SOVIET PHYSICS JETP VOLUME 31, NUMBER 1 JULY, 1970

THE PHONON GREEN FUNCTION FOR SMALL ENERGIES AND MOMENTA

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Submitted July 3, 1969

Zh. Eksp. Teor. Fiz. 58, 257–263 (February, 1970)

A sequence of perturbation theory diagrams, giving the main contribution to the asymptotic form of the phonon Green function for small energies and momenta, is summed. The summation reduces to the solution of a system of kinetic equations by the Chapman-Enskog method. As a result the Green function acquires new poles when compared with the unperturbed function; a pole appears which corresponds to thermal conduction and, in the model in which no account is taken of Coulomb interaction and Umklapp processes, further poles appear corresponding to second sound.

1. The model of an electron-phonon system in which no account is taken of Coulomb interaction or Umklapp processes is often used and has been studied in detail. The Green functions of this model at zero temperature have been investigated by Migdal.[1]

In the present paper, the phonon Green function of this model is considered for T = 0 in the hydrodynamic regime ($E\tau$, $kl \ll 1$, where τ is the relaxation time and l is the mean free path). In this regime a sequence of the diagrams giving the main contribution to the Green function is distinguished. It is shown that summation of this sequence reduces to the solution of a system of equations of the kinetic type. The solution of this system is found by the Chapman-Enskog method. It is shown that, in the second approximation of this method, the Green function $G_{\text{ph}}(E, \mathbf{k})$ has poles corresponding to first and second sound and to thermal conduction. The thermal conduction pole remains even when Coulomb interaction and Umklapp processes are taken into account. The poles corresponding to second sound are characteristic only for a model without the latter interactions and disappear even if only one of them is included.

In Sec. 2 the main sequence of diagrams is distinguished and the system of kinetic equations is derived. The derivation follows from the fundamental work of Éliashberg,[2] in which kinetic equations are derived for a Fermi system. In Sec. 3 we consider the solution of the resulting system by the Chapman-Enskog method and find an expression for the function $G_{\text{ph}}(E, \mathbf{k})$ in the hydrodynamic regime. A limit on the coupling constant, which follows from the stability condition for the energy spectrum, is also discussed in Sec. 3. In Sec. 4 we consider the effect of Coulomb interaction and Umklapp processes on the hydrodynamic asymptotic form of the Green function $G_{\text{ph}}(E, \mathbf{k})$.

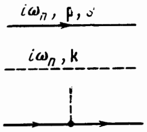
2. We shall consider the diagram summation using the example of a model of an electron-phonon system with the Hamiltonian[1)]

$$H' = H - \lambda N = \sum_{\mathbf{p},s}\left(\frac{p^2}{2m} - \lambda\right) a^+_{\mathbf{p},s} a_{\mathbf{p},s} + \sum_{\mathbf{k}<k_0} \omega_0(\mathbf{k}) b_{\mathbf{k}}^+ b_{\mathbf{k}} \qquad (2.1)$$
$$+ \frac{g}{\sqrt{2V}} \sum_{s,\,\mathbf{p},\,\mathbf{k}<k_0} \omega_0^{1/2}(\mathbf{k}) a_{\mathbf{p}+\mathbf{k},s} a_{\mathbf{p},s}(b_{\mathbf{k}} + b_{-\mathbf{k}}^+).$$

Here λ is the chemical potential, $s = \pm$ is the spin index, and $\omega_0(\mathbf{k}) = c_0 k$. The sums over the phonon momenta are truncated at the upper limit k_0.

The Hamiltonian (2.1) does not take account of the Coulomb interaction between the electrons or of the Umklapp processes. The results which are obtained on inclusion of these are discussed in Sec. 4.

The temperature diagram technique is constructed according to the known rules.[3] The expressions for the lines and vertices have the form:

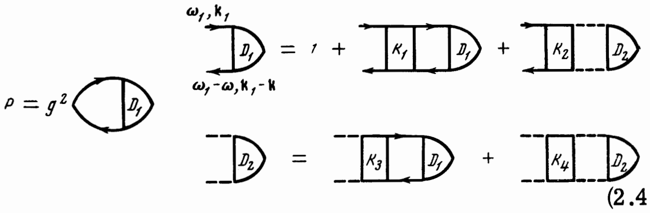


$$G_e^{(0)}(i\omega_n, \mathbf{p}, s) = [i\omega_n - p^2/2m + \lambda]^{-1} \qquad (2.2)$$
$$G_\varphi^{(0)}(i\omega_n, \mathbf{k}) = \omega_0^2(\mathbf{k})[(i\omega_n)^2 - \omega_0^2(\mathbf{k})]^{-1}$$
$$V = g$$

The exact electron and phonon Green functions G_e and G_{ph} are depicted by heavy continuous lines and heavy dotted lines respectively. They are expressed in terms of the self-energy parts:

$$G_e = \left(\frac{1}{G_e^{(0)}} - \Sigma\right)^{-1}, \quad G_{\text{ph}} = \left(\frac{1}{G_{\text{ph}}^{(0)}} - P\right)^{-1}. \qquad (2.3)$$

The following exact diagrammatic equalities



(2.4)

show that the function P (the polarization operator) is expressed in terms of the vertex part D_1 and the exact electron function, while the vertex parts D_1 and D_2 satisfy a system of linear equations.

The electron and phonon lines in the diagrams (2.4) are heavy (i.e., they correspond to exact Green functions). Irreducible diagrams, which cannot be cut vertically by breaking two lines, contribute to the blocks K_1 to K_4.

We are interested in the analytic continuation $i\omega_n \to E$ of the temperature phonon function in the hydrodynamic regime ($E\tau \ll 1$). In this case the equations (2.4) for the vertex parts reduce to kinetic equations. Following Éliashberg,[2] we obtain their explicit form.

First we select approximations for the self-energy parts Σ and P and the diagrams K_i in (2.4). We con-

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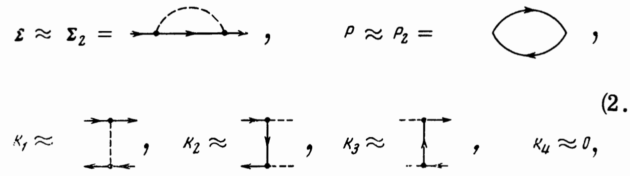
[1)] We use a system of units with ℏ = k = 1 (ℏ and k are Planck's and Boltzmann's constants).

fine ourselves to the simplest perturbation theory diagrams of the form



(2.5)

where the lines and vertices correspond to the unperturbed functions (2.2) (in contrast to the diagrams (2.4)).

The approximation $P \approx P_2$ ceases to be true for $E\tau \lesssim 1$. However, this approximation is valid in the equations (2.5) for the vertex parts D_1 and D_2, since the phase volume of the regime in which it is not valid is small. In the hydrodynamic regime, as we shall see below, P differs essentially from P_2.

The vertex parts D_1 and D_2 as functions of $i\omega_1$ are defined, on analytic continuation $i\omega_1 \to z$, in a plane with two branch cuts, Im z = 0, ω. The system of equations for D_1 and D_2 reduces to equations for the boundary values of these functions at the edges of the branch cuts and takes the following form

$$\begin{aligned} D_1^{(i)} &= 1 + L_{11}^{(ie)}D_1^{(e)} + L_{11}^{(ii)}D_1^{(i)} + L_{12}^{(ie)}D_2^{(e)} + L_{12}^{(ii)}D_2^{(i)},\\ D_1^{(e)} &= 1 + L_{11}^{(ee)}D_1^{(e)} + L_{11}^{(ei)}D_1^{(i)} + L_{12}^{(ee)}D_2^{(e)} + L_{12}^{(ei)}D_2^{(i)}\\ D_2^{(i)} &= L_{21}^{(ie)}D_1^{(e)} + L_{21}^{(ii)}D_1^{(i)} + L_{22}^{(ie)}D_2^{(e)} + L_{22}^{(ii)}D_2^{(i)},\\ D_2^{(e)} &= L_{21}^{(ee)}D_1^{(e)} + L_{21}^{(ei)}D_1^{(i)} + L_{22}^{(ee)}D_2^{(e)} + L_{22}^{(ei)}D_2^{(i)}. \end{aligned} \tag{2.6}$$

Here $D_{1,2}^{(i)}$ are the boundary values of $D_{1,2}$ at the internal edges of the branch cuts (Im z = +0, ω − 0) and $D_{1,2}^{(e)}$ are those at the external edges (Im z = −0, ω + 0). L denotes integral operators acting on the functions $D_{1,2}$; the expressions for L depend on the choice of approximations for Σ, P and K_i (see (2.5)).

On analytic continuation $i\omega \to E \lesssim \tau^{-1}$, the products of Green functions contained in the expressions for the operators L have closely-spaced singularities. These singularities lie on the same side of the contour of integration over the energy variable in operators acting on $D_{1,2}^{(e)}$ and on different sides in operators acting on $D_{1,2}^{(i)}$. The contribution of expressions containing $D_{1,2}^{(e)}$ to the system (2.6) turns out to be small, since the contour can be deformed away from singularities positioned on the same side of the contour. One can show that the contribution of expressions containing the operators $L^{(ei)}$ is also small. Therefore, in the first approximation

$$D_1^{(e)} = 1, \quad D_2^{(e)} = 0. \tag{2.7}$$

For the functions $D_{1,2}^{(i)}$, the system turns out to be nontrivial. The small region in the neighborhood of the Green function singularities makes the main contribution to the expressions $L_{jk}^{(ii)}D_k^{(i)}$. On integration over the energy variable, all functions apart from the product of Green functions can be taken outside the integral. The remaining integrals, each of a product of electron or phonon Green functions, are given by

$$\int G_e(x+E, \mathbf{k}_1+\mathbf{k})G_e(x,\mathbf{k}_1)\,dx = \frac{2\pi i}{E-\varepsilon_1(\mathbf{k}_1+\mathbf{k})+\varepsilon_1(\mathbf{k}_1)+2i\Delta_1},$$

$$\int G_{\mathrm{ph}}(x+E, \mathbf{k}_1+\mathbf{k})G_{\mathrm{ph}}(x,\mathbf{k}_1)\,dx = \left(\frac{\varepsilon_2(\mathbf{k}_1)}{2}\right)^2 \times \frac{2\pi i}{E-\varepsilon_2(\mathbf{k}_1+\mathbf{k})+\varepsilon_2(\mathbf{k}_1)+2i\Delta_2} \tag{2.8}$$

In these formulas $\epsilon_1(\mathbf{k}_1) = (k_1^2/2m) - \lambda$, and $\epsilon_2(\mathbf{k}_1) = c_0k_1$ are the energy spectra of the electrons and phonons respectively; Δ_1 and Δ_2 are their imaginary parts (defined by the diagrams (2.5)).

As the unknown quantities it is natural to introduce the functions

$$\begin{aligned} h(\mathbf{k}_1;E,\mathbf{k}) &= \frac{D_1^{(i)}(i\omega_1\to\varepsilon_1(\mathbf{k}_1), i\omega\to E, \mathbf{k}_1, \mathbf{k})}{E-\varepsilon_1(\mathbf{k}_1+\mathbf{k})+\varepsilon_1(\mathbf{k}_1)+2i\Delta_1},\\ g(\mathbf{k}_1;E,\mathbf{k}) &= \frac{\varepsilon_2(\mathbf{k}_1)}{2}\,\frac{D_2^{(i)}(i\omega_1\to\varepsilon_2(\mathbf{k}_1), i\omega\to E, \mathbf{k}_1, \mathbf{k})}{E-\varepsilon_2(\mathbf{k}_1+\mathbf{k})+\varepsilon_2(\mathbf{k}_1)+2i\Delta_2} \end{aligned} \tag{2.9}$$

for which the system (2.6) appears in the following form

$$\begin{aligned} (E-2\varepsilon_1'(\mathbf{k}_1\mathbf{k}))h + iI_1(h,g) &= 1,\\ (E-2\varepsilon_2'(\mathbf{k}_1\mathbf{k}))g + iI_2(h,g) &= 0. \end{aligned} \tag{2.10}$$

In (2.10) we have made the replacement

$$\varepsilon_i(\mathbf{k}_1+\mathbf{k}) - \varepsilon_i(\mathbf{k}_1) \to (\partial\varepsilon_i/\partial k_1^2)\cdot 2(\mathbf{k}_1\mathbf{k}) \equiv 2\varepsilon_i'(\mathbf{k}_1\mathbf{k})$$

for $k_1 \gg k$. I_1 and I_2 denote the expressions

$$\begin{aligned} I_1(h,g) = {}&(1+e^{-\beta\varepsilon_1(\mathbf{k}_1)})\frac{\pi g^2}{(2\pi)^3}\int d^3\mathbf{k}_2\,d^3\mathbf{k}_2\,\varepsilon_2(\mathbf{k}_3)n_1(\mathbf{k}_2)n_2(\mathbf{k}_3)\\ &\times\{e^{\beta\varepsilon_1(\mathbf{k}_2)}\delta(\varepsilon_1(\mathbf{k}_1)-\varepsilon_1(\mathbf{k}_2)+\varepsilon_2(\mathbf{k}_3))\delta(\mathbf{k}_1-\mathbf{k}_2+\mathbf{k}_3)[h(\mathbf{k}_1)-h(\mathbf{k}_2)+g(\mathbf{k}_3)]\\ &+2e^{\beta\varepsilon_1(\mathbf{k}_1)}\delta(\varepsilon_1(\mathbf{k}_1)-\varepsilon_1(\mathbf{k}_2)-\varepsilon_2(\mathbf{k}_3))\delta(\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)[h(\mathbf{k}_1)-h(\mathbf{k}_2)-g(\mathbf{k}_3)]\},\\ I_2(h,g) = {}&(1-e^{-\beta\varepsilon_2(\mathbf{k}_1)})\frac{2\pi g^2}{(2\pi)^3}\int d^3\mathbf{k}_2\,d^3\mathbf{k}_3\,\varepsilon_2(\mathbf{k}_1)n_1(\mathbf{k}_2)n_1(\mathbf{k}_3)\\ &\times e^{\beta\varepsilon_1(\mathbf{k}_2)}\delta(\varepsilon_2(\mathbf{k}_1)-\varepsilon_1(\mathbf{k}_2)+\varepsilon_1(\mathbf{k}_3))\delta(\mathbf{k}_1-\mathbf{k}_2+\mathbf{k}_3)[g(\mathbf{k}_1)-h(\mathbf{k}_2)+h(\mathbf{k}_3)], \end{aligned} \tag{2.11}$$

where

$$n_1(\mathbf{k}) = (e^{\beta\varepsilon_1(\mathbf{k})}+1)^{-1}, \quad n_2(\mathbf{k}) = (e^{\beta\varepsilon_2(\mathbf{k})}-1)^{-1}.$$

The equations (2.8) strongly resemble the system of linearized kinetic equations for the electron and phonon distribution functions. The functions I_1 and I_2 play the role of collision integrals. Their form is determined by the imaginary parts Δ_1 and Δ_2 of the energy spectrum by the following rule. We write out $2\Delta_1$ and $2\Delta_2$ in the form of integrals with δ-functions expressing the law of conservation of energy-momentum. We multiply $2\Delta_1$ by $h(\mathbf{k}_1)$, $2\Delta_2$ by $g(\mathbf{k}_1)$, bring h and g under the integral signs and then make the replacement

$$h(\mathbf{k}_1) \to h(\mathbf{k}_1) - h(\mathbf{k}_2) \pm g(\mathbf{k}_3), \quad g(\mathbf{k}_1) \to g(\mathbf{k}_1) - h(\mathbf{k}_2) + h(\mathbf{k}_3).$$

The expressions (2.11) are then obtained for I_1 and I_2.

The polarization operator P for $E\tau \lesssim 1$ is expressed in terms of the function L, introduced above, by the formula

$$P = \frac{2g^2\beta}{(2\pi)^3}\int d^3\mathbf{k}_1\frac{e^{\beta\varepsilon_1(\mathbf{k}_1)}}{(e^{\beta\varepsilon_1(\mathbf{k}_1)}+1)^2}(Eh(\mathbf{k}_1)-1). \tag{2.12}$$

To derive (2.12) one must write out the diagrammatic equality for P in (2.4), reduce the sums over the frequencies to an integral of the boundary values of the function D_1, and then take into account that, at the external edges of the branch cuts, according to (2.7), $D_1^{(e)} = 1$, whilst at the internal edges, the main contribution is given by the neighborhood of the closely-spaced singularities of the Green functions.

3. To solve the system (2.10) in the hydrodynamic

regime ($E\tau \ll 1$, $kl \ll 1$) one can use the Chapman-Enskog method.[4] We seek the first approximation for the functions h and g in the form of linear combinations of functions which cause the collision integrals to vanish:

$$h = a + (\mathbf{bk}_1) + c\varepsilon_1(\mathbf{k}_1), \quad g = (\mathbf{bk}_1) + c\varepsilon_2(\mathbf{k}_1). \tag{3.1}$$

The orthogonality conditions

$$(I_1)_1 = 0, \quad (I_1\mathbf{k}_1)_1 + (I_2\mathbf{k}_1)_2 = 0, \quad (I_1\varepsilon_1(\mathbf{k}_1))_1 + (I_2\varepsilon_2(\mathbf{k}_1))_2 = 0. \tag{3.2}$$

hold for I_1 and I_2. Here we have used the notations

$$\begin{gathered}(\varphi(\mathbf{k}_1))_1 = \frac{2}{(2\pi)^3}\int d^3\mathbf{k}_1 \frac{e^{\beta\varepsilon_1(\mathbf{k}_1)}}{(e^{\beta\varepsilon_1(\mathbf{k}_1)}+1)^2}\varphi(\mathbf{k}_1),\\ (\psi(\mathbf{k}_1))_2 = \frac{1}{(2\pi)^3}\int d^3\mathbf{k}_1 \frac{e^{\beta\varepsilon_2(\mathbf{k}_1)}}{(e^{\beta\varepsilon_2(\mathbf{k}_1)}-1)^2}\psi(\mathbf{k}_1)\end{gathered} \tag{3.3}$$

(the factor 2 in the definition of $(\varphi(\mathbf{k}_1))_1$ takes account of summation over the spin index).

The system of linear equations following from the conditions (3.2) gives for a, b and c the expressions

$$a = \Delta_1/\Delta, \mathbf{b} = \mathbf{k}\Delta_2/\Delta, c = \Delta_3/\Delta. \tag{3.4}$$

For the determinants Δ, Δ_1, Δ_2 and Δ_3 we obtain the formulae

$$\begin{gathered}\Delta = E[E^2(k^2)_{12}((1)_1(\varepsilon^2)_{12} - (\varepsilon)_1{}^2) - \\ - {}^4/_3k^2((\varepsilon^2)_{12}(\varepsilon'k^2)_1{}^2 + (1)_1(\varepsilon\varepsilon'k^2)_{12}{}^2 - 2(\varepsilon)_1(\varepsilon'k^2)_1(\varepsilon\varepsilon'k^2)_{12})] \\ \equiv (k^2)_{12}((1)_1(\varepsilon^2)_{12} - (\varepsilon)_1{}^2)E(E^2 - u^2k^2), \\ \Delta_1 = E^2(k^2)_{12}((1)_1(\varepsilon^2)_{12} - (\varepsilon_1{}^2)) - \\ -{}^4/_3k^2[(1)_1(\varepsilon\varepsilon'k^2)_{12}{}^2 - (\varepsilon)_1(\varepsilon'k^2)_1(\varepsilon\varepsilon'k^2)_{12}], \\ \Delta_2 = 2E(\varepsilon'k^2)_1((1)_1(\varepsilon^2)_{12} - (\varepsilon)_1{}^2), \\ \Delta_3 = {}^4/_3k^2(\varepsilon'k^2)_1[(1)_1(\varepsilon\varepsilon'k^2)_{12} - (\varepsilon)_1(\varepsilon'k^2)_1],\end{gathered} \tag{3.5}$$

where, e.g., $(\epsilon)_1 = (\epsilon_1(\mathbf{k}_1))_1$ and $(\epsilon^2)_{12} = (\epsilon_1^2(\mathbf{k}_1))_1 + (\epsilon_2^2(\mathbf{k}_1))_2$.

Knowing h, we find from formula (2.12) the first approximation for P in the Chapman-Enskog method:

$$P = \frac{4g^2\beta(\varepsilon'k^2)_1{}^2}{3(k^2)_{12}}k^2\frac{1}{E^2-u^2k^2} \equiv \frac{\delta^2k^2}{E^2-u^2k^2}. \tag{3.6}$$

Equating the denominator of the Green function G_{ph} to zero, we obtain the equation

$$(E^2 - c_0{}^2k^2)(E^2 - u^2k^2) - c_0{}^2\delta^2k^4 = 0, \tag{3.7}$$

giving two branches of the energy spectrum.

The expression $c_0^2(u^2 - \delta^2)$ is a product of the squares of the first and second sound velocities and so must be positive (otherwise the system will be unstable). Hence follows an inequality which at low temperatures takes the form

$$g^2 < (\beta(1)_1)^{-1} = \pi^2/m^2v_F, \tag{3.8}$$

where v_F is the velocity at the Fermi surface. The inequality (3.8) coincides with the limit on the coupling constant obtained in Migdal's paper.[1] We see that it guarantees the existence of the second sound branch. The derivation given here for this inequality formally uses approximations for Σ, P and K_i (see (2.5)) which are valid only for $g^2 \ll \pi^2/m^2v_F$. However, it can be justified by noting that the kinetic equations (2.10) can also be obtained for $g^2 \sim \pi^2/m^2v_F$ if we replace the electron and phonon spectra ϵ_1 and ϵ_2 by renormalized quantities.

In the second approximation of the Chapman-Enskog method it is necessary to find corrections δh and δg to the functions (3.1) of the first approximation and to calculate the coefficients a, b and c with greater precision. The functions δh and δg, though formally obtainable by transformation of the collision integral operator, in practice must be calculated by approximate methods. The coefficients a, b and c in the second approximation are obtained from the orthogonality conditions (3.2). The second approximation for the polarization operator P is given by the formula (2.12) in terms of the second approximation for h and is found to be

$$P = \frac{\delta^2k^2(E - i\alpha E^2 + i\varkappa_2k^2)}{(E + i\varkappa_1k^2)[(E + i\eta k^2)^2 - u^2k^2]}. \tag{3.9}$$

The constants α, η, κ_1 and κ_2 are expressed in terms of kinetic coefficients and must be calculated by approximate methods. As a result the Green function acquires a further (thermal) pole.

$$E = -i\varkappa k^2, \quad \varkappa = (u^2\varkappa_1 - \delta^2\varkappa_2)/(u^2 - \delta^2) \tag{3.10}$$

(for $\delta^2 \ll u^2$, we shall have $\kappa = \kappa_1$).

In the sound branches imaginary contributions $\sim k^2$ appear which determine the attenuation.

A superfluid Fermi-Bose system, in the description of which the Bose-particles are replaced by phonons (a valid replacement at sufficiently low temperatures), is a real system for which Coulomb interaction and Umklapp processes are excluded.[5] For such a system the boson Green function, in addition to the poles corresponding to the two sound branches, must also have a pole corresponding to thermal conduction. We recall that, as was shown by Bogolyubov,[6] the Green function of a superfluid Bose system does not have a thermal pole. The inequality (3.8) may be considered in this case as an upper limit for $k_F \sim \rho_F^{1/3}$, i.e., as a limit on the concentration of Fermi-particles in the Fermi-Bose system.

4. We shall discuss now how the results are changed when Coulomb interaction and Umklapp processes are taken into account.

For a system with Coulomb interaction it is advisable to separate out the "irreducible Coulomb" diagrams contributing to P; these cannot be cut at the Coulomb vertex $4\pi e^2k^{-2}$, which is singular for small k. We denote the sum of these diagrams by $\tilde{P}$. Then P is expressed in terms of $\tilde{P}$ by the formula

$$P = \tilde{P}[1 - (4\pi e^2k^{-2})g^{-2}\tilde{P}]^{-1}. \tag{4.1}$$

A formula of the type (3.9) for P is now true for $\tilde{P}$ as well. As a result we obtain for the function P the expression

$$P = \frac{\delta^2k^2(E - i\alpha E^2 + i\varkappa_2k^2)}{(E+i\varkappa_1k^2)((E+i\eta k^2)^2-u^2k^2) - 4\pi e^2\delta^2g^{-2}(E - i\alpha E^2 + i\varkappa_2k^2)} \tag{4.2}$$

The second acoustic branch is then converted into a plasma branch with $\omega = (4\pi e^2\delta^2g^{-2})^{1/2}$.

In the presence of Umklapp processes, momentum is not conserved (it is conserved to within a reciprocal lattice vector). We therefore seek the first approximation of the Chapman-Enskog method in the form

$$h = a + c\varepsilon_1(\mathbf{k}_1), \quad g = c\varepsilon_2(\mathbf{k}_1). \tag{4.3}$$

The further calculations are performed according to the usual scheme of the Chapman-Enskog method. The first approximation for P is found to be zero and the second approximation has the following form:

$$P = -i\varkappa_2 k^2 (E + i\varkappa_1 k^2). \tag{4.4}$$

As a result the phonon Green function acquires a thermal pole.

$$E = -i\varkappa k^2, \quad \varkappa = \varkappa_1 - \varkappa_2.$$

We see that the thermal pole in the energy spectrum is present in each case, whereas the second sound branch disappears when Coulomb interaction or Umklapp processes are included. If the latter interactions are for some reason small compared to the main one, so that the corresponding relaxation time τ_1 is significantly greater than the time τ_0 of the main interaction ($\tau_1 \gg \tau_0$), second sound may exist in the interval $\tau_1^{-1} \ll E \ll \tau_0^{-1}$.

The author is grateful to L. D. Faddeev for discussions.

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Translated by P. J. Shepherd
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