OSCILLATIONS OF ULTRASOUND ABSORPTION BY A THIN PLATE IN A TRANSVERSE QUANTIZING MAGNETIC FIELD

B. A. TAVGER and V. A. MARGULIS

Institute of the Physics of Semiconductors, Siberian Branch, USSR Academy of Sciences, Mordovian State University

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Ultrasound absorption in a metallic film (thin plate) located in a transverse quantizing magnetic field is considered. The effect of the boundaries on the ultrasonic wave is taken into account. It is shown that giant absorption oscillations arise, just as in the case of a longitudinal field in a bulky sample.

1. INTRODUCTION. STATEMENT OF THE PROBLEM

IN the well-known research of Gurevich, Skobov and Firsov,[1] it was shown that giant oscillations in the absorption of ultrasound by metals takes place in a quantizing magnetic field, if the sound is propagated along the magnetic field. These oscillations arise upon change in the magnetic field and are connected with the fact that the absorption increases sharply if the projection of the Fermi velocity of the electron on the direction of the magnetic field becomes equal to the phase velocity c_l of the wave. In subsequent researches, the theory of the giant oscillations underwent further development (Skobov,[2] Zyryanov,[3] Pustovoit;[4] for details, see the review[5]).

Korolyuk and Prushchak[6] first discovered the giant oscillations in Zn. The oscillations in ultrasonic absorption have also been observed in Bi,[7] Ga,[8] Mg,[9] PbTe,[10] PbSe.[11] The essential feature of the observed oscillations is that they arise when the wave vector **q** is parallel or almost parallel to the magnetic field. If q is perpendicular to H, this effect is entirely absent.

In this research, ultrasound absorption is considered for metallic or semimetallic films placed in a transverse quantized magnetic field. It is assumed that the film thickness is of the order of the ultrasonic wavelength. This should lead to a significant effect of the boundaries on the ultrasound propagation. Thus the arrangement in this research will differ from[1] in two respects: 1. The ultrasound is propagated transverse to the magnetic field, along the film. 2. The ultrasound propagation depends significantly on the boundaries of the sample.

As will be shown, in spite of the mutual perpendicularity of **q** and $\mathscr{H}$, there will be giant oscillations in absorption in this case, too, owing to the effect of the boundaries. The transverse orientation of the film relative to the magnetic field is more convenient for experimental observation of the effect since it enables us to use more powerful fields.

Oscillations of a film (plate) with account of the boundary conditions for an arbitrary relation between the wavelength λ and the thickness L were studied in[12]. In that research, dispersion curves were obtained, the form of which was used for qualitative consideration and for estimates.

At a definite ultrasonic frequency, a number of modes are excited in the film, corresponding to different dispersion branches. The boundary conditions will be important when the number of excited branches is small. This occurs, as is seen from the graphs in[12], when $L/c_T \lesssim 20$–40. For the ultrasonic frequencies used, $\omega \sim 10^7$–10^9 Hz, $L \sim 10^{-1} - 10^{-3}$ cm. Thus the consideration applies to rather thick metallic or semimetallic films, essentially to thin plates. Inasmuch as the given thicknesses are much greater than the de Broglie wavelength even for the case of a semimetal, the boundedness of the plate is not essential for the electron states.

The boundedness of the plate also does not lead to the appearance of oscillations of the classical size effect, since the radius of the Larmor orbit is much less than L in the given case.

2. ELECTRON TRANSITION PROBABILITY

Let the magnetic field and the normal to the film be directed along the z axis. The direction of the propagation of the ultrasonic wave will be the x axis. We shall assume that the condition of a quantizing magnetic field

$$\hbar\omega_c \gg k_B T, \quad \omega_c\tau \gg 1 \tag{1}$$

(the cyclotron frequency $\omega_c = e\mathscr{H}/mc$) is satisfied. The wave functions and the spectrum of the electrons in the magnetic field without account of the perturbing effect of the ultrasonic wave have the form

$$\psi_{Mx_0k_z}(x, y, z) = \frac{1}{\sqrt{lL_y L_z}} \varphi_M\left(\frac{x - x_0}{l}\right) \exp\left[i(k_y y + k_z z)\right], \tag{2}$$

$$\varepsilon_{Mk_z} = (M + 1/2)\hbar\omega_c + \hbar^2 k_z^2 / 2m, \tag{3}$$

where M is the magnetic quantum number, x_0 the coordinate of the center of oscillations, $l = (\hbar/m\omega_c)^{1/2}$ is the magnetic length, and φ_M is the oscillator function.

The action of the ultrasonic field on the electron is determined by the perturbation operator

$$W = E_1 \operatorname{div} \mathbf{u}. \tag{4}$$

A contribution to the divergence is made only by that part of the displacement **u** which determines the compression (u_l), but u_T, which gives the shear, is unimportant here. In a plate with free boundaries, horizontal waves are possible, the displacement in which takes place along the y axis, and also the so-called vertical displaced waves, the displacement in which is located in the xz plane (see[12]). The first type of waves does not contribute to the divergence and for the second, u_l, can be written in the form

$$u_l = \operatorname{grad}\Phi, \quad \Phi = A \exp iqx \begin{cases} \operatorname{ch} \alpha z \\ \operatorname{sh} \alpha z \end{cases}, \tag{5}$$

where the hyperbolic cosine corresponds to symmetric and the hyperbolic sine to the antisymmetric oscillations, $\alpha^2 = q^2 - \omega^2/c_l^2$; z is measured from the center of the plate.

The quantity α plays in some sense the role of the transverse quasimomentum q_z. For values $q < \omega/c_l$, α is purely imaginary and the part of the displacement u_l, which gives the compression, represents a standing wave, for which, however, the wave number $\alpha_0 = i\alpha$ depends on q. It will be shown below that acoustical branches with small q are important in absorption. Therefore one can limit the discussion to the case in which α is purely imaginary.

The matrix element defining the electron transition from one state to another under the action of the sound wave is obtained if we use (2) and (4) in the form

$$|\langle M, x_0, k_z|\hat{W}|M', x_0', k_z'\rangle|^2 \sim \delta_{MM'}\delta_{x_0x_0'}\chi(k_z - k_z')F(ql). \tag{6}$$

For the computation of (6), it was assumed that $M = M'$ (this assumption will be established below) and we use the formula[13]

$$\int_{-\infty}^{\infty} e^{-x^2} H_M(x+y) H_M(x+z)\,dx = 2^M M! \sqrt{\pi}\, L_M(-2yz). \tag{7}$$

The integration with respect to z in getting (6) was carried out in the interval $-L/2$, $L/2$. The function $\chi(k_z - k_z')$ is equal to

$$\chi(k_z - k_z') = \frac{1}{[\alpha_0^2 - (k_z - k_z')^2]^2}\Big[(k_z - k_z')\sin\frac{L}{2}(k_z - k_z') \times\cos\frac{\alpha_0 L}{2} - \alpha_0 \sin\frac{\alpha_0 L}{2}\cos\frac{L}{2}(k_z - k_z')\Big]^2. \tag{8}$$

It expresses the approximate conservation law of the z-th projection of the quasimomentum in the case of a bulky sample; integration with respect to z between infinite limits would yield a δ-shaped function, expressing the conservation of k_z, in place of $\chi(k_z - k_z')$.

The maxima of $\chi(k_z - k_z')$ lie at the points $\pm\alpha_0$. The width of the region in which χ differs appreciably from zero is approximately equal to $\sim\pi/L$ for all branches. This is connected with the uncertainty principle. Thus, in the interaction of an electron with a wave, the most probable value of the transferred momentum is equal to $\pm\alpha_0$.

The dependence of the matrix element (6) on q is determined by the function $F(ql)$

$$F(ql) = \frac{\omega^4}{c_l^4}\exp\left(-\frac{q^2 l^2}{2}\right) L_M^2\left(\frac{l^2 q^2}{2}\right), \tag{9}$$

where L_M is the Laguerre polynomial. Upon increase in ql, this function falls off rapidly, since the magnetic quantum number M is not large. Therefore, in the interaction, the upper modes are the most important from among the perturbed acoustic branches. For them, $q < \omega/c_l$ and α is imaginary. Moreover, the oscillations with large ql are very "transverse" and give a smaller contribution to the absorption for this reason.[12]

We write down the energy conservation law for an electronic transition with absorption of a phonon

$$\hbar\omega_c(M + 1/2) + \frac{\hbar^2 k_z^2}{2m} + \hbar\omega = \hbar\omega_c(M' + 1/2) + \frac{\hbar^2 k_z'^2}{2m}, \tag{10}$$

where $\hbar\omega$ is the energy of the ultrasonic phonon.

Transitions for which $M \neq M'$, connected with a change in the magnetic energy by $\hbar\omega$, which cannot be compensated either by the phonon energy $\hbar\omega$ or by a change in the kinetic energy of motion of the electron along the field, because of its smallness. Therefore, only transitions without change in the magnetic quantum number are possible.

In the case of a bulky sample, $k_z = k_z'$ and Eq. (10) is not satisfied for any value of k_z, i.e., sound absorption cannot occur in the bulky sample in a quantizing magnetic field as the result of an electron transition. In the case of the plate $k_z \neq k_z'$. Even this circumstance should lead to the appearance of absorption.

By denoting the momentum change by Δ, we obtain the result that electrons with $k_z^0 \ll k_F$ take part in the absorption, since

$$k_z^0 = \frac{m}{\hbar}\frac{\omega}{\Delta}, \tag{11}$$

the quantities $\pm\Delta$ lie in the range $(\alpha_0 - \pi/2,\ \alpha_0 + \pi/2)$.

Because of the diffuseness of the value of Δ, k_z also has an uncertainty, equal to

$$\frac{m\omega}{\hbar}\frac{\pi}{L}\frac{1}{\alpha_0(\alpha_0 + \pi/L)}. \tag{12}$$

3. DEPENDENCE OF THE ABSORPTION ON THE VALUE OF THE MAGNETIC FIELD

The only electrons that can take part in the absorption are those in the interval $k_B T$ at the Fermi surface. Keeping in mind the dispersion law for electrons in a magnetic field, we obtain the following equation for the possible k_z:

$$\varepsilon_F \pm k_B T = \hbar\omega_c(M + 1/2) + \hbar^2 k_z^2/2m. \tag{13}$$

The obtained intervals of k_z are shown in the drawing. With increase of the magnetic field, the parabolas in the drawing are raised upward while the range of possible values of k_z are shifted to the left. Whenever any of these intervals intersects the region of variation of k_z^0, resonance absorption should appear. The distance between maxima on the curve of the dependence of the absorption on $\mathscr{H}$ is easily determined:

$$\frac{\Delta\mathscr{H}}{\mathscr{H}} = \frac{1}{M - 1/2}$$

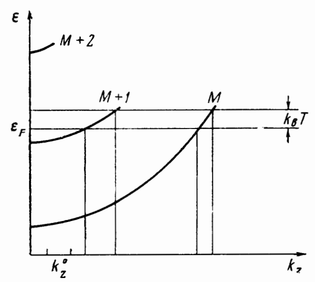
or

$$\Delta\mathscr{H} = \frac{e\hbar}{mc}\frac{\mathscr{H}^2}{\varepsilon_F - \hbar\omega_c}. \tag{14}$$

In the last expression we discarded the term with k_z^0 because of its smallness.

In the case in which the magnetic quantum number is comparatively large (metal), Eq. (14) is simplified:

$$\Delta\left(\frac{1}{\mathscr{H}}\right) = \frac{e\hbar}{mc\varepsilon_F}, \quad \Delta\mathscr{H} = \frac{\hbar\omega_c}{\varepsilon_F}\mathscr{H}. \tag{15}$$



The width of the absorption resonance peak is determined by two factors—the value of the interval of k_z associated with thermal diffuseness, $mk_BT/\hbar^2k_z^0$, and the indeterminacy of k_z^0 itself (12).

Inasmuch as α_0 is of the order of $n\pi/L$[12] (n is the number of the acoustic branch), the ratio of the noted diffusenesses is $n^3kT/mL^2\omega^2$. It follows from the graphs of[12] that $L\omega/n \sim 10c_T$. For the electron mass and the temperatures at which the magnetic field can be quantizing, the noted diffusenesses will be of the same order. For smaller effective masses, the interval of k_z will be more significant, due to thermal scatter. Therefore, it can be assumed that this determines the width of the peak of resonance absorption. Simple calculations lead to the expression

$$\delta\mathscr{H} = \mathscr{H}k_BT / \varepsilon_F. \tag{16}$$

It is seen from a comparison with (15) that for a quantized field, the peak width is much less than the distance between neighboring peaks.

For an estimate of the relative value of the maxima of the absorption, we write down the absorption coefficient[14]

$$\Gamma = \frac{\pi A}{V_0\rho c_l^2}\sum\frac{\partial f}{\partial\varepsilon_F}|\langle M, x_0, k_z|W|M'x_0'k_z'\rangle|^2 \times\delta\left[\frac{\hbar^2}{2m}(k_z^2 - k_z'^2) + \hbar\omega\right], \tag{17}$$

where f is the Fermi function and ρ is the density. Summation in (17) is carried out over the initial and final electron states.

Simple calculations show that $\Gamma \sim H$ at the maxima. Finding the absolute value of the maxima of Γ/Γ_0 is more difficult since a complicated expression is obtained for Γ_0 (the absorption coefficient of ultrasound in the film in the absence of a field). However, this is not necessary in principle, since in the case of a bulky sample, when Γ/Γ_0 is easily calculated, there is no good quantitative agreement of theory with experiment.[4]

To find the shape of the resonance peaks, one must take into account the electron scattering processes in the film. Here, for estimating purposes, we must replace the δ function in (17) by

$$\frac{1}{\pi}\frac{1/\tau}{\hbar/\tau^2 + (\hbar^2qk_z/m + \hbar^2q^2/2m - \hbar\omega)^2}$$

both in the case of a bulky sample and for longitudinal waves.[1] Unfortunately, this procedure is not reliable in our case, since there is no satisfactory expression for the relaxation time in the film. Therefore we shall not discuss the question of the shape of the resonance curves, and limit ourselves to the experimentally observed parameters obtained above: the period of oscillation (14), (15), the width of the resonance peaks (16), and the dependence of the height of the peaks on the applied magnetic field.

As in the case of a massive sample for longitudinal direction of the magnetic field, the observation of the dependence of the absorption coefficient on the magnetic field in the case considered gives information concerning the Fermi surface. As follows from the well-known expression[15],

$$\Delta\left(\frac{1}{\mathscr{H}}\right) = 2\pi e\hbar/cS(\varepsilon, k_z). \tag{18}$$

Here $S(\epsilon, k_z)$ is the cross section area of the Fermi surface parallel to the plane of the film.

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