I_{\nu\nu}^{ep}(f) + I_{\nu\nu}^{ee}(f) = 0,$$

in the case of small ξ has the approximate solution

$f_{vv} \sim \exp(-\varepsilon_v / T_e)$, $N_q = [\exp(\hbar\omega_q / T) - 1]^{-1}$.

APPENDIX

The dielectric constant of a spatially uniform system in a quantizing magnetic field can be represented by

$$\varepsilon(\omega, k) = 1 - \frac{4\pi e^2}{k^2} \frac{2}{(2\pi l)^2 \hbar} \sum_{NN'} \int dp_z \times \frac{f_{N'}(p_z + \hbar k_z) - f_N(p_z)}{\varepsilon_{N'}(p_z + \hbar k_z) - \varepsilon_N(p_z) - \hbar\omega} \frac{(\overline{N}!)^2}{N!N'!} \left(\frac{lk_\perp}{\sqrt{2}}\right)^{2|N'-N|} \times \exp\left(-\frac{l^2 k_\perp^2}{2}\right) \left[L_{\overline{N}}^{|N'-N|}\left(\frac{l^2 k_\perp^2}{2}\right)\right]^2, \quad \text{(A.1)}$$

where $\overline{N} = \min(N, N')$.

We shall henceforth neglect frequency dispersion, assuming that the external fields vary slowly with time. Let us consider the case of strongly quantized cyclotron orbits ($\hbar\Omega \gg T$). Omitting exponentially small terms in (A.1), we obtain

$$\varepsilon(k) \approx 1 - \frac{4\pi e^2}{k^2} \frac{2}{(2\pi l)^2 \hbar} \sum_{N=0}^{\infty} \int dp_z \frac{1}{N!} \left(\frac{lk_\perp}{\sqrt{2}}\right)^{2N} \times \exp\left(-\frac{l^2 k_\perp^2}{2}\right) \left[\frac{f_0(p_z + \hbar k_z)}{(p_z + \hbar k_z)^2/2m - p_z^2/2m - N\hbar\Omega} - \frac{f_0(p_z)}{(p_z + \hbar k_z)^2/2m - p_z^2/2m + N\hbar\Omega}\right]. \quad \text{(A.2)}$$

The long-range character of Coulomb fields permits us to assume $\hbar k_z \ll p_z$, i.e., in the majority of collisions the change in the longitudinal component of the momentum is much smaller than p_z itself. We expand to the second power of $\hbar k_z$ inclusively and obtain, to the second order in $T/\hbar\Omega$:

$$\varepsilon(k) \approx 1 + \frac{4\pi e^2 n_0}{k^2 T} \exp\left(-\frac{l^2 k_\perp^2}{2}\right) \left[1 + 2\frac{T}{\hbar\Omega} \sum_{N=1}^{\infty} \frac{1}{N!N} \left(\frac{lk_\perp}{\sqrt{2}}\right)^{2N}\right] = 1 + \frac{k_D^2}{k^2} e^{-x} \left\{1 + 2\frac{T}{\hbar\Omega}[\mathrm{Ei}^*(x) - \ln x - C]\right\}, \quad \text{(A.3)}$$

where

$$\mathrm{Ei}^*(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt \quad (x > 0);$$

$C = 0.577$ is Euler's constant; $x = (lk_\perp/\sqrt{2})^2$; k_D^{-1} is the Debye radius.

It follows from (A.3) that in a quantizing magnetic field the dielectric constant is essentially anisotropic. If $k_\perp \ll l^{-1}$, then $\mathrm{Ei}^*(x) \approx \ln(x) + C$ and

$$\varepsilon(k) \approx 1 + k_D^2 / k^2. \quad \text{(A.4)}$$

If $k_\perp \gg l^{-1}$, then $\mathrm{Ei}^*(x) \approx x^{-1} e^x$ and

$$\varepsilon(k) \approx 1 + \frac{k_D^2}{k^2} \frac{4mT}{\hbar^2 k_\perp^2}. \quad \text{(A.5)}$$

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