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BAND TAILING AND ABSORPTION OF LIGHT IN SEMICONDUCTORS

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The density of states arising in the forbidden band of a strongly doped semiconductor as a result of charged-impurity concentration fluctuations is considered. It is shown that sufficiently deep in the forbidden band the state density is determined by clusters of attracting impurities; these clusters form multicharged nuclei of atoms whose ground hydrogen-like states realize the energy required. The state density decreases with increasing tail depth like $\ln\rho(\epsilon) \sim \sqrt{\epsilon}\ln\epsilon$ for random distribution of the impurities and like $\ln\rho(\epsilon) \sim -\epsilon^{3/2}$ if a correlation exists in the impurity distribution due to mutual repulsion during preparation of the sample. It is shown that the frequency dependence of the interband light absorption coefficient in the deep tail reproduces the state density.

1. INTRODUCTION

IN strongly doped semiconductors at low temperatures, there exists intraband absorption of light at frequencies smaller than the width of the forbidden band. This absorption is connected with the electronic states that arise as a result of fluctuations of the concentration of the charged impurities.

We shall show that at a sufficient depth in the forbidden band, the density of states is determined by clusters of attracting impurities, producing multiply charged nuclei of atoms whose ground hydrogenlike states realize the required energy. The Bohr radius of such a state turns out to be large compared with the optimal dimension of the nucleus. It is important that the dimension of the cluster and the Bohr radius are small compared with the screening radius, so that the nucleus can be assumed to have a Coulomb potential. (Screening in strongly doped semiconductors is effected by electrons located near the Fermi surface.) The principal term of the logarithm of the density of states $\rho(\epsilon)$ then takes the form

$$\ln\frac{\rho(\varepsilon)}{\rho(0)} = -\sqrt{\frac{\varepsilon}{E_B}}\ln\left[\left(\frac{\varepsilon}{E_B}\right)^2\frac{D(\varepsilon)}{Na^3}\right], \tag{1}$$

where ϵ is the energy reckoned downward from the bottom of the conduction band, $E_B = \hbar^2/2ma^2$ is the ionization energy of the isolated singly-charged impurity atom, $a = \hbar^2\kappa/me^2$ is the Bohr radius of this atom, κ is the dielectric constant, m is the effective mass, N is the concentration of the attracting impurities, and $D(\epsilon)$ is a dimensionless quantity that depends logarithmically on the energy.

In an earlier paper[1] we presented a qualitative derivation of (1), where it was assumed that the fluctuations are uniformly charged spheres. This method makes it possible to determine the factor preceding the logarithm in (1), but does not make it possible to determine the function $D(\epsilon)$. We show in the present paper that the axial distribution of the impurities in the nucleus is Gaussian.

Formula (1) is essentially connected with the proposed random distribution of the impurities. In semiconductors, however, this assumption is not always satisfied. Usually the samples are made of an alloy or are subjected to high-temperature treatment. The diffusion coefficient of the impurities depends exponentially on the temperature, decreasing rapidly at a certain temperature T_0 ($T_0 = 900°\mathrm{K}$ for Cd impurities in CaAs). At low temperatures, therefore, the spatial correlation of the impurities can be taken into account by assuming it to be the same as in an equilibrium gas with temperature T_0. Naturally, the probability of production of the nuclei that lead to the density of states (1) at sufficiently high energies ϵ will be greatly reduced by the Coulomb repulsion of the impurities making up the nucleus (this effect was discussed in [2]). We shall show that when

$$\varepsilon > T_0 L, \quad L = \ln\left[\left(\frac{T_0}{E_B}\right)^2\frac{D(T_0)}{Na^3}\right] \tag{2}$$

the correlation in the arrangement of the impurities causes the dimension of the nucleus to become of the order of the corresponding Bohr radius, and (1) is replaced by

$$\ln\frac{\rho(\varepsilon)}{\rho(0)} = -C\frac{\varepsilon^{3/2}}{E_B^{1/2}T_0}, \tag{1'}$$

where C is a number determined by solving the dimensionless equation (26). The derivation of formulas (1) and (1′) is obtained within the framework of the effective-mass method. Their validity is therefore limited to energies that are small compared with the width of the forbidden band. We shall prove in Sec. 5 that the frequency dependence of the coefficient of interband absorption of light at frequencies below the Berstein-Moss threshold reproduces directly the density of states (1) and (1′).

2. PRINCIPAL EQUATIONS

Let us consider an electron in the field of attracting disordered impurities whose potential is of the Coulomb type at small distances. It will be clear from the following that the interaction between the electrons does not influence the results. Our theory will be based on the method proposed by I. Lifshitz.[3]

We are interested in energies $\epsilon \gg E_B$. Such levels can be obtained only as a result of clusters of a large number of impurities. Obviously, the wave function of the electron in the ground state changes little over lengths on the order of the distance between the impurities and the clusters. Therefore the fluctuation can be characterized by the macroscopic concentration of the excess impurities $\xi(\mathbf{r})$. The probability of the fluctuations $\xi(\mathbf{r})$ is $\exp(-\Omega\{\xi\})$, where in accordance with the minimum-work principle, the functional $\Omega\{\xi\}$ is given by

$$\Omega\{\xi\} = \frac{e^2}{2T_0\varkappa}\int\frac{\xi(\mathbf{r})\,\xi(\mathbf{r}')\,d^3r\,d^3r'}{|\mathbf{r}-\mathbf{r}'|} - \int[\sigma(N+\xi)-\sigma(N)]\,d^3r, \tag{3}$$

where $\sigma(n) = -n[\ln(n/N) - 1]$. (We assume that the distances between impurities in the clusters are large compared with the lattice constant.) The first term in (3) describes the repulsion of the impurities, and, in accordance with the statements made in the introduction, T_0 is the temperature at which the impurity diffusion becomes quenched. The second term in (3) is combinatorial. In accordance with [3], we have for the principal term of the logarithm of the density of states

$$\ln\frac{\rho(\varepsilon)}{\rho(0)} = -\Omega\{\xi\}. \tag{4}$$

Here $\tilde{\tilde{\xi}}(\mathbf{r})$ is a function minimizing the functional (3) subject to the additional condition $\lambda_0\{\xi\} = -\epsilon$, where $\lambda_0\{\xi\}$ is the energy of the ground state, determined by the Schrödinger equation for the electron in the field of the fluctuation $\xi(\mathbf{r})$. From the equation $\delta(\Omega\{\xi\} + \beta\lambda_0\{\xi\}) = 0$, recognizing that

$$\delta\lambda_0\{\xi\} = -\frac{e^2}{\varkappa}\int|\psi(\mathbf{r}')|^2\,d^3r'\int\frac{\delta\xi(\mathbf{r}'')}{|\mathbf{r}'-\mathbf{r}''|}\,d^3r'', \tag{5}$$

we get

$$\ln\frac{N+\tilde{\tilde{\xi}}}{N} + \frac{e^2}{T_0\varkappa}\int\frac{\tilde{\tilde{\xi}}(\mathbf{r}')\,d^3r'}{|\mathbf{r}-\mathbf{r}'|} - \frac{\beta e^2}{\varkappa}\int\frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\,d^3r' = 0, \tag{6}$$

where β is an undetermined coefficient. Adding to (6) the Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\psi + \left(\varepsilon - \frac{e^2}{\varkappa}\int\frac{\tilde{\tilde{\xi}}(\mathbf{r}')\,d^3r'}{|\mathbf{r}-\mathbf{r}'|}\right)\psi = 0, \tag{7}$$

we obtain a system of two equations for the determination of $\tilde{\tilde{\xi}}$ and ψ. We have left out from (7) the potential produced by the average distribution of impurities. If the neutrality condition is satisfied in the semiconductor, then this potential is compensated by the potential of the electrons and of the impurities of opposite sign. At high energies, when formulas (1) and (1′) are valid, an important role is played by fluctuations whose dimension is small compared with the screening radius (both at T_0 and at lower temperatures). We therefore assume a Coulomb potential in (3) and (5)–(7). An important factor in what follows, however, is that the potential decreases rapidly at large distances. If we discard the second term of (6), which takes into account the correlation, we obtain Eq. (2.31) of [3]. However, the investigation of the equations in [3] was based on the assumption that the potential of the fluctuations is locally connected with the concentration $\xi(\mathbf{r})$. It was also indicated in [3] that at sufficiently high energies ϵ, the fluctuations of small dimension become significant, and this assumption is not satisfied for such fluctuations. These are precisely the fluctuations which we shall investigate.

3. RANDOM DISTRIBUTION OF IMPURITIES

If the temperature T_0 is high enough (an exact criterion will be indicated below), then the second term of Eq. (5) can be neglected. We then get from (6)

$$\tilde{\tilde{\xi}}(\mathbf{r}) = N(e^{\chi(\mathbf{r})} - 1), \tag{8}$$

where

$$\chi(\mathbf{r}) = \frac{\beta e^2}{\varkappa}\int\frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\,d^3r'. \tag{9}$$

We shall show that $\tilde{\xi}(\mathbf{r})$ has a maximum at $r = 0$ and decreases rapidly at distances $\tilde{\rho}$ that are small compared with the electron wavelength $\hbar/(2m\epsilon)^{1/2}$. At distances larger than the wavelength but smaller than the screening radius r_0,

$$\tilde{\tilde{\xi}}(r) = N\beta e^2/\varkappa r \quad (r < r_0). \tag{10}$$

We shall show subsequently that the potential produced by this part of the charge has little effect on the position of the level.

We introduce the quantity

$$Z = \int\limits_{|r|<R}\tilde{\tilde{\xi}}(\mathbf{r})\,d^3r, \tag{11}$$

with R satisfying the dual inequality $\tilde{\rho} \ll R \ll r_\epsilon$, where $r_\epsilon = \hbar/(2m\epsilon)^{1/2}$. Neglecting in (7) the contribution made to the potential by the large distances, we obtain an equation for the electron in the field of an almost pointlike nucleus with charge Z. We need the wave function in the zeroth approximation, and the eigenvalue in the first approximation in $(\tilde{\rho}/r_e)^2$, i.e., with a correction for the fact that the nucleus is not pointlike,

$$\psi(r) = \frac{1}{\sqrt{\pi}\,r_\varepsilon^{3/2}}e^{-r/r_\varepsilon} \tag{12}$$

and

$$Z = \sqrt{\frac{\varepsilon}{E_B}}\left(1 + \frac{2}{3}\frac{\langle r^2\rangle}{r_\varepsilon^2}\right), \tag{13}$$

where

$$\langle r^2\rangle = \frac{1}{Z}\int\limits_{|r|<R}\tilde{\xi}(\mathbf{r})\,r^2\,d^3r. \tag{14}$$

Substituting (12) in (9) and expanding in r/r_e, we obtain

$$\chi(r) = t - r^2/\tilde{\rho}^2,$$

where

$$t = \beta e^2/r_\varepsilon\varkappa, \quad \tilde{\rho}^2 = {}^3/_2 r_\varepsilon^2/t. \tag{15}$$

We see that the assumption made above, namely that $\tilde{\xi}(\mathbf{r})$ decreases sharply at small distances, is equivalent to the assumption $t \gg 1$. If indeed $t \gg 1$, then at small distances

$$\tilde{\tilde{\xi}}(r) = Ne^t\exp\{-r^2/\tilde{\rho}^2\}. \tag{16}$$

Substituting (15) and (13) in (11) and extending the integration to infinity with accuracy to terms of order t^{-1}, we obtain a transcendental equation for t

$$t = \ln\left[\left(\frac{\varepsilon}{E_B}\right)^2 \frac{t^{3/2}}{Na^3}\left(\frac{2}{3\pi}\right)^{3/2}\right] . \tag{17}$$

From this, at sufficiently high energies,

$$t \approx \ln\left[\left(\frac{\varepsilon}{E_B}\right)^2 \frac{1}{Na^3}\right] \gg 1, \tag{18}$$

which justifies the assumption made. Thus, according to (16), the distribution of the impurities in the optimal fluctuation of $\tilde{\xi}$ is spherically symmetrical, the concentration at the center is large compared with average value, and decreases in Gaussian fashion at a distance that is small compared with the Bohr radius of the cluster.

We now calculate $\Omega\{\tilde{\tilde{\xi}}\}$. Recognizing that $\tilde{\tilde{\xi}} \gg N$, discarding the first term in (3) and substituting (16), we obtain

$$\Omega\{\tilde{\xi}\} = \int \tilde{\xi} \ln \frac{\tilde{\xi}}{Ne} d^3r = Z(t-1) - \frac{3}{2} Z. \tag{19}$$

Substituting (16) and (14) in (13), we get

$$Z = \sqrt{\frac{\varepsilon}{E_B}}\left(1 + \frac{3}{2t}\right). \tag{20}$$

We then find from (20) and (19), taking into account terms of order $t\sqrt{\epsilon/E_B}$ and $\sqrt{\epsilon/E_B}$,

$$\Omega\{\tilde{\xi}\} = \sqrt{\varepsilon / E_B}(t-1). \tag{21}$$

Substitution of (21) in (4) leads to formula (1), with

$$D(\varepsilon) = \left(\frac{2}{3\pi}\right)^{3/2} \frac{t^{3/2}}{e}, \tag{22}$$

and t is determined by Eq. (17).

Let us estimate the potential at the point r_ϵ produced by a charge located at distances larger than r_ϵ from the fluctuation center. Using formulas (10) and (15), we obtain

$$\frac{e^2}{\varkappa} \int \frac{\tilde{\xi}(r')}{r'} d^3r' \approx t \frac{e^2}{\varkappa} N r_\varepsilon r_0, \tag{23}$$
$$r_\varepsilon < |r'| < r_0.$$

The theory developed above is valid if the right side of (23) is small compared with ϵ. To this end it is necessary to have

$$\varepsilon > E_{\mathrm{cl}} = E_B (a / r_0)^{4/3} (N r_0^3)^{2/3} t(E_{\mathrm{cl}}).$$

In addition, of course, it is necessary to satisfy the condition $r_\epsilon < r_0$, i.e., $\epsilon > \hbar^2/mr_0^2$. These two conditions suffice also to make the expression under the logarithm sign in (18) large. The low-energy region, when at least one of these inequalities is violated, is treated in [1].

Formula (1) was derived above in the effective-mass and isotropic-spectrum approximation. We note, however, that within the framework of the effective-mass method, the factor in front of the logarithm in (1) has the same form regardless of the spectrum (e.g., degenerate bands are admissible). Indeed, the form of this factor is a consequence of only the fact that $\epsilon \sim Z^2$. This is always satisfied when the Hamiltonian is a quadratic form of the momentum components. In the derivation of (1) it was also assumed that the distances between the impurities in the clusters are large compared with the lattice constant. With the aid of (15) and (20) it is easy to verify that the energies at which this condition is violated are comparable with the width of the forbidden band.

The entire foregoing analysis was based on the single-electron approximation. It is easy to see, however, that the presence of other electrons does not change the results. The formation considered by us is a multi-electron atom, the lowest term of which realizes the required energy ϵ. It is known, however, that in the case of a large nuclear charge the outer electrons change the energy of this term little.

4. CORRELATED DISTRIBUTION OF THE IMPURITIES

If the temperature T_0 is small and the probability of the fluctuations is determined by the mutual repulsion of impurities, it is possible to neglect the first term in (6). Then

$$\psi(\mathbf{r}) = \left(\frac{1}{T_0\beta}\right)^{1/2} \tilde{\xi}^{1/2}. \tag{24}$$

(The wave function of the ground state has no nodes.) Substituting (24) in (7) and introducing the variables

$$\eta = \tilde{\xi}^{1/2} \frac{e\hbar}{\varepsilon\sqrt{2m\varkappa}}, \qquad \mathbf{x} = \frac{\mathbf{r}\sqrt{2m\varepsilon}}{\hbar}, \tag{25}$$

we obtain an equation without parameters

$$-\Delta\eta(\mathbf{x}) + \left(1 - \int \frac{\eta^2(\mathbf{x}')\, d^3x'}{|\mathbf{x} - \mathbf{x}'|}\right)\eta(\mathbf{x}) = 0 \tag{26}$$

with boundary condition $\eta \to 0$ as $x \to \infty$. In this case it is necessary to retain in formula (3) only the first term. Substituting (25) in it, we obtain

$$\Omega\{\tilde{\xi}\} = C\varepsilon^{3/2} / E_B^{1/2} T_0,$$

where

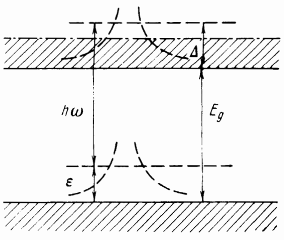
$$C = \frac{1}{4} \int \frac{\eta^2(\mathbf{x})\eta^2(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x\, d^3x'. \tag{27}$$

Substituting $\Omega\{\xi\}$ in (4) we get formula (1′).

Let us estimate now the energy at which formula (1) is replaced by (1′). To this end, we substitute the function ξ from (25) in the first and second terms of (6), and compare them. As a result we get that formula (1′) is valid when $\epsilon \gg T_0 L$, where L is defined in (2). At smaller values of ϵ, formula (1) holds.

5. ABSORPTION OF LIGHT

In this section we construct a theory of interband absorption of light at frequencies below threshold. We consider a degenerate strongly doped semiconductor (for concreteness, n-type) at zero temperature. The final state of the electron cannot have an energy below the Fermi energy μ (see the figure). Therefore the absorption coefficient begins to decrease exponentially



Scheme of transition upon absorption of a quantum hω. The solid lines show the boundaries of the bands; dash-dot – Fermi level, dash – potential produced by the fluctuations, and the energy of the initial and final states. The occupied states are shaded.

when the energy of the quantum $\hbar\omega$ becomes smaller than the energy $\mu + E_g$, which we shall call the threshold energy (the threshold for direct transitions is, as is well known, the energy $\mu(1 + m_e/m_h) + E_g$, where m_e and m_h are the masses of the electron and of the hole. However, indirect transitions within an initial state in the allowed band have no experimental small quantity).

An important role in the absorption is played only by the tails of the valence band, since the tails of the conduction band are filled. We assume that the semiconductor is compensated. Were it to contain only positively charged donors, then the spectrum of the holes would have an end point and would not extend deep into the forbidden band (for more details see [1]). Therefore the deep tails of the valence band are produced by the acceptors.

For absorption of a quantum with energy much lower than threshold, it is necessary to have a cluster of a large number of impurities, which can be described by a concentration $\xi(\mathbf{r})$. The contribution of such a cluster to the absorption coefficient $\alpha_\omega\{\xi\}$ is proportional to the product of the probability of occurrence of this cluster $W\{\xi\}$ by the probability of the quantum-mechanical transition $P\{\xi\}$. (In $P\{\xi\}$ we also take into account the filling of the final state.) The total absorption coefficient $\alpha(\omega)$ is a functional integral of $\alpha_\omega\{\xi\}$. We are interested in the principal term of $\ln\alpha(\omega)$.

Since the fluctuations are macroscopic, we can assume, in analogy with (4), that in a sufficiently deep tail

$$\ln\frac{\alpha(\omega)}{\alpha_0} = \ln\frac{\alpha_\omega\{\tilde{\xi}\}}{\alpha_0} = \ln W\{\tilde{\xi}\} + \ln P\{\tilde{\xi}\}, \tag{28}$$

where $\tilde{\xi}$ is a function realizing the maximum of the functional $\alpha_\omega\{\xi\}$ at a fixed transition frequency ω, and α_0 is the absorption coefficient at the threshold frequency.

Since $\ln W\{\xi\} = -\Omega\{\xi\}$, the problem formulated now differs from the problem concerning the density of states, solved in Secs. 2–4. Of course, the fluctuation, which is optimal for $\rho(\epsilon)$, generally speaking should not be optimal for $\alpha(\omega)$. The functional (28) could be varied in analogy with the functional (3). We, however, present a simpler method of solving this problem.

We vary the functional $\alpha_\omega\{\xi\}$, remaining in the class of functions that realize the maximum of the functional (3). These functions depend only on one parameter ϵ, which is the energy of the hole state, reckoned upward from the top of the valence band. At fixed frequency ω, the variation of ϵ is equivalent to a decrease of the electron energy in the final state $\Delta = \epsilon + \hbar\omega - E_g$ (see the figure). Of course, $\Delta \geq \mu$. We shall show that the quantity ϵ_{opt}, which realizes the maximum of $\alpha_\omega(\epsilon)$, is equal to

$$\varepsilon_{\text{opt}} \approx E_g + \mu - \hbar\omega, \quad |\ln P(\varepsilon_{\text{opt}})| \ll \Omega(\varepsilon_{\text{opt}}). \tag{29}$$

Then, if we confine ourselves only to such fluctuations

$$\ln\frac{\alpha(\omega)}{\alpha_0} = \ln\frac{\rho(E_g + \mu - \hbar\omega)}{\rho(0)}, \tag{30}$$

where $\rho(\epsilon)$ is given by (1) and (1′). We shall prove that (30) is the maximum value of the function $\alpha_\omega\{\xi\}$ relative to any class of functions. Contributions to the absorption are made only by the functions $\xi(r)$ that realize the hole states with energy $\epsilon \geq E_g + \mu - \hbar\omega$. Recognizing that $\ln P\{\tilde{\xi}\} < 0$ and the function $\rho(\epsilon)$ is monotonic, in accordance with the definition (4), we obtain

$$\ln\frac{\alpha_\omega\{\tilde{\xi}\}}{\alpha_0} \leqslant -\Omega\{\tilde{\xi}\} < \ln\frac{\rho(\varepsilon)}{\rho(0)} \leqslant \ln\frac{\rho(E_g + \mu - \hbar\omega)}{\rho(0)}. \tag{31}$$

Thus, it follows from (29) that the absorption coefficient is determined by (30).

Let us obtain now formulas (29). We consider the case when there is no correlation. As shown in Sec. 3, the fluctuations realizing the extremum of $\Omega\{\tilde{\xi}\}$ are almost pointlike nuclei of atoms with a charge $Z = (\epsilon/E_{Bh})^{1/2}$, where E_{Bh} is the Bohr energy of a singly-charged acceptor. We must find the maximum of the function

$$\ln\frac{\alpha_\omega(\varepsilon)}{\alpha_0} = -\Omega(\varepsilon) + \ln P(\varepsilon), \tag{32}$$

where $\Omega(\epsilon)$ is given by (21). As is well known, the ϵ-dependent factor in the transition probability is given by

$$P(\varepsilon) = \theta(\varepsilon + \hbar\omega - E_g - \mu)\left|\int \psi_h(r)\psi_e(r)\,d^3r\right|^2 \tag{33}$$

The wave function of the hole $\psi_h(r)$ is given by formula (12) with $r_\epsilon = \hbar/(2m_h\epsilon)^{1/2}$, and the wave function of the electron in the field of the fluctuation repelling it is given by the formula

$$\psi_e = \left[\frac{\delta}{\pi(e^{-2\pi/\delta} - 1)}\right]^{1/2} \exp\left\{\frac{ir\sqrt{2m_e\Delta}}{\hbar}\right\} \cdot F\left(\frac{i}{\delta} + 1,\ 2,\ -\frac{2ir}{\hbar}\sqrt{2m_e\Delta}\right), \tag{34}$$

where $\delta = (\Delta/\epsilon)^{1/2}(m_h/m_e)^{1/2}$, and F is the confluent hypergeometric function. The exponential smallness of the quantity P may result from the depletion of the wave function of the electron (34) in the region of hole localization. Investigating integral (33), we can readily verify that this smallness takes place only if the following inequalities are satisfied

$$\left(\frac{\hbar\omega + \varepsilon - E_g}{\varepsilon}\frac{m_h}{m_e}\right)^{1/2} < 1, \quad \left(\frac{\hbar\omega + \varepsilon - E_g}{\varepsilon}\frac{m_e}{m_h}\right)^{1/2} < 1. \tag{35}$$

Since the density of states of the holes in the forbidden band decreases rapidly, it is clear that if the probability of the transition with a final state on the Fermi surface is not small, then such an approach is optimal. Substituting $\hbar\omega + \epsilon - E_g = \mu$ in (35), we find that such a situation takes place when the frequency is close to the threshold value

$$E_g + \mu - \hbar\omega < \mu\max\left(\frac{m_h}{m_e}, \frac{m_e}{m_h}\right). \tag{36}$$

Obviously, in this case the conditions (29) are satisfied.

To investigate the region of frequencies lying much lower than the threshold, it is necessary to calculate the integral (33) under the condition (35). This yields

$$\ln\frac{\alpha_\omega(\varepsilon)}{\alpha_0} = -\frac{2\pi\sqrt{\varepsilon}}{(\hbar\omega - E_g + \varepsilon)^{1/2}}\left(\frac{m_e}{m_h}\right)^{1/2} - \sqrt{\frac{\varepsilon}{E_{Bh}}}(t - 1), \tag{37}$$
$$\varepsilon > E_g - \hbar\omega + \mu.$$

The maximum of (37) is reached when

$$\varepsilon_{\text{opt}} = \begin{cases} E_g - \hbar\omega + \Delta_0, & \Delta_0 > \mu, \\ E_g - \hbar\omega + \mu, & \Delta_0 \leqslant \mu, \end{cases} \tag{38}$$

where

$$\Delta_0=\left(\frac{2\pi}{t-1}\right)^{2/3}(E_g-\hbar\omega)^{2/3}(E_{Be})^{1/3}. \tag{39}$$

It is seen that at sufficiently large $E_g - \hbar\omega$ the optimal energy of the final state is $\Delta_0 > \mu$. However, in this case $\Delta_0 \ll E_g - \hbar\omega$. Therefore the first condition of (29) is satisfied at all frequencies. Substituting (38) and (39) in (37), we can easily verify that the second condition of (29) is satisfied.

An analogous proof of (29) can be obtained also for the case when the correlation in the locations of the impurities is significant. We obtain ultimately

$$\ln\frac{\alpha(\omega)}{\alpha_0}=-\left(\frac{E_g+\mu-\hbar\omega}{E_{Bh}}\right)^{1/2}\ln\left[\left(\frac{E_g+\mu-\hbar\omega}{E_{Bh}}\right)^2\frac{D(E_g+\mu-\hbar\omega)}{Na_h^3}\right],$$

$$E_g+\mu-\hbar\omega\ll T_0L_h; \tag{40}$$

$$\ln\frac{\alpha(\omega)}{\alpha_0}=-C\frac{(E_g+\mu-\hbar\omega)^{3/2}}{E_{Bh}^{1/2}T_0}, \tag{41}$$

$$E_g+\mu-\hbar\omega\gg T_0\ln\left[\left(\frac{T_0}{E_{Bh}}\right)^2\frac{D(T_0)}{Na_h^3}\right],$$

where N is the acceptor concentration.

The most complete experimental data on optical absorption at frequencies below the threshold have been obtained for GaAs.[4] The $\alpha(\omega)$ dependence is given by

$$\alpha(\omega)=\alpha_0\exp\left(-\frac{\omega_{\text{thr}}-\omega}{E_0}\right), \tag{42}$$

where ω_{thr} is the threshold frequency, and the characteristic energy E_0 depends on the impurity concentration. Estimates show that in the observed frequency region the correlation is insignificant and the function $\ln(\alpha(\omega)/\alpha_0)$, which follows from (40), is close to linear. Calculations by means of formula (40) for n-GaAs at reasonable assumptions concerning the degree of compensation (the ratio of the number of acceptors to the number of donors is of the order of 0.1) yield values of E_0 close to those observed (10–20 MeV). A more detailed comparison is difficult, since the degree of the compensation of the samples of [4] is not known.

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